```
DSolve[m v'[x] == -η v[x] + ξ[x], v[x], x]
```
求解微分方程

$$\left\{\left\{v[x] \to e^{-\frac{x \eta}{m}} C[1] + e^{-\frac{x \eta}{m}} \int_1^x \frac{e^{\frac{\eta K[1]}{m}} \xi[K[1]]}{m} \, dK[1]\right\}\right\}$$

```
DSolve[m v'[t] == -η v[t] + ξ[t], v[t], t]
```
求解微分方程

$$\left\{\left\{v[t] \to e^{-\frac{t \eta}{m}} C[1] + e^{-\frac{t \eta}{m}} \int_1^t \frac{e^{\frac{\eta K[1]}{m}} \xi[K[1]]}{m} \, dK[1]\right\}\right\}$$

```
A = {{a1, a2}, {a3, a4}}
A // MatrixForm
```
矩阵格式

```
B = {{b1, b2}, {b3, b4}}
B // MatrixForm
```
矩阵格式

```
{{a1, a2}, {a3, a4}}
```

$$\begin{pmatrix} a1 & a2 \\ a3 & a4 \end{pmatrix}$$

```
{{b1, b2}, {b3, b4}}
```

$$\begin{pmatrix} b1 & b2 \\ b3 & b4 \end{pmatrix}$$

```
A.B
A * B
```

$$\{\{a1\ b1 + a2\ b3,\ a1\ b2 + a2\ b4\},\ \{a3\ b1 + a4\ b3,\ a3\ b2 + a4\ b4\}\}$$

$$\{\{a1\ b1,\ a2\ b2\},\ \{a3\ b3,\ a4\ b4\}\}$$

```
A.B // MatrixForm
```
矩阵格式

```
A * B // MatrixForm
```
矩阵格式

$$\begin{pmatrix} a1\ b1 + a2\ b3 & a1\ b2 + a2\ b4 \\ a3\ b1 + a4\ b3 & a3\ b2 + a4\ b4 \end{pmatrix}$$

$$\begin{pmatrix} a1\ b1 & a2\ b2 \\ a3\ b3 & a4\ b4 \end{pmatrix}$$

```
Exp[A] // MatrixForm
```
指数形式 矩阵格式

$$\begin{pmatrix} e^{a1} & e^{a2} \\ e^{a3} & e^{a4} \end{pmatrix}$$

```
MatrixExp[A] // MatrixForm
```

矩阵指数 矩阵格式

$$\begin{pmatrix} -\frac{\left(a1-a4-\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}\right)\,e^{\frac{a1}{2}+\frac{a4}{2}-\frac{1}{2}\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}}}{2\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}}+\frac{\left(a1-a4+\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}\right)\,e^{\frac{a1}{2}+\frac{a4}{2}+\frac{1}{2}\sqrt{a1^2+4\,a2\,a3-2\,a1} \\ -\frac{a3\,e^{\frac{a1}{2}+\frac{a4}{2}-\frac{1}{2}\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}}}{\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}}+\frac{a3\,e^{\frac{a1}{2}+\frac{a4}{2}+\frac{1}{2}\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}}}{\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}} \end{pmatrix}$$

```
L = FullSimplify[MatrixExp[A] // MatrixForm]
```

完全简化 矩阵指数 矩阵格式

$$\begin{pmatrix} \frac{e^{\frac{a1+a4}{2}}\left(\sqrt{4\,a2\,a3+(a1-a4)^2}\,\text{Cosh}\left[\frac{1}{2}\sqrt{4\,a2\,a3+(a1-a4)^2}\right]+(a1-a4)\,\text{Sinh}\left[\frac{1}{2}\sqrt{4\,a2\,a3+(a1-a4)^2}\right]\right)}{\sqrt{4\,a2\,a3+(a1-a4)^2}} & \\ \frac{2\,a3\,e^{\frac{a1+a4}{2}}\,\text{Sinh}\left[\frac{1}{2}\sqrt{4\,a2\,a3+(a1-a4)^2}\right]}{\sqrt{4\,a2\,a3+(a1-a4)^2}} & e^{\frac{a1+a4}{2}}\left(\sqrt{4\,a2\,a3+(a1-a4)^2}\,C \end{pmatrix}$$

```
L /. {Sqrt[4 a2 a3 + (a1 - a4)^2] → X}
```

