2022.9.7. 第2周第1次课

上节课回顾:

Newton方程 ⇒ Lagrange方程 ⇒ 计算

直角 ✓

其它 ✗

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}}\right) = \frac{\partial L}{\partial q}$$

广义坐标(…)

Newton方程 $\dot{p}_i = F_i$

这次课的内容:

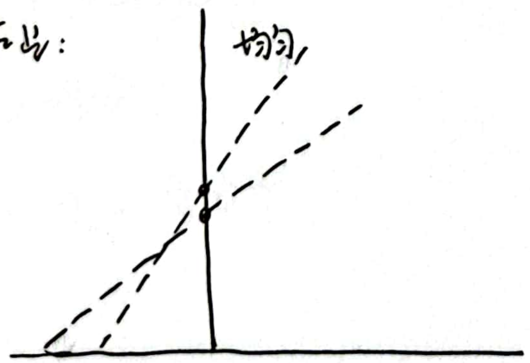
证明拉氏方程与广义坐标的选取无关;

最好的坐标选择 ⟺ 守恒量;

Fourier变换 (场论, $N\to\infty$).

$$L = \sum_{ij} g_{ij}(q)\underbrace{\dot{q}_i\dot{q}_j}_{\text{二次型}} - U(q)$$

作业:



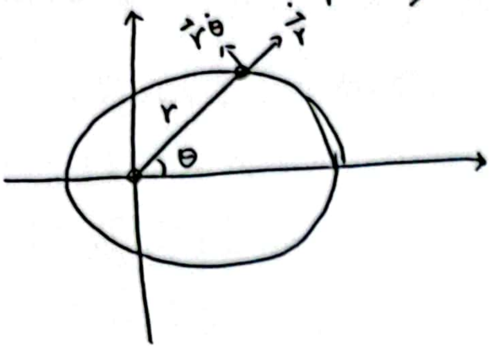
• 求均匀杆倒下的方式(轨迹)

1) 用一个广义坐标;

2) 用两个广义坐标;

3) 用Mathematica作模拟.

第二章. 守恒定律: (~~衡~~).

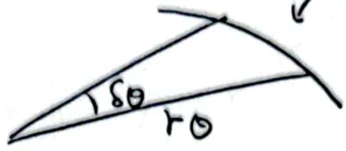


$$L=\frac{1}{2}m(\dot r^2+r^2\dot\theta^2)-U(r)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot\theta}\right)=\frac{\partial L}{\partial\theta}=0$$

$$\frac{d}{dt}(mr^2\dot\theta)=0$$

$\delta t \propto r^2\delta\theta$



$\delta\theta=$ ~~$\delta(\theta+$~~

$\theta(t+\delta t)-\theta(t)$

$$\boxed{\begin{array}{l} L(q_1,q_2,q_3,\dot q_1,\dot q_2,\dot q_3,\dot q_4) \\ \dfrac{\partial L}{\partial\dot q_3}=C_3 \\ \dfrac{\partial L}{\partial\dot q_4}=C_4 \end{array}}$$

几类守~~衡~~恒性:

① 能量;

② 动量.角动量;

$$L(q,\dot q)\Leftrightarrow\frac{dL}{dt}=\frac{\partial L}{\partial q}\frac{dq}{dt}+\frac{\partial L}{\partial\dot q}\frac{d\dot q}{dt}$$

$$=\frac{d}{dt}\left(\frac{\partial L}{\partial\dot q}\dot q\right)$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial\dot q}\dot q-L\right)=0$$

$$H=\frac{\partial L}{\partial\dot q}\dot q-L$$

$$=Pq-L$$

$$L=\frac{1}{2}\dot q^2-U$$

$$\frac{\partial L}{\partial\dot q}\dot q=2T$$

$$\Rightarrow\quad\frac{\partial L}{\partial\dot q}\dot q-L=2T-(T-U)=T+U=\text{Energy},$$

