1d SSH model

已知实验结果

↓猜测

代入模型

$x_n = (-1)^n u$

motivation 聚乙炔

$$H = \frac{1}{2} m \dot{x}_n^2 + \frac{1}{2} k (x_{n+1} - x_n)^2$$

$$- [t_0 - \alpha (x_{n+1} - x_n)] C^{+}_{n+1,s} C_{n,s} + h.c.$$

$- u C^{+}_{ns} C_{ns}$

$\overset{\shortparallel}{0}$

$\begin{cases} \text{不可求解} \\ x_n = (-1)^n u \text{ 假设} \end{cases}$

Lemons. 打开 gap $\begin{cases} \text{BCS pairing} \\ \text{不同种原子} \\ \text{结构相变} \end{cases}$

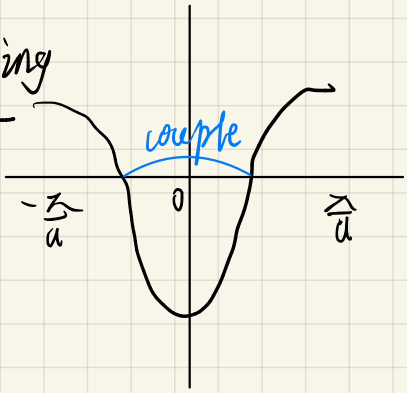
$\alpha = 0$

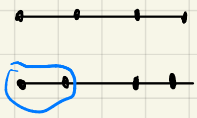
$\Downarrow$

$2t \cos k \, C_k^{+} C_k$

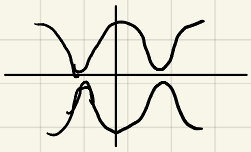
$- u C^{+}_{ks} C_{ks}$

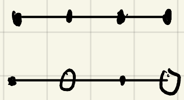
$\overset{\shortparallel}{0}$





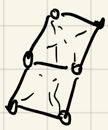
结构相变





不同种类的原子

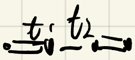
技巧.



$\longleftrightarrow$ k空间

$$\sum t_{\alpha}^{\beta} = \sum_{k} C_{\alpha k}^{+} C_{\beta k} e^{i\vec{k}\cdot\vec{R}}$$ ←相位差

平移对称性



$t_1 A_k^\dagger B_k$

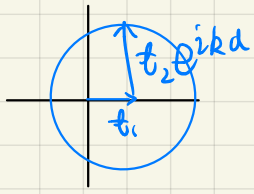
$t_2 A_k^\dagger B_k e^{ik\cdot a}$

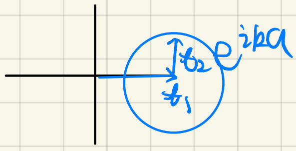
$$H = \begin{pmatrix} 0 & t_1 + t_2 e^{ika} \\ t_1 + t_2 e^{-ika} & 0 \end{pmatrix} = \begin{pmatrix} 0 & q \\ q^* & 0 \end{pmatrix}$$

$\frac{i}{2\pi} \oint \frac{dq}{q}$ winding #.

$q = t_1 + t_2 e^{ika}$

$t_1 < t_2$ $\qquad\qquad$ $t_1 > t_2$





$E_q = \frac{1}{2} k (2u)^2 L$ ⇐ 形变

$\sim \sum_k \sqrt{|t_1 + t_2 e^{ika}|^2}$

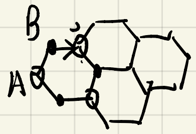
$t_1 = t_0 (1 + 2\alpha u)$

$t_2 = t_0 (1 - 2\alpha u)$

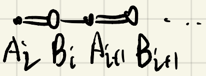
2004 单层 graphene.

Haldane. $\qquad$ 2005 Mele, Kane

曹原.



非对角 A — B t_1



对角 A ◁ A t_2 $A_k^\dagger A_k e^{i\phi/3}$

$\begin{pmatrix} 0 & q \\ q^* & 0 \end{pmatrix}$

B ▷ B $\quad B_k^\dagger B_k e^{-i\phi/3}$

$\phi = 0$

$$\Rightarrow \begin{pmatrix} \varepsilon_k & q \\ q^* & \varepsilon_k \end{pmatrix}$$

$$= \varepsilon_k + \begin{pmatrix} 0 & q \\ q^* & 0 \end{pmatrix}$$

$\phi \neq 0$

$$\varepsilon_k + \begin{pmatrix} 0 & q \\ q^* & -\Delta \end{pmatrix}$$

$\Downarrow$

$\sqrt{|q|^2 + |\Delta|^2}$ 打开 gap.

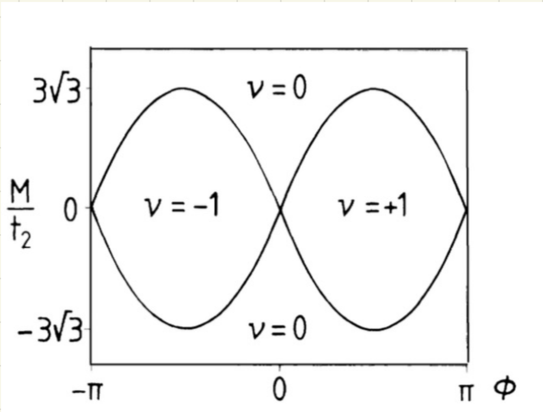
$\Sigma \quad / / / \quad + \quad \backslash \backslash \backslash \quad + \quad = \sim$

$$H = 2t_2 \cos\phi \sum_k \cos(\vec{k}\cdot\vec{b}_i) + t_1 \sum_i [\cos(\vec{k}\cdot\vec{d}_i)\,\sigma^1 + \cos(\vec{k}\cdot\vec{d}_i)\,\sigma^2]$$

$$+ [M - 2t_2 \sin(\phi) \sum \sin(\vec{k}\cdot\vec{b}_i)]\,\sigma^3$$

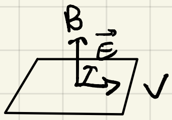
$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \qquad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$B_x \sigma_x + B_y \sigma_y + B_z \sigma_z = 0 \Rightarrow \varepsilon = \pm \sqrt{B_x^2 + B_y^2 + B_z^2} = 0$$



