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Note: Signal amplification and filtering with a tristable stochastic resonance cantilever

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This Note reports a tristable cantilever that exploits stochastic resonance (SR) phenomenon for a study of signal amplification and filtering. The tristable device system combines the benefits of bistable system (wide interwell spacing) and monostable system (smooth motion in potential). The prototype tristable cantilever exhibits 42 times root-mean-square amplitude, 35.86 dB power gain, advance of 15 dB signal-to-noise ratio, and twice fidelity at around 7.6 Hz as compared to the input signal. In a wide operating bandwidth [5.5 Hz, 8.2 Hz], the tristable SR cantilever outperforms the traditional monostable cantilever and bistable SR cantilever in these characteristics. © 2013 American Institute of Physics. [http://dx.doi.org/10.1063/1.4792603]

During the past three decades, stochastic resonance (SR) being one of the most exciting nonlinear phenomena has attracted considerable attentions in a wide range of research. Via the SR, the output signal of a nonlinear dynamic system can be enhanced by means of noise addition to the system.1,2 The phenomenon of SR benefits the nonlinear energy harvesting.3–5 Moreover, SR could be used for signal amplification,6 detecting signals buried in noise,7 and target detection.8 Previous works show that the SR could be affected by the nonlinear models. The most typical model is the bistable SR system. Different structures and different scale of mechanisms have been proposed to realize the bistable oscillators.3–5 However, the bistable system has only two potential wells, and the range of particle motion will be limited to the interwell spacing. In addition, the zero position of bistable potential is a barrier, which implies that the velocity of the oscillator will be decreased when it crosses the zero position,9,10 so the system response amplitude along with the efficiency of the SR will be further affected.

In this Note we demonstrate a tristable cantilever to further broaden the motion range and smoothen the motion curve of the particle, and to enhance the performance of the SR system in signal amplification and filtering. The structure of the device is shown in Fig. 1. The device has a permanent magnet (denoted by \(m_0\)) being mounted at the tip of the elastic cantilever as an oscillator, and the other two fixed magnets (denoted by \(m_1\) and \(m_2\)) with the same scale as \(m_0\). At y-axis, magnets \(m_1\) and \(m_2\) have the same polarity, which is opposed to magnet \(m_0\). A similar mechanism that a ferromagnetic cantilever attracts two magnets is proposed by Moon and Holmes in 1979,11 and Erturk et al. fabricated a patch of piezoelectric on the attractor system as a bistable energy harvester in 2009.5 In our model, the repulsive force replaces the attractive force between the oscillator magnet and fixed magnets and thus a potential with three wells and two barriers is firmly formed.

Assume that the motion direction of the oscillator is x-axis, and \(U(x)\) is the potential of the system that consists of three parts: the elastic potential energy of the cantilever, the magnetic field potential energy for magnet pair \(m_0\) and \(m_1\), and magnet pair \(m_0\) and \(m_2\), respectively. On the basis of SR theory, the oscillator motion in a potential is considered in the presence of noise and periodic force and can be depicted as follows:

\[
\frac{dx}{dt} = -U'(x) + A_0 \sin(2\pi f_0 t + \phi) + D\xi(t),
\]

where \(A_0\) is the periodic signal amplitude, \(f_0\) is the driving frequency, \(D\) is the noise intensity, and \(\xi(t)\) represents a Gaussian white noise with zero mean and unit variance. Assume that \(U(x) = 0\) when the cantilever does not have deformation (\(x = 0\)), then the potential function can be written as below:

\[
U(x) = \frac{EwR^3}{8l^3}x^2 - \mu_0 ab \int_0^x (H_{x1}|H_{x1}| + H_{x2}|H_{x2}|)dx,
\]

where \(E = 113\) GPa is Young’s Modulus of the cantilever, \(\mu_0 = 4\pi \times 10^{-7}\) N/A² is the permeability, \(H_{x1}\) and \(H_{x2}\) are the magnetic field of \(m_1\) and \(m_2\) at x-axis, respectively. In our coordinate system, \(H_{x1} = H_x(x - pl/2, d - h/2, b/2)\) and \(H_{x2} = H_x(x + pl/2, d - h/2, b/2)\), where \(H_x(x,y,z)\) is

