Second Order Predicting-Error Sorting for Reversible Data Hiding

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Introduction of Reversible Data Hiding (RDH)

Proposed Method

Second Order Predicting-Error Based on Correlation Among Color Channels
Second Order Predicting-Error Sorting Based on Generalized Normal Distribution

Experiment & Conclusion
Framework of RDH

1. Predicting
2. Sorting
3. Coding
4. Embedding
5. Transmission
6. Recovering
7. Extracting
Related Work

- **Prediction**
  - median edge detection (MED)
  - gradient adjusted prediction (GAP)
  - differential adaptive run coding (DARC)
  - extended gradient-based selection and weighting (EGBSW)

- **Sorting**
  - reduce the location map by arranging the pairs of pixels in order
  - use local variance values to sort the predicted errors
  - use the sum of absolute differences between diagonal blank pixels in local region

- **RDH for color images**
  - based on prediction-error expansion that can enhance the prediction accuracy in one color channel through exploiting the edge information from another channel
Proposed Method

- Second order predicting-error
  - A double layered embedding method
  - $D_{avg} = |\sum_{k=1}^{8} a^k_{avg} p^k_{r} - p_{r}|$
  - $D_{dir} = \min \left\{ \left| \frac{p^w_{r} + p^e_{r}}{2} - p_{r} \right|, \left| \frac{p^n_{r} + p^s_{r}}{2} - p_{r} \right|, \left| \frac{p^{nw}_{r} + p^{se}_{r}}{2} - p_{r} \right|, \left| \frac{p^{ne}_{r} + p^{sw}_{r}}{2} - p_{r} \right| \right\}$
  - $|D_{avg} - D_{dir}|$ indicates whether the reference sample is located on an edge.

- $\hat{p}_c = \begin{cases} \frac{1}{4}((p^w_c + p^e_c + p^n_c + p^s_c)/4 + 0.5)|D_{avg} - D_{avg}| \leq \rho \\ \left[ P(p^k_c|D_{dir}) + 0.5 \right] \quad |D_{avg} - D_{avg}| > \rho \end{cases}$

- For instance, when $D_{dir} = |\frac{p^n_{r} + p^s_{r}}{2} - p_{r}|$, then $P(p^k_c|D_{dir}) = \frac{p^n_r + p^s_r}{2}$. 
First order prediction error:
\[ \Delta e_c = p_c - \hat{p}_c \]
\[ \Delta e_r = p_r - \hat{p}_r \]

Second order prediction error:
\[ \Delta^2 e = \Delta e_c - \Delta e_r \]

Prediction error sequence:
\[ \Delta^2 e = (\Delta^2 e_1, ..., \Delta^2 e_N) \]

Entropy is used to evaluate the performance of prediction.
Generalized normal distribution density function:

\[
f(\Delta e | u, \alpha, \beta) = \frac{\beta}{2\alpha \Gamma \left(\frac{1}{\beta}\right)} \exp \left\{ - \left| \frac{\Delta e - u}{\alpha} \right|^\beta \right\}
\]

Since \(\Delta e_c \sim GND(u_c, \alpha_c, \beta_c)\) and \(\Delta e_r \sim GND(u_r, \alpha_r, \beta_r)\),

\[
f(\Delta^2 e) = - \int_{-\infty}^{+\infty} f_{\Delta e_c - \Delta e_r}(\Delta e_c, \Delta^2 e - \Delta e_c) d\Delta e_c
\]

\[
= - \int_{-\infty}^{+\infty} \frac{\beta_c \beta_r}{4 \alpha_c \alpha_r \Gamma \left(\frac{1}{\beta_c}\right) \Gamma \left(\frac{1}{\beta_r}\right)} \times \exp \left( - \left| \frac{\Delta e_c - u_c}{\alpha_c} \right|^{\beta_c} \right)
\]

\[
- \left| \frac{(\Delta^2 e - \Delta e_r - u_r)}{\alpha_r} \right|^{\beta_r} d\Delta e_c
\]
\[ \beta = 1: \]

\[
f(\Delta^2 e) = -\int_{-\infty}^{+\infty} f_{\Delta e_c-\Delta e_r}(\Delta e_c, \Delta^2 e - \Delta e_c) d\Delta e_c
\]

\[
= \frac{\alpha_c}{2(\alpha^2_c - \alpha^2_r)} \exp\left( - \frac{|\Delta^2 e - (u_c - u_r)|}{\alpha_c} \right) - \frac{\alpha_r}{2(\alpha^2_c - \alpha^2_r)} \exp\left( - \frac{|\Delta^2 e - (u_c - u_r)|}{\alpha_r} \right)
\]
Sorting Based on Generalized Normal Distribution

- $\Phi(\Delta^2 e) = |f(\Delta^2 e)|$, and the expectation of function $E[\Phi(\Delta^2 e)]$ has positive correlation to $u_c - u_r$.

- $\Phi(\Delta^2 e) = \frac{a}{2\alpha_c} \left( \exp\left( - \frac{|u - \alpha_c|}{\alpha_c} \right) + \exp\left( - \frac{|u + \Delta^2 e|}{\alpha_c} \right) \right) - \frac{b}{2\alpha_r} \left( \exp\left( - \frac{|u - \Delta^2 e|}{\alpha_r} \right) + \exp\left( - \frac{|u + \Delta^2 e|}{\alpha_r} \right) \right)$

- $E[\Phi(\Delta^2 e)] = - \int_0^{\infty} \Delta^2 e \Phi(\Delta^2 e) \, d\Delta^2 e$
  
  $= a \times \exp\left( - \frac{u}{\alpha_c} \right) - b \times \exp\left( - \frac{u}{\alpha_r} \right) + u$

- $E[\Phi(\Delta^2 e)] \leq \lambda$ are used in data embedding
Framework of Data Hiding for the 3 Channels
Experiment & Analysis

(a) Lena

(b) Barbara


Propose a novel second order predicting and sorting technique for reversible data hiding. The method make full use of the feature of the edge information obtained from another color channel and high correlation between adjacent pixels. So it will reflect the local context complexity for pixel and prediction accuracy of prediction-error. Experimental results show that the proposed method has better results compared to the other six recent works of Li et al. [11], Li et al.[12], Sachnev et al. [13], Alattar [14], Hu et al. [15], Yang and Huang [16].
THANKS FOR LISTENING!

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