ALGEBRAIC GEOMETRY: FINAL EXAM

1 (30). (1) Ample invertible sheaf, very ample invertible sheaf.

(2) The relation between ample and very ample invertible sheaves on projective schemes over a Noetherian ring.

(3) Serre vanishing theorem, Serre duality for a Cohen-Macaulay projective variety.

2 (10). (If you can prove assertion (2), omit (1)).

Let X be a topological space and $\mathcal{U}$ a covering of X. Let $\mathcal{F}$ be a flasque sheaf (namely, for any open subset $U \subset X$, $\mathcal{F}(X) \to \mathcal{F}(U)$ is surjective).

(1) Assume $\mathcal{U}$ consists of 2 open sets. Show that $H^1(\mathcal{U}, \mathcal{F}) = 0$;

(2) Assume $\mathcal{U}$ consists of 3 open sets, show that $H^1(\mathcal{U}, \mathcal{F}) = 0$.

3 (10). Let $X \subset P = \mathbb{P}^2_{\mathbb{Z}}$ be the closed subsheme (over Spec $\mathbb{Z}$) given by the homogenous equation $x_0^3 + 2x_1^3 + 3x_2^3 = 0$. Find two closed fibers such that, one is non-reduced, the other one is reducible.

4 (15). Show that: (1) An affine scheme is quasi-compact;

(2) A scheme is quasi-compact if and only if it is a union of finite affine open subschemes;

(3) For a Noetherian scheme X and $x \in X$, the closure $\overline{\{x\}}$ contains a closed point (In fact, this is true for a quasi-compact and T_0 topological space).

5 (25). Let k be a field, and let $Z \subset P = \mathbb{P}^2_k$ be a closed subscheme defined by the homogenous equation $x_0^5 + x_1^5 + x_2^5 = 0$.

(1) Is Z regular?

(2) Show that the ideal sheaf $\mathcal{I}_Z$ is invertible on P.

(3) Find n such that $\mathcal{I}_Z \cong \mathcal{O}_P(n)$ and explain why.

(4) Show that for a coherent sheaf $\mathcal{F}$ on Z, $H^i(Z, \mathcal{F}) = 0$ if $i > 2$. (short exact seq)

(5) Compute $\dim_k H^0(Z, \mathcal{O}_Z(1))$ and $\dim_k H^1(Z, \mathcal{O}_Z(1))$.

6 (10). (1) Let X be a projective variety, $\mathcal{O}_X(1)$ a very ample invertible sheaf and $\mathcal{F}$ a coherent sheaf. Show that for sufficiently large n, $H^1(X, \mathcal{F}(n)) = 0$. Is this true for a quasi-coherent sheaf?

(2) Let X be a Cohen-Macaulay projective variety in $P = \mathbb{P}^n_k$ of pure codimention r. Show that $\mathcal{E}xt^i_P(\mathcal{O}_X, \omega_P) = 0$ if $i \neq r$ (Use Serre duality).

7 (2). 请给代数几何课程提出中肯、有效、善意的建议（如果你讲代数几何课程的话你会讲什么内容，侧重内容？）