Kitaev model

① Topo QC ⇒ 有一定纠错能力 ⇒ motivation 1995.

② Kitaev

③ Kitaev model / hamiltonian

Kitaev – Heisenberg model

motivation: α-$RuCl_3$ (?)

④ Entanglement Entropy ··· ⇒ Category theory

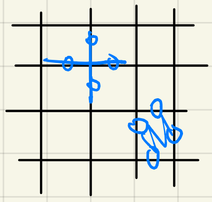
Levin-Wen model

Kitaev model (初步)

目的: 了解其基本特点

① Toric-code model

$$H = -\sum_s A_s - \sum_p B_p \qquad A_s = \prod_{j\in s} \sigma_j^x$$



S: star

P: plaquette

坐标在 Bond

$$B_p = \prod_{j\in p} \sigma_j^z$$

$\sigma_x \sigma_z = -\sigma_z \sigma_x$

0/2个相同点(j)

基本性质: $[A_s, A_{s'}] = [B_p, B_{p'}] = [A_s, B_p] = 0$

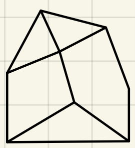
$A_s = A_s^\dagger, \quad B_p = B_p^\dagger$

$A_s^2 = B_p^2 = 1$

$\prod_s A_s = \prod_p B_p = 1$

(所有j计算2次).

推广到∀结构

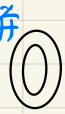


所有算子对易(仅限2d)

求解 ⇒ { 困难 / sym　一般 少量，如平移

⇒ { ① WF　(严格可解)
② ε_0, $\varepsilon_{excited}$ ...
③ GS Degeneracy ⇒ Topo 有关
几何

简并 Toric



e.g. TR ⇔ $|\phi\rangle$ 和 $T|\phi\rangle$

⇒ { $[A_s, H] = [B_p, H] = 0$
A, B 对易

$|GS\rangle$ ⇒ A_s, B_p, H 共同本征态

$\boxed{P}$　$N\times N$ ⇒ $4N^2/2 = 2N^2$ (边/自由度) ⇒ Hilbert space 2^{2N}

周期性边界条件　　real $2N^2-2$　　2^{2N-2}

⇓

4重简并

$$E_g = -2N^2\times 1 - 2N^2\times 1$$
$$= -4N^2 \qquad \text{令 } A_s = B_p = 1$$

$A_s: 1 \to -1$　　$\varepsilon_1 - \varepsilon_0 \sim 2$.　　gapped.

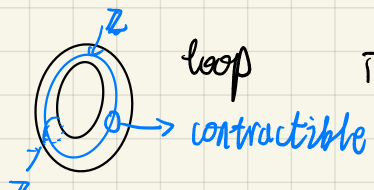
GS ⇒ projector　写不出来，构造出来

$$|G\rangle = \prod_p \left(\frac{1+B_p}{2}\right)\prod_s\left(\frac{1+A_s}{2}\right)|\phi\rangle$$ (非0的基态)

$B_p|G\rangle = |G\rangle$

$B_p(1+B_p) = B_p + \underbrace{B_p^2}_{1} = 1 + B_p$.

体现其几何性质 / Topo 性质　Degeneracy



loop　　$\pi_1(T^2) = \mathbb{Z}\times\mathbb{Z}$

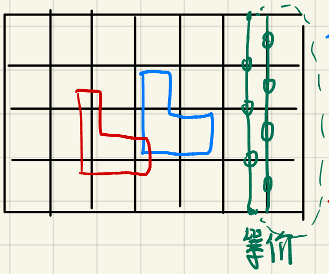
$\{0,1\}\times\{0,1\} \Rightarrow 4$.

几何 loop { contractible / not ...

① 物理上 loop

② 两种 loop 为何不同.

周期性边界条件



—: $t_p = \prod B_p$ loop contractible

$t_p|\omega\rangle = |\omega\rangle$

Trivial

—: $t_s = \prod A_s$

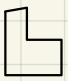
$t_s|\omega\rangle = |\omega\rangle$

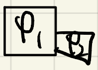
代数: $t_s\, t_{s'}$ (独立 扩大) $t_p\, t_{p'}$

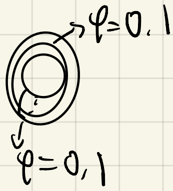
—: 不能写成 B_p, A_s 相乘的形式

\+ 一个变量

loop 意义 ⇒ Wilson loop
geometry phase

 $\prod \sigma_i^z = e^{i\varphi}$ $e^{i2\varphi} = 1 \Rightarrow \varphi = 0, \pi$.

