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Midterm examination of *Geometric Analysis*

Autumn semester 2020

1. Let M be an n-dimensional complete Riemannian manifold with Ricci curvature bounded from below $Ric(M) \geq -(n-1)k^2$ $(k \geq 0)$. If u is a positive solution of

$$\Delta u = \lambda u, \quad \lambda \text{ is constant},$$

then we have

$$\frac{|\nabla u|}{u} \leq C(n, k, \lambda),$$

where $C(n, k, \lambda)$ is a constant depending on n, k, λ.

2. Let M be an n-dimensional complete Riemannian manifold with Ricci curvature bounded from below $Ric(M) \geq -(n-1)k^2$ $(k \geq 0)$. Then

$$\frac{Vol_M(B_{2R})}{Vol_M(B_R)} \leq 2^n e^{(n-1)kR}.$$

3. Let M be a compact n-dimensional Riemannian manifold without boundary. Suppose the Ricci curvature $R_{ij} \geq 0$ and ω is a harmonic 1-form. Then

$$|\nabla \omega| = 0.$$