Homotopy groups of spheres

$$\pi_1\left(S^1\right) = Z, \quad \pi_k\left(S^1\right) = 0, \text{ for } k \geq 2 \tag{1}$$

$$\pi_n\left(S^n\right) = Z, \quad \pi_k\left(S^n\right) = 0, \text{ for } k < n \tag{2}$$

Homotopy groups of spheres

	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}	π_{11}	π_{12}
S^1	Z	0	0	0	0	0	0	0	0	0	0	0
S^2	0	$\boxed{Z}$	Z	Z_2	Z_2	Z_{12}	Z_2	Z_2	Z_3	Z_{15}	Z_2	$Z_2 \times Z_2$
S^3	0	0	Z	$\boxed{Z_2}$	Z_2	Z_{12}	Z_2	Z_2	Z_3	Z_{15}	Z_2	$Z_2 \times Z_2$
S^4	0	0	0	Z	Z_2	$\boxed{Z_2}$	$Z \times Z_{12}$	$Z_2 \times Z_2$	$Z_2 \times Z_2$	$Z_{24} \times Z_3$	Z_{15}	Z_2
S^5	0	0	0	0	Z	Z_2	Z_2	$\boxed{Z_{24}}$	Z_2	Z_2	Z_2	Z_{30}
S^6	0	0	0	0	0	Z	Z_2	Z_2	Z_{24}	$\boxed{0}$	Z	Z_2
S^7	0	0	0	0	0	0	Z	Z_2	Z_2	Z_{24}	0	$\boxed{0}$
S^8	0	0	0	0	0	0	0	Z	Z_2	Z_2	Z_{24}	0

Bott periodicity theorem for unitary groups: for $k > 1, n \geq \frac{k+1}{2}$

$$\pi_k(U(n)) = \pi_k(SU(n)) = \begin{cases} 0, & \text{if } k \text{ -even} \\ Z, & \text{if } k \text{ -odd} \end{cases} \quad (3)$$

对于基本群 $\pi_1(SU(n)) = 0$, $\pi_1(U(n)) = 1$

Homotopy groups of unitary groups

	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}	π_{11}	π_{12}
$U(1)$	$\boxed{Z}$	0	0	0	0	0	0	0	0	0	0	0
$U(2)$	0	$\boxed{0}$	$\boxed{Z}$	Z_2	Z_2	Z_{12}	Z_2	Z_2	Z_3	Z_{15}	Z_2	$Z_2 \times Z_2$
$U(3)$	0	0	Z	$\boxed{0}$	$\boxed{Z}$	Z_6						
$U(4)$	0	0	Z	0	Z	$\boxed{0}$	$\boxed{Z}$					
$U(5)$	0	0	Z	0	Z	0	Z	$\boxed{0}$	$\boxed{Z}$			

Homotopy groups of Lie groups

Bott periodicity theorem for orthogonal groups: for $n \geq k+2$

$$\pi_k(O(n)) = \pi_k(SO(n)) = \begin{cases} 0, & \text{if } k = 2,4,5,6 \pmod{8} \\ Z_2, & \text{if } k = 0,1 \pmod{8} \\ Z, & \text{if } k = 3,7 \pmod{8} \end{cases} \tag{4}$$

Homotopy groups of orthogonal groups									
	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9
$SO(2)$	Z	0	0	0	0	0	0	0	0
$SO(3)$	$\boxed{Z_2}$	0	Z	Z_2	Z_2	Z_{12}	Z_2	Z_2	Z_3
$SO(4)$	Z_2	$\boxed{0}$	$(Z)^{\times 2}$	$(Z_2)^{\times 2}$	$(Z_2)^{\times 2}$	$(Z_{12})^{\times 2}$	$(Z_2)^{\times 2}$	$(Z_2)^{\times 2}$	$(Z_3)^{\times 2}$
$SO(5)$	Z_2	0	$\boxed{Z}$	Z_2	Z_2	0	Z	0	0
$SO(6)$	Z_2	0	Z	$\boxed{0}$	Z	0	Z	Z_{24}	Z_2
$SO(n),\ n>6$	Z_2	0	Z	0	0	0			

Bott periodicity theorem for symplectic groups: for $n \geq \frac{k-1}{4}$

$$\pi_k(Sp(n)) = \begin{cases} 0, & \text{if } k = 0,1,2,6(\mathrm{mod} 8) \\ Z_2, & \text{if } k = 4,5(\mathrm{mod} 8) \\ Z, & \text{if } k = 3,7(\mathrm{mod} 8) \end{cases} \quad (5)$$

Homotopy groups of symplectic groups

	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}	π_{11}	π_{12}
$Sp(1)$	0	$\boxed{0}$	$\boxed{Z}$	$\boxed{Z_2}$	$\boxed{Z_2}$	Z_{12}	Z_2	Z_2	Z_3	Z_{15}	Z_2	$Z_2 \times Z_2$
$Sp(2)$	0	0	Z	Z_2	Z_2	$\boxed{0}$	$\boxed{Z}$	$\boxed{0}$	$\boxed{0}$	Z_{120}	Z_2	$Z_2 \times Z_2$
$Sp(3)$	0	0	Z	Z_2	Z_2	0	Z	0	0	$\boxed{0}$	$\boxed{Z}$	$\boxed{Z_2}$
$Sp(4)$	0	0	Z	Z_2	Z_2	0	Z	0	0	0	Z	Z_2
$Sp(5)$	0	0	Z	Z_2	Z_2	0	Z	0	0	0	Z	Z_2

Homotopy groups of real projective spaces

Real projective spaces $RP^n := S^n/Z_2$

$$\begin{aligned} \pi_1\left(RP^1\right) &= Z \\ \pi_1\left(RP^n\right) &= Z_2, \quad \text{for } n \geq 2 \\ \pi_k\left(RP^n\right) &= \pi_k\left(S^n\right), \quad \text{for } k \geq 2 \end{aligned} \tag{6}$$

Homotopy groups of real projective spaces

	π_1	π_2	π_3	π_4	π_5	π_6	π_7	π_8	π_9	π_{10}	π_{11}	π_{12}
RP^1	Z	0	0	0	0	0	0	0	0	0	0	0
RP^2	Z_2	Z	Z	Z_2	Z_2	Z_{12}	Z_2	Z_2	Z_3	Z_{15}	Z_2	$Z_2 \times Z_2$
RP^3	Z_2	0	Z	Z_2	Z_2	Z_{12}	Z_2	Z_2	Z_3	Z_{15}	Z_2	$Z_2 \times Z_2$
RP^4	Z_2	0	0	Z	Z_2	Z_2	$Z \times Z_{12}$	$Z_2 \times Z_2$	$Z_2 \times Z_2$	$Z_{24} \times Z_3$	Z_{15}	Z_2