$$\begin{pmatrix} \frac{e^{\frac{a1+a4}{2}}\left(X\,\text{Cosh}\left[\frac{X}{2}\right]+(a1-a4)\,\text{Sinh}\left[\frac{X}{2}\right]\right)}{\sqrt{4\,a2\,a3+(a1-a4)^2}} & \frac{2\,a2\,e^{\frac{a1+a4}{2}}\,\text{Sinh}\left[\frac{X}{2}\right]}{\sqrt{4\,a2\,a3+(a1-a4)^2}} \\ \frac{2\,a3\,e^{\frac{a1+a4}{2}}\,\text{Sinh}\left[\frac{X}{2}\right]}{\sqrt{4\,a2\,a3+(a1-a4)^2}} & \frac{e^{\frac{a1+a4}{2}}\left(X\,\text{Cosh}\left[\frac{X}{2}\right]+(-a1+a4)\,\text{Sinh}\left[\frac{X}{2}\right]\right)}{\sqrt{4\,a2\,a3+(a1-a4)^2}} \end{pmatrix}$$

```
L /. {Sqrt[4 a2 a3 + (a1 - a4)^2] → X, 1/Sqrt[4 a2 a3 + (a1 - a4)^2] → 1/X}
```

$$\begin{pmatrix} \frac{e^{\frac{a1+a4}{2}}\left(X\,\text{Cosh}\left[\frac{X}{2}\right]+(a1-a4)\,\text{Sinh}\left[\frac{X}{2}\right]\right)}{X} & \frac{2\,a2\,e^{\frac{a1+a4}{2}}\,\text{Sinh}\left[\frac{X}{2}\right]}{X} \\ \frac{2\,a3\,e^{\frac{a1+a4}{2}}\,\text{Sinh}\left[\frac{X}{2}\right]}{X} & \frac{e^{\frac{a1+a4}{2}}\left(X\,\text{Cosh}\left[\frac{X}{2}\right]+(-a1+a4)\,\text{Sinh}\left[\frac{X}{2}\right]\right)}{X} \end{pmatrix}$$

```
KroneckerProduct[A, B] // MatrixForm
```

克罗内克积 矩阵格式

$$\begin{pmatrix} a1\,b1 & a1\,b2 & a2\,b1 & a2\,b2 \\ a1\,b3 & a1\,b4 & a2\,b3 & a2\,b4 \\ a3\,b1 & a3\,b2 & a4\,b1 & a4\,b2 \\ a3\,b3 & a3\,b4 & a4\,b3 & a4\,b4 \end{pmatrix}$$

```
KroneckerProduct[B, A] // MatrixForm
```

克罗内克积 矩阵格式

$$\begin{pmatrix} a1\,b1 & a2\,b1 & a1\,b2 & a2\,b2 \\ a3\,b1 & a4\,b1 & a3\,b2 & a4\,b2 \\ a1\,b3 & a2\,b3 & a1\,b4 & a2\,b4 \\ a3\,b3 & a4\,b3 & a3\,b4 & a4\,b4 \end{pmatrix}$$

```
Eigenvalues[A]
```

特征值

$$\left\{\frac{1}{2}\left(a1+a4-\sqrt{\left(a1^2+4\,a2\,a3-2\,a1\,a4+a4^2\right)}\right),\ \frac{1}{2}\left(a1+a4+\sqrt{\left(a1^2+4\,a2\,a3-2\,a1\,a4+a4^2\right)}\right)\right\}$$

FullSimplify[Eigenvalues[A]]

完全简化 特征值

$$\left\{\frac{1}{2}\left(a1-\sqrt{4\,a2\,a3+(a1-a4)^2}+a4\right),\ \frac{1}{2}\left(a1+\sqrt{4\,a2\,a3+(a1-a4)^2}+a4\right)\right\}$$

Eigensystem[A]