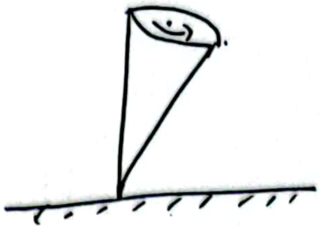
角动量. $M = \vec{r} \times \vec{p}$

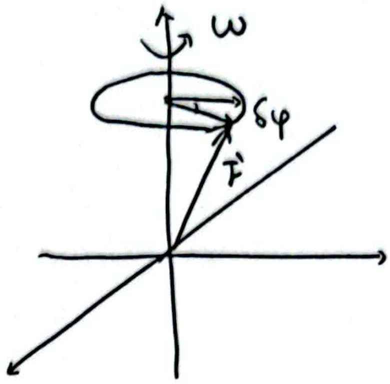
$(\vec{a}\times\vec{b})\cdot\vec{c}$ ⇒ 体积

$= (\vec{b}\times\vec{c})\cdot\vec{a}$

$= (\vec{c}\times\vec{a})\cdot\vec{b}$

$(\vec{a}\times\vec{b})\cdot\vec{a} = 0$





L在转动下不变, ⇒ 证明角动量守恒.

⇕ 即

$$L(\vec{r},\dot{\vec{r}}) = L(\vec{r}+\delta\vec{r}, \dot{\vec{r}}+\delta\dot{\vec{r}})$$

又 $\begin{cases} \delta\vec{r} = \vec{\omega}\times\vec{r}\,dt = \delta\varphi\times\vec{r} \\ \delta\vec{\varphi} = \vec{\omega}\,dt \\ |\delta\vec{\varphi}| = |\vec{\omega}\,dt| = \omega\,dt \end{cases}$

即 $\delta\vec{r} = \delta\vec{\varphi}\times\vec{r}$

$\delta\dot{\vec{r}} = \delta\vec{v} = \delta\vec{\varphi}\times\dot{\vec{r}}$

则⇒ $$\delta L = \frac{\partial L}{\partial\vec{r}}\cdot\delta\vec{r} + \frac{\partial L}{\partial\dot{\vec{r}}}\delta\dot{\vec{r}}$$

$$= \frac{\partial L}{\partial\vec{r}}\cdot(\delta\vec{\varphi}\times\vec{r}) + \frac{\partial L}{\partial\dot{\vec{r}}}(\delta\vec{\varphi}\times\dot{\vec{r}})$$

$$= \dot{\vec{p}}\cdot(\delta\vec{\varphi}\times\vec{r}) + \vec{p}\cdot(\delta\vec{\varphi}\times\dot{\vec{r}})$$

又 $\delta L = 0$. 且与 $\delta\varphi$ 无关,

则 $\delta\vec{\varphi}\cdot(\vec{r}\times\dot{\vec{p}} + \dot{\vec{r}}\times\vec{p}) = 0$ $\quad M = \text{const}.$

$\delta\vec{\varphi}\cdot\left[\frac{d}{dt}(\vec{r}\times\vec{p})\right] = 0 \Rightarrow \vec{r}\times\vec{p} = \text{Const}(t)$;

1) 时间平移不变性： $\Leftrightarrow$ E守恒

$$L(t) = L(t+c)$$

2) 空间平移不变性； $\Leftrightarrow$ 动量守恒.

$$L(q,\dot{q}) = L(q+c,\dot{q})$$

3) 转动不变性. $\Leftrightarrow$ 角动量守恒.

$$L(\varphi,\dot{\varphi}) = L(\varphi+c,\dot{\varphi})$$

层次

用公式 $\longrightarrow$ 会推导 $\longrightarrow$ 讲逻辑

Lagrange 方程

$$m\ddot{\vec{r}}_i = \vec{F}_i \xleftrightarrow{\text{读出来}} \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{r}_i}\right) = \frac{\partial L}{\partial r_i}$$

$\Updownarrow$ textbook

达朗贝尔（Newton方程的微分形式）

$$\sum_i \left(\vec{F}_i - m\ddot{\vec{r}}_i\right)\delta \vec{r}_i = 0$$

$$\downarrow$$

$$r_i = r_i(q_1, q_2, \cdots, q_s)$$

$$\Downarrow$$

$$\Longrightarrow \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_\alpha}\right) = \frac{\partial L}{\partial q_\alpha}$$

$P_{12} \sim P_{13}$. 复习