 $\varphi = \varphi_1 + \varphi_2 \mod 2$.



4重简并.

Z_2 gauge theory   Ising model

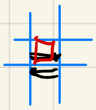
$$Z = \mathrm{Tr}[e^{-\beta J\sum_{\langle ij\rangle}\sigma_i\cdot\sigma_j}] \quad \text{loop}$$

$$= \mathrm{Tr}[1 - \beta J\sum_{\langle ij\rangle}\sigma_i\cdot\sigma_j + \frac{(\beta J)^2}{2}\Big(\sum_{\langle ij\rangle}\sigma_i\cdot\sigma_j\Big)^2 - \cdots]$$

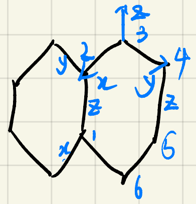

1阶=0


$\sigma_i\sigma_j\ \sigma_i\sigma_j$

3阶=0


4阶


Kitaev honeycomb lattice



$$H = -J_x \sum \sigma_i^x \sigma_j^x - J_y \sum \sigma_i^y \sigma_j^y - J_z \sum_{\langle ij \rangle} \sigma_i^z \sigma_j^z$$

$$W_p = \sigma_1^z \sigma_2^y \sigma_3^z \sigma_4^x \sigma_5^y \sigma_6^z$$

① $W_p = W_p^+$ Wilson loop

② $W_p^2 = 1$ geometry phase

③ $[W_p, W_{p'}] = 0$ 0/2个公共点

④ $[W_p, H] = 0$

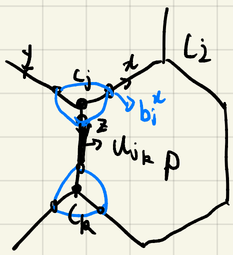
$$\sigma^x = i b^x c$$
$$\sigma^y = i b^y c$$
$$\sigma^z = i b^z c$$

b, c majorana fermion

$$\begin{cases} \gamma_i \gamma_j = -\gamma_i \gamma_j & i \neq j \\ \gamma_i^2 = 1 \end{cases}$$

$$\sigma^x \sigma^x = i b^x c \, i b^x c = (+1) b^x b^x c c = 1$$

$$\sigma^x \sigma^y \sigma^z = i \sigma^{z\,2} = i D$$
$$= i b^x c \, i b^y c \, i b^z c$$
$$= i b^x b^y b^z c$$ 约束



$$\sigma_j^x = i b_j^x c_j$$

$$\sigma_j^z \sigma_k^z = i b_j^z c_j \, i b_k^z c_j$$
$$= b_j^z b_k^z c_j c_k$$

$$\sigma_j^x \sigma_i^x = b_j^x b_i^x c_j c_i$$

$$W_p = \prod u_{jk}$$

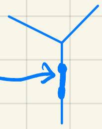
e.g $H = -\sum_{ij} t\, e^{-i\int_i^j \vec{A}\cdot d\vec{l}}\, c_i^\dagger c_j$

$= -\sum_{ij} t\, e^{i\theta_{ij}}\, c_i^\dagger c_j$

Wen RVB

$= \sum_{ij} u_{ij}\, c_i^\dagger c_j$

$H = \frac{i}{4}\sum_{ij} A_{ij}\, c_i c_j$



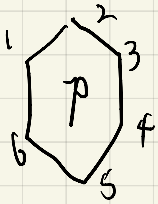
$A_{ij} = 2i\, J_\alpha\, b_j^\alpha b_k^\alpha$

$\Downarrow$

$[\, b_i^\alpha b_j^\alpha, H\,] = 0$

of GS. $A_{ij} = J^\alpha \Rightarrow i b_i^\alpha b_j^\alpha = 1$

$H = \frac{1}{4}\sum J_\alpha c_i c_j$



$\frac{J_x}{4} c_1 c_2 + \frac{J_y}{4} c_2 c_3 + \cdots$

$$H_q = \begin{pmatrix} 0 & f_q \\ f_q^* & 0 \end{pmatrix}$$

$$f_q = J_z + J_x e^{i\vec{q}\cdot\vec{R}_1} + J_y e^{i\vec{q}\cdot\vec{R}_2}$$