70~80 年代 Witten
Parisi
Haldane
Wilzeck

名词 Hall effect 导体



$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

$$\vec{E} = vB$$

$$V = Ed = vBd$$

$$I = nevd \qquad R_H = \frac{V}{I} = \frac{vBd}{nevd} = \frac{B}{ne} \Rightarrow \begin{cases} n \\ e \ \text{sgn} \end{cases}$$

$\Rightarrow$ Quantum HE 单

$$\begin{cases} G = n\, e^2/h \qquad n,\ \text{int} \ \sim 10^{12}\ \text{cm}^{-3} \\ R = \frac{1}{n}\frac{h}{e^2} \end{cases}$$

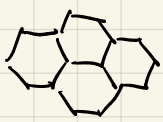
$\Rightarrow$ Fractional HE 多体

Anormalous HE Haldane. 单体

反常

$\Rightarrow$ spin HE 单体

Mele, Kane.

 + spin-orbit coupling

① no SOC

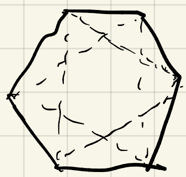
$H_k^\sigma \quad \sigma: \uparrow \downarrow.$

$$H = \begin{pmatrix} H_k^\uparrow & 0 \\ 0 & H_k^\downarrow \end{pmatrix} \Rightarrow \begin{pmatrix} H_k^\uparrow & \\ & H_{-k}^{\uparrow *} \end{pmatrix}.$$

$$H_k^\downarrow = (H_{-k}^\uparrow)^*$$

$$H = \begin{pmatrix} H_k^\uparrow & \\ & H_k^\downarrow \end{pmatrix} \Rightarrow \begin{pmatrix} H_k^\uparrow & \\ & H_{-k}^{\uparrow *} \end{pmatrix}.$$

$\begin{cases} \text{No } \vec{B} \\ \text{TRS}. \end{cases}$

引入  2倍 copy.

$$\frac{d\sigma}{dt} = k\Delta. \quad \Delta \sim 15K.$$

Yugui Yao, Qian Niu

Qi - Wu - Zhang model PRB. 74. 085308 (2006)

Xiao liang S.C

Yong shi

正方形格子.

结构: I 反常 HE

II. spin HE.

$$H = \begin{pmatrix} H_k & \\ & H_{-k}^{*} \end{pmatrix} \quad \text{eq. 15.}$$

spin Hall quantilized

$$\sigma = \sigma^{\uparrow} - \sigma^{\downarrow} \mod 2$$

$$\{0, 1\}$$

反常 HE

$$H = d_x \sigma_x + d_y \sigma_y + d_z \sigma_z$$

$$= \sin k_x \sigma_x - \sin k_y \sigma_y + c(2 - \cos k_x - \cos k_y - e_s) \sigma_z .$$

$$\sigma_{xy} = \begin{cases} \frac{1}{2\pi} & 0 < e_s < 2 \\ -\frac{1}{2\pi} & 2 < e_s < 4 \\ 0 & e_s < 0 \text{ 或 } e_s > 4 \end{cases}$$

证明: $C = \mathrm{TKNN}$ 立体角

$$= -\frac{1}{8\pi^2} \int \underbrace{dk_x dk_y (\hat{d}_x \times \hat{d}_y) \cdot \hat{d}}_{d\Omega} \qquad \hat{d} = \frac{\vec{d}}{|\vec{d}|}$$

Spin HE

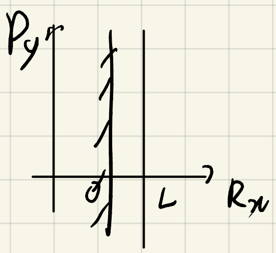
$$H_k = \begin{pmatrix} \epsilon_k + V d_\alpha(k)\sigma^\alpha & \\ & \epsilon_k - V d_\alpha(k)\sigma^\alpha \end{pmatrix}$$

$$\sigma_{xy}^{s} = \sigma_{xy}^{\uparrow} - \sigma_{xy}^{\downarrow} = \frac{n}{\pi} \qquad n = 0, \pm 1$$

作业. 推导 Zhou Bin, Shunqin Shen PRL. 101. 246807 (2008).

$$H = v\vec{p}\cdot\vec{\sigma} + (mv^2 - Bp^2)\sigma_z$$

$$= v p_x \sigma_x + v p_y \sigma_y + (mv^2 - Bp^2)\sigma_z$$



$$\psi = \varphi(x) e^{iqy}$$

$$\varphi(0) = \varphi(L) = 0$$

无限长. $\psi \sim \begin{pmatrix} a \\ b \end{pmatrix} e^{-\lambda x + iqy}$.