A New Image Filtering Method: Nonlocal Image Guided Averaging

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Motivation

Filtering a map under the guidance of a color image

- Joint bilateral filter
  - Spatial filter kernel
  - Range filter kernel (guidance image information)
- Image guided filter
  - Local linear model
- To design a new one
  - Nonlocal self-similarity of natural image
  - Nonlocal linear model?
Related work

Joint bilateral filter:

\[ q_i = \frac{1}{k_i} \sum_{j \in \Omega} p_{ji} f(||i - j||) g(||I_i - I_j||) \]
Related work

\[
q_i = \frac{1}{|\omega|} \sum_{k:i \in \omega_k} (a_k^T I_i + b_k)
\]

Image guided filter kernel:

\[
W_{ij}(I) = \frac{1}{|\omega|^2} \sum_{k:(i,j) \in \omega_k} \left(1 + (l_i - \mu_k)^T (\Sigma_k + \epsilon)^{-1} (l_j - \mu_k)\right)
\]

See NLGF kernel
Nonlocal linear model: \( Q_{ij} \mathbf{q}_j = Q_{ij} \left( a_i^T \mathbf{l}_j + b_i \right), j \in N(i) \)

Here \( Q_{ij} = \sqrt{w_{ij} / \sum_{j \in N(i)} w_{ij}} \), \( w_{ij} \) is the nonlocal weight.

Energy function: 
\[
E(\mathbf{a}, \mathbf{b}) = \sum_{i \in \Lambda} \left[ \sum_{j \in N(i)} w_{ij} \left\| \mathbf{p}_j - \mathbf{q}_j \right\|^2 + \varepsilon \mathbf{a}_i^2 \right] \\
= \sum_{i \in \Lambda} \left[ \sum_{j \in N(i)} w_{ij} \left\| \mathbf{p}_j - \left( a_i^T \mathbf{l}_j + b_i \right) \right\|^2 + \varepsilon \mathbf{a}_i^2 \right] 
\]

See regression
Nonlocal linear model

\[
Q_{ij} \left( a_i^T I_j + b_i \right) = Q_{ij} q_j
\]
Remarks on the energy function:

- Weighted quadratic form of $a_i$ and $b_i$,
- Applying the first-order condition on every pair of $a_i$ and $b_i$ separately.

Coefficients

\[
\frac{\partial E(a_i, b_i)}{\partial b_i} = 0 \Rightarrow b_i^* = \frac{p_w(N(i)) - a_i^T I_w(N(i))}{\Sigma_i, w + \varepsilon}
\]

\[
\frac{\partial E(a_i, b_i^*)}{\partial a_i} = 0 \Rightarrow a_i^* = \left(\Sigma_i, w + \varepsilon\right)^{-1} \times \left(\sum_{j \in N(i)} (w_{ij} p_j \times l_j) - \frac{p_w(N(i)) \times I_w(N(i))}{\Sigma_i, w + \varepsilon}\right)
\]
New filters v.s. GF

\[ q_i = \sum_{j \in N(i)} w_{ij} \left( a_j^T l_j + b_j \right) \]

Filter kernel:
\[ W_{ij} (l) = \frac{\partial q_i}{\partial p_j} = \sum_{k: (i,j) \in N(k)} w_{ik} w_{kj} \times \]
\[ 1 + \left( l_k - \bar{l}_w (N(k)) \right)^T (\Sigma_{k,w} + \varepsilon)^{-1} \left( l_j - \bar{l}_w (N(k)) \right) \]

Relation with guided filter: See GF kernel

\[ q_i = \sum_{j \in N(i)} w_{ij} \left( a_j^T l_j + b_j \right) = \sum_{j \in N(i)} (w_{ij} a_j)^T l_i + (w_{ij} b_j) \]

Filter kernel:
\[ W_{ij} (l) = \frac{\partial q_i}{\partial p_j} = \sum_{k: (i,j) \in N(k)} w_{ik} w_{kj} \times \]
\[ 1 + \left( l_i - \bar{l}_w (N(k)) \right)^T (\Sigma_{k,w} + \varepsilon)^{-1} \left( l_j - \bar{l}_w (N(k)) \right) \]
A New Image Filtering Method: Nonlocal Image Guided Averaging (9/16)

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Introduction

Related Work

NLGA
Nonlocal Linear Model
Regression
A New Filter

Experiments
Depth SR
Dehazing
Denoising

Conclusion

Kernel comparisons

Figure: Kernels of GF, JBF, NLGF and NLGA.
Experiments: depth SR

Figure: Depth super-resolution results of GF, NLGF and NLGA, down-sampling factor: 10.
Experiments: depth SR

Table: PSNR (dB) of super-resolution results obtained by different methods: Nearest Interpolation, GF, NLGF and NLGA. The down-sampling factor is 10. $r = 20$, $\varepsilon = 10^{-3}$.

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Experiments: dehazing

(a) The hazy image. (e) Raw transmission map. (f)-(h) Filtered transmission map using GF, NLGF and NLGA respectively. (b)-(d) Recovered image using (f)-(h) respectively.
Experiments: denoising

Figure: Image denoising results of GF, NLGF, NLM and NLGA. (a) The original color test image: “House”. (b) Noisy image with Gaussian noise: $\sigma=20$, PSNR: 22.10dB. (c) Image Guided filtering result, PSNR: 25.13dB. (d) Result of NLGF, PSNR: 27.32dB. (e) Result of NLM, PSNR: 31.59dB. (f) Result of NLGA, PSNR: 32.04dB.
Experiments: denoising

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— A new image filtering method: NLGA
  ▶ exploiting the nonlocal self-similarity of the color image
  ▶ nonlocal linear model: adaptive regression
  ▶ robust to small textures and noise
  ▶ relation with image guided filter (NLGF v.s GF)

— Applications
  ▶ depth super-resolution
  ▶ image dehazing
  ▶ image denoising
Thank you!