![FIG. 1. Schematic structure of tristable stochastic resonance cantilever.](image)

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given as
\[ H(x, y, z) = \frac{B_0}{\mu_0} \sum_{k,l,m=1}^2 (-1)^{k+l+m} \ln \left( z + (-1)^m \frac{b}{2} \right) \]
\[ + \sqrt{\left( x + (-1)^k \frac{a}{2} \right)^2 + \left( y + (-1)^l \frac{h}{2} \right)^2 + \left( z + (-1)^m \frac{b}{2} \right)^2} \]
\[ \text{(3)} \]

where \( B_0 = 0.5 \) T is the residual flux density on the magnet polarity surface. Thus, the system structure can be adjusted by the parameters \( d \) and \( p \). When \( d \) is set to a small value in a specific range, the system is tristable; while the system becomes monostable when \( d \) is set to an exceedingly large value. When \( p \) is set to zero, which means that \( m_1 \) and \( m_2 \) are merged into one magnet, the system turns into the bistable situation.

Fig. 2(a) demonstrates the shape of \( U(x) \) of the system with tuning of \( d \) and \( p \). It can be seen that the number of wells and barriers, the depth of well and the height of barrier, and the distance between wells or barriers depend on the value of \( d \) and \( p \). Moreover, the resultant force \( F_x \) of \( m_0 \) at \( x \)-axis is plotted in Fig. 2(b). In the typical tristable system as shown in Fig. 2(b3), the varying trend of \( F_x \) is smoother than the bistable system as shown in Fig. 2(b2), and the tendency of the curve is similar to the monostable system as shown in Fig. 2(b1), which implies that the acceleration of the tristable oscillator is more stable in comparison with the bistable oscillator. This will affect the shape of system response and the efficiency of oscillation as discussed in Fig. 5.

A prototype tristable cantilever for experiment and the signal flow in system are shown in Fig. 3 (connecting wires between the instruments are not shown in the figure). The cantilever is made of phosphor copper with the size \( l = 145 \) mm, \( w = 10 \) mm, and \( t = 0.4 \) mm. The magnets used here are the \( NdFeB \) permanent magnets with the size \( h = 3 \) mm, \( a = 3 \) mm, and \( b = 3 \) mm. \( m_0 \) is glued to the cantilever tip, and \( m_1 \), \( m_2 \) are glued to an adjustable platform. A plat loudspeaker is used as an electric shaker that is driven by the audio signal. A 50 mm \( \times \) 50 mm \( \times \) 10 mm carbon fiber hollow block being glued between the loudspeaker and the cantilever guarantees the free deflection of the cantilever. The total mass of the cantilever, magnet \( m_0 \), and hollow block is 7.8 g, implying that they can be easily driven by the loudspeaker. The parameter \( d \) can be adjusted via a screw in the adjustable platform. In the experiments, the vibration at the loudspeaker surface (which represents the base vibration) with an equivalent load (7.8 g) is measured as the system input signal. The displacements of \( m_0 \) at \( x \)-axis are recorded as the monostable, bistable, and tristable system output signals at \( (d = 25 \) mm, \( p = 10 \) mm), \( (d = 6 \) mm, \( p = 0) \), and \( (d = 6 \) mm, \( p = 10 \) mm), respectively. Those signals are recorded by a laser sensor. The audio signal is provided by a signal generator with the parameters \( A_0 = 1 \), \( D = 8 \), and the noise bandwidth being [0 Hz, 35 Hz] as presented in Eq. (1). The driving frequency \( f_0 \) is taken from the range of [2 Hz, 12 Hz].