特征系统

$$\left\{\left\{\frac{1}{2}\left(a1+a4-\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}\right),\ \frac{1}{2}\left(a1+a4+\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}\right)\right\},\right.$$
$$\left\{\left\{-\frac{1}{2\,a3}\left(-a1+a4+\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}\right),\ 1\right\},\right.$$
$$\left.\left.\left\{-\frac{1}{2\,a3}\left(-a1+a4-\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}\right),\ 1\right\}\right\}\right\}$$

Eigenvectors[A]

特征向量

$$\left\{\left\{-\frac{1}{2\,a3}\left(-a1+a4+\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}\right),\ 1\right\},\right.$$
$$\left.\left\{-\frac{1}{2\,a3}\left(-a1+a4-\sqrt{a1^2+4\,a2\,a3-2\,a1\,a4+a4^2}\right),\ 1\right\}\right\}$$

A[[1, 1]]

```
a1
```

A[[1, 2]]

```
a2
```

Inverse[A]

逆

$$\left\{\left\{\frac{a4}{-a2\,a3+a1\,a4},\ -\frac{a2}{-a2\,a3+a1\,a4}\right\},\left\{-\frac{a3}{-a2\,a3+a1\,a4},\ \frac{a1}{-a2\,a3+a1\,a4}\right\}\right\}$$

FullSimplify[QRDecomposition[A]]

完全简化 QR分解

$$\left\{\left\{\left\{\frac{\text{Conjugate}[a1]}{\sqrt{\text{Abs}[a1]^2+\text{Abs}[a3]^2}},\ \frac{\text{Conjugate}[a3]}{\sqrt{\text{Abs}[a1]^2+\text{Abs}[a3]^2}}\right\},\right.\right.$$
$$\left\{\left(a3\sqrt{\left((a2\,a3-a1\,a4)\left(\text{Conjugate}[a2\,a3]-\text{Conjugate}[a1]\,\text{Conjugate}[a4]\right)\right)/\left(\text{Abs}[a1]^2+\text{Abs}[a3]^2\right)}\right)/(a2\,a3-a1\,a4),\right.$$
$$\left.\left.\left(a1\sqrt{\left((a2\,a3-a1\,a4)\left(\text{Conjugate}[a2\,a3]-\text{Conjugate}[a1]\,\text{Conjugate}[a4]\right)\right)/\left(\text{Abs}[a1]^2+\text{Abs}[a3]^2\right)}\right)/(-a2\,a3+a1\,a4)\right\}\right\},$$
$$\left\{\left\{\sqrt{\text{Abs}[a1]^2+\text{Abs}[a3]^2},\ \left(a2\,\text{Conjugate}[a1]+a4\,\text{Conjugate}[a3]\right)/\left(\sqrt{\text{Abs}[a1]^2+\text{Abs}[a3]^2}\right)\right\},\right.$$
$$\left.\left\{0,\ \sqrt{\left((a2\,a3-a1\,a4)\left(\text{Conjugate}[a2\,a3]-\text{Conjugate}[a1]\,\text{Conjugate}[a4]\right)\right)/\left(\text{Abs}[a1]^2+\text{Abs}[a3]^2\right)}\right\}\right\}$$

```
FullSimplify[QRDecomposition[A], Element[{a1, a2, a3, a4}, Reals]]
```

完全简化 QR分解 属于 实数域

$$\left\{\left\{\left\{\frac{a1}{\sqrt{a1^2+a3^2}},\ \frac{a3}{\sqrt{a1^2+a3^2}}\right\},\ \left\{\frac{a3\ \text{Sign}[a2\ a3-a1\ a4]}{\sqrt{a1^2+a3^2}},\ -\frac{a1\ \text{Sign}[a2\ a3-a1\ a4]}{\sqrt{a1^2+a3^2}}\right\}\right\},\right.$$
$$\left.\left\{\left\{\sqrt{a1^2+a3^2},\ \frac{a1\ a2+a3\ a4}{\sqrt{a1^2+a3^2}}\right\},\ \left\{0,\ \frac{\text{Abs}[a2\ a3-a1\ a4]}{\sqrt{a1^2+a3^2}}\right\}\right\}\right\}$$