2022年10月9日. 第六周（国庆之后）.

①

$U(x) = kx^6$. (4.6.8…)

$m\ddot{x} = -6kx^5$;

1) 有周期运动. $x \simeq A\cos\omega t$;

2) ω和A.k等有关系;

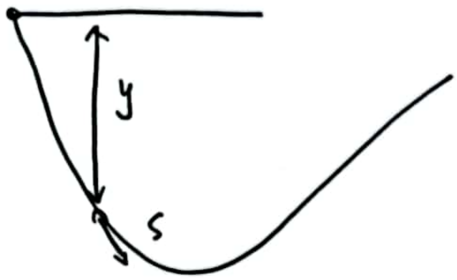
$m\delta\ddot{x} = -\frac{\partial^2 U}{\partial x^2}\delta x^2$

②

$m\ddot{x} = -kx$

ω与A无关

重力场



$(ds)^2 = (dx)^2 + (dy)^2$

$U = mgy = \frac{1}{2}m\dot{s}^2 = \frac{1}{2}ks^2$. 保证振动周期与A无关.

$y = \frac{ks^2}{2mg}$

$s = \sqrt{\frac{2mgy}{k}}$

⇒ 建立 x和y的关系.

$x = \int \sqrt{(\frac{\partial s}{\partial y})^2 - 1}\, dy$

U是否为保守力场? ⟸

$\omega = \sqrt{\frac{k}{m}}$

③. 22节. 受迫振动.

两种典型的运动.

1) $m\ddot{x} = -kx + f(t)$

2) $m\ddot{x} = -kx + f(t)\cdot x$ ，参量共振.

3) 阻尼运动.

$$m\ddot{x} = -kx + \eta\dot{x}$$

4) 非线性.

$$m\ddot{x} = -kx + hx^2 + gx^3 + \cdots$$

$U = \frac{1}{2}kx^2 \nearrow \frac{1}{2}(k+\gamma f(t))x^2$ ，$F = -\frac{\partial U}{\partial x} = -(k+\gamma f)x$;

$U = \frac{1}{2}kx^2 \searrow \frac{1}{2}k(x+\gamma f(t))^2$ ，$F = -\frac{\partial U}{\partial x} = -k(x+\gamma f(t))$.

$$L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 + xF(t)$$

$$m\ddot{x} = -kx + F(t)$$

齐次解: $x_0 = a\cos(\omega x + \alpha)$

$\omega = \sqrt{k/m}$ ， a 幅度. α 相位;

考虑: $m(\ddot{x}+\ddot{x}_0) = -k(x+x_0)+F$

令 $F = f\cos(\gamma t+\beta)$.

求 $m\ddot{x} = -kx + f\cos(\gamma t+\beta)$

$$x = A\cos(\gamma t+\beta)$$

$$A = \frac{f}{k-\gamma^2 m}$$

1) 有两个频率. (ω和γ)

2) $\gamma\to\omega$. A发散. (这里已经可以看出A与ω的依赖关系).

3) $\gamma\to\omega$. 解 = ?

令 $\gamma=\omega+\eta$. $\eta\to 0$.

可以求极限解~

$$x = A\cos(\omega t+\alpha) + \frac{ft}{2\omega}\sin(\omega t+\beta).$$

幅度随t线性增加.

$$m\ddot{x} = -kx + F(t).$$

定义 $z = \dot{x} + i\omega x \in \mathbb{C}$

$$\frac{d}{dt}z - i\omega z = \frac{F(t)}{m}$$

将2阶方程转化为1阶复域方程；

① 齐次解.

$$\frac{d}{dt}z = -i\omega z$$

$$z = e^{-i\omega t}$$

② 非齐次解.

$$z = A(t)\, e^{-i\omega t}$$

$$\dot{A}(t) = \frac{F}{m} e^{-i\omega t}$$

$$\Leftrightarrow \quad A(t) = A_0 + \int_0^t \frac{F}{m} e^{-i\omega\tau}\, d\tau$$

$$\begin{aligned} |z(t)|^2 &= \dot{x}^2 + \omega^2 x^2 \\ &= \frac{2}{m}\left(\frac{1}{2}m\dot{x}^2 + \frac{1}{2}kx^2\right) \\ &= \frac{2E}{m} \end{aligned}$$

幅度$^2 \propto$ 能量；

IF. $F(t) = f\cos(\gamma t+\beta)$, 但 $\gamma \neq \omega$

$$\int_0^{+\infty} f\cos(\gamma t+\beta)\, e^{-i\omega t}\, dt \sim 0$$

利用 $\int_0^{+\infty} \cos(\gamma t+\beta)\, dt = 0$ (ill-defined)

= 小量;

$\int_0^{t} \cos\beta\, dt \propto (\cos\beta)t$

幅度 $\propto$ 能量 $\propto t^2$

$\int_0^{t} \cancel{\cos}\,\sin\beta\, dt \propto (\sin\beta)t$

(像Dirac矩阵~)

$\square^2 = (P+i\gamma)(P-i\gamma)$

z 非共振. $z \sim$ finite;

共振. $z \propto t e^{i\beta}$

$|z|^2 \propto t^2$ (E)

23节. 多自由度.

～振子一样. $\omega \sim \sqrt{\frac{k}{m}}$;

24节. 应用:

振动.转动;

原子物理;

数学基础: 矩阵;

$$T = \frac{1}{2}\Sigma_\alpha m_\alpha \dot{x}_\alpha^2$$

$$U = \frac{1}{2}\Sigma_{\alpha\beta} k_{\alpha\beta} x_\alpha x_\beta$$

对易关系: $k_{\alpha\beta} = k_{\beta\alpha}$.

$$m_\alpha \ddot{x}_\alpha = -\frac{\partial U}{\partial x_\alpha} = -k_{\alpha\beta} x_\beta$$,(一般不加 m_α. 将 $\sqrt{m_\alpha} x_\alpha \to q_\alpha$

($\ddot{q}_\alpha = -k_\alpha{}^\beta q_\beta$)

$$\begin{pmatrix} m_1 & & \\ & \ddots & \\ & & m_N \end{pmatrix}\begin{pmatrix} \ddot{x}_1 \\ \vdots \\ \ddot{x}_N \end{pmatrix} = -\begin{pmatrix} k_{11} & k_{12} & \cdots & k_{1N} \\ k_{21} & \ddots & & \\ \vdots & & & \\ k_{N1} & \cdots & & k_{NN} \end{pmatrix}\begin{pmatrix} x_1 \\ \vdots \\ x_N \end{pmatrix}$$

$$x_\alpha(t) = x_\alpha^q \cos(\omega_q t + \theta_q)$$

$$x_\alpha \sim A_\alpha \cos(\omega t + \theta).$$

↑简正频率;

$\det(\omega^2 m - k) = 0$

$\underline{\omega^2 m} = k$

$\omega^2 m X^q = kX$

矩阵方程.