2022 春季最优化算法考试

(凸优化部分)

1．证明 QCQP 都是 SOCP

2．证明对于c-强凸函数$f:\mathbb{R}^d \to \mathbb{R}$，$\forall w \in \mathbb{R}^d$，

$$f(w) - \inf f \leq \frac{1}{2c}\|\nabla f(w)\|^2$$

(稀疏优化部分)

3. 求$\min_x (\|x\|_p + \frac{1}{2\tau}\|z - x\|_2^2)$ 的解

4. 默写 ADMM 算法

(机器学习中的优化部分)

5.证明

定理 (Strongly Convex Objective, Diminishing Stepsizes)

Under the assumptions of Lipschitz-continuous objective gradients, first and second moment limits and strong convexity, suppose that SG method is run with a step size sequence such that, for all $k \in \mathbb{N}$,

$$\alpha_k = \frac{\beta}{\gamma + k} \text{ for some } \beta > \frac{1}{c\mu} \text{ and } \gamma > 0 \text{ such that } \alpha_0 \leqslant \frac{\mu}{LM_G}. \tag{123}$$

Then, for all $k \in \mathbb{N}$, the expected optimality gap satisfies

$$E\left[F(w_k) - F_*\right] \leqslant \frac{\nu}{\gamma + k}, \tag{124}$$

where

$$\nu := \max\left\{\frac{\beta^2 LM}{2(\beta c\mu - 1)}, (\gamma + 1)(F(w_1) - F_*)\right\}. \tag{125}$$

6.证明

定理 (Strongly Convex Objective, Noise Reduction)

Under the assumptions of Lipschitz-continuous objective gradients and first and second moment limits and strong convexity, but with (116) refined to the existence of constants $M \geqslant 0$ and $\zeta \in (0, 1)$ such that

$$Var_{\xi_k}[g(w_k, \xi_k)] \leqslant M\zeta^{k-1}, \quad \forall k \in \mathbb{N}. \tag{131}$$

In addition, suppose that the SG method is run with a fixed stepsize, $\alpha_k = \bar{\alpha}$ satisfying

$$0 < \bar{\alpha} \leqslant \min\left\{\frac{\mu}{L\mu_G^2}, \frac{1}{c\mu}\right\}. \tag{132}$$