Typical signals of the device system input and outputs at the driving frequency 7 Hz are shown in Fig. 4, where the function \( y = a \sin(2\pi ft + b) \) is used to fit the raw signals via
the principle of nonlinear least squares. As indicated in Fig. 4(a), the driving sinusoid is screened by the noise in the input signal from the surface of the loudspeaker. As seen from the monostable output waveform in Fig. 4(b), the signal has been amplified and the noise effect is lower, but the output frequency is not consistent with the driving frequency 7 Hz. In Fig. 4(c), the signal is further amplified by the bistable system, and the shape of waveform is similar to the fitted curve, but the noise effect can still be noticed from the result. Finally as found in Fig. 4(d), the output waveform from tristable system is smooth and nearly coincides with the fitted curve, and the signal amplitude is the highest in comparison with the previous systems. This shows that the periodic signal is successfully enhanced by the appropriate noise\(^2\) in the tristable cantilever, which is due to that the smooth tristable potential leads to stable motion of the oscillator as compared to the bistable potential.

To further evaluate the performance of the system, four characteristics based on the frequency response are analyzed as shown in Fig. 5. Fig. 5(a) shows the root-mean-square (RMS) amplitude of different systems. The monostable output indicates that the best performance of system response is at its fundamental resonance frequency (8.8 Hz). If the driving frequency deviates slightly from the resonance frequency then the output amplitude decreases rapidly.\(^5\) For the bistable system, in the range from 2 Hz to 7.5 Hz below the resonance frequency, the cantilever gives a better result in comparison with the monostable system. However, once the driving frequency approaches the resonance frequency, the motion of oscillator is limited to a range around the equilibrium position following a locally linear behavior. When the oscillator toggles into the tristable system, in the range [5.5 Hz, 8.2 Hz], the RMS value of the system output is larger than that of both monostable and bistable systems, which is benefited from a wider interwell spacing (between state 1 and state 2) and a higher average oscillation velocity as demonstrated in Fig. 2. Furthermore, the power gain \( G = 10 \log_{10}(P_{\text{output}}/P_{\text{input}}) \) is used to evaluate the amplifying capability at each frequency. As shown in Fig. 5(b), the output from the tristable cantilever gives 35.86 dB (at 7.6 Hz) amplification factor. The synthetic action of the cantilever structure and the nonlinear force leads to that the tristable system acts like a band-pass amplifier.

To evaluate the filtering performance of the cantilever system, the signal-to-noise ratio (SNR) of each system is first calculated by SNR = \(10 \log_{10}(P_{\text{signal}}/P_{\text{noise}})\). As shown in Fig. 5(c), it can be found that in the bandwidth [5.5 Hz, 8.2 Hz], the SNR of tristable system output is beyond that of the system input, which indicates that the tristable system amplifies the amplitude and improves the SNR synchronously. In this case, the oscillator has been well forced to track the potential curve and bounce among the wells and the barriers continuously under the periodic force by the aid of noise. Moreover, the fidelity is also introduced to evaluate the filtering capability of the device system. A function group \( y_i = a \sin(2\pi f_i t + b) \) (\(f_i\) equals the driving frequency \(f_0\) in the range of [2 Hz, 12 Hz]) is used to calculate the adjusted R-square coefficients. As seen from the results in Fig. 5(d), different output signal has a specific bandwidth beyond the input signal on the fidelity aspect. Due to the distinguished smoothness of \(U(x)\), the tristable and the monostable systems show a better performance than the bistable system. Especially, the tristable cantilever demonstrates a good performance at different frequencies from the structural resonance frequency.

In conclusion, both the bistable and the tristable SR cantilevers verify the theory of SR, but the tristable cantilever gives a better performance in amplification and filtering in the specific frequency band. It should be noted that the effective bandwidths of the bistable and tristable cantilevers are both limited to its fundamental resonance frequency. Hence, to get a wider operating bandwidth, the resonance frequency of the cantilever should be shifted to a higher frequency (e.g., a smaller size of device), which will be a further study in the future. In general, this Note indicates that the tristable SR cantilever has valuable advantages in comparison with the bistable and monostable cantilevers. The principle of the tristable SR cantilever device shows a potential in a wide range of applications related to signal amplification and filtering, such as mechanical resonator, energy harvesting, and weak signal detection.

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