You can use all results proved in classes, but if you want to use some homework problems then you need to give a full proof.

1. Find the coefficient of x^n in the product

$$\left(\sum_{k=0}^{+\infty} \frac{x^k}{6^{k+1}}\right)\left(\sum_{k=0}^{+\infty}(-1)^k 6^{n-k}\binom{n}{k}x^k\right),$$

where $n > 0$ is an integer. Your answer should be in closed form.

2. Let $t \geq 2$ be a fixed integer. Let $S_1, S_2, \ldots, S_n$ be n subsets of $[n]$ such that for any $1 \leq i < j \leq n$, we have

$$|S_i \cap S_j| \leq t - 1.$$

Prove that there exist some set S_i and some absolute constant C satisfying that $|S_i| \leq C\sqrt{t \cdot n}$.

3. How many permutations π of $[2n]$ are there so that no odd number is mapped to itself?

4. Let $X = \{x_1, ..., x_n\}$ where $x_i \in \mathbb{R}$ for each $i \in [n]$. A subset S of X is called *nonnegative*, if the sum of the elements contained in S is nonnegative. Find the minimum number of nonnegative subsets of $X = \{x_1, ..., x_n\}$, subject to the condition that

$$x_1 + x_2 + ... + x_n \geq 0.$$

And then give an example which achieves this minimum.

For example, if $X = \{x_1, x_2, x_3\}$ with $x_1 = -4, x_2 = 1$ and $x_3 = 4$, then the nonnegative subsets are:

$$\{x_2\}, \{x_3\}, \{x_1, x_3\}, \{x_2, x_3\}, \{x_1, x_2, x_3\}.$$

5. Let $n \geq 3$. Prove that any n-vertex graph G with at least $\lfloor \frac{n^2}{4} \rfloor + 1$ edges contains at least $\lfloor \frac{n}{2} \rfloor$ triangles.