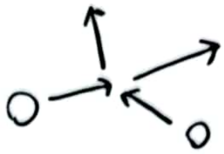
散射问题：（Landau书. §18） 9月23日.

1) 两体之间相互作用；

☆ $M \gg m$；

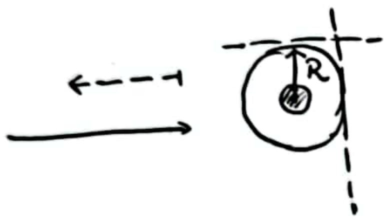
☆ $m_1 \sim m_2$，质心 + 相对； $U(r)$



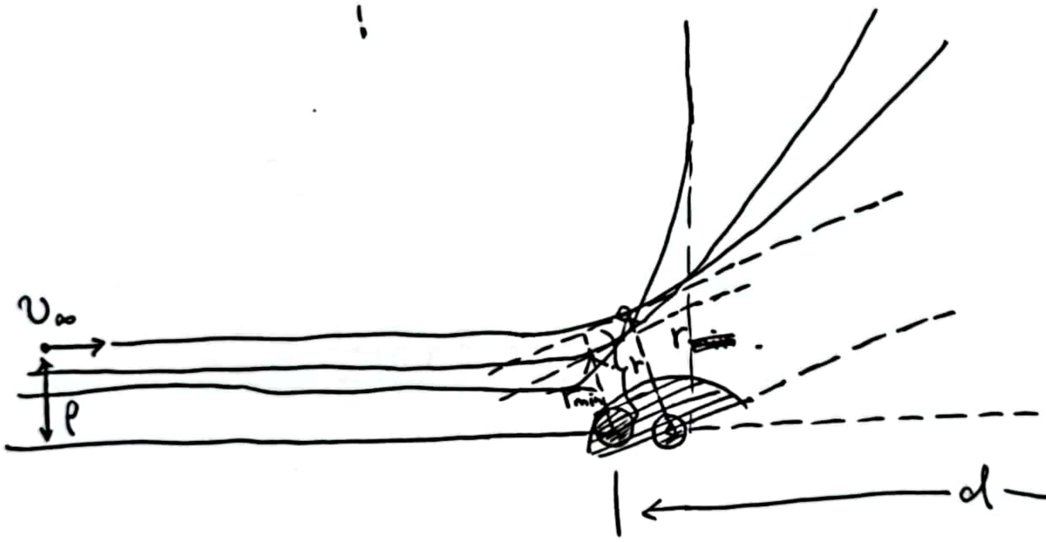
2) Ruthefold 散射；

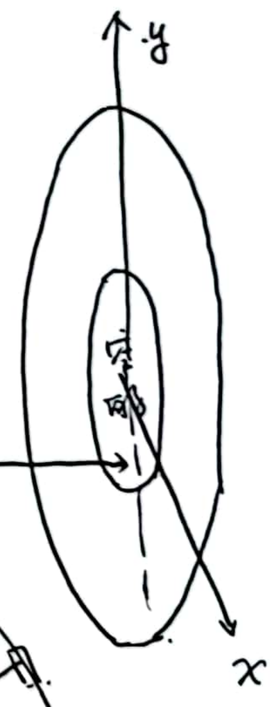
原子物理，排斥相互作用.

核心概念：散射截面 $\sigma \sim \pi R^2$.



图像：

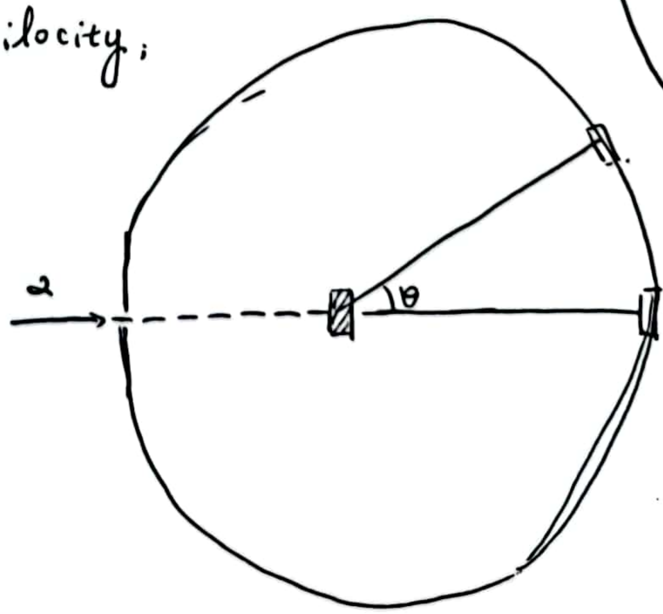




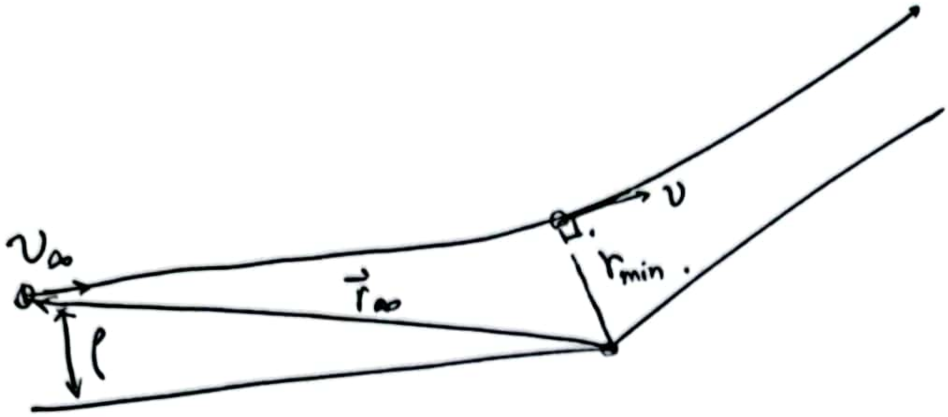
V_∞，$r \to \infty$ 时. ~~Vilocity~~. vilocity；

角动量守恒；

角度的分布；

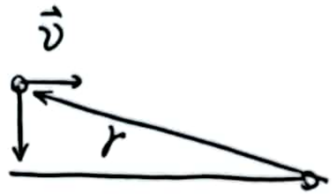


探测角分布；



角动量守恒.

$$L = \frac{1}{2}m(\dot{r}^2 + r^2\dot{\theta}^2) - U(r)$$



$$mr^2\dot{\theta} = M(\rho)$$

$$E = \frac{1}{2}mv_\infty^2$$

求M的值. $\vec{M} = \vec{r} \times m\vec{v}$

$$= \vec{r}_\infty \times m\vec{v}_\infty$$

$$= m\rho v_\infty$$

$$E = \frac{1}{2}mv_\infty^2 = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\dot{\theta}^2 + U(r)$$

$$= \frac{1}{2}m\dot{r}^2 + \frac{1}{2}mr^2\frac{M^2}{m^2r^4} + U(r)$$

$$= \frac{1}{2}m\dot{r}^2 + \frac{M^2}{2mr^2} + U(r)$$

在 r_{min} 处; $\dot{r} = 0$;

$$E = \frac{M^2}{2mr^2} + U(r)$$

2)

$$\frac{dr}{dt}=\dot{r}$$

$$\frac{1}{2}m\dot{r}^2 = E-U(r)-\frac{m^2\rho^2 v_\infty^2}{2mr^2}$$

$$=\frac{1}{2}mv_\infty^2-U(r)-\frac{m\rho^2 v_\infty^2}{2r^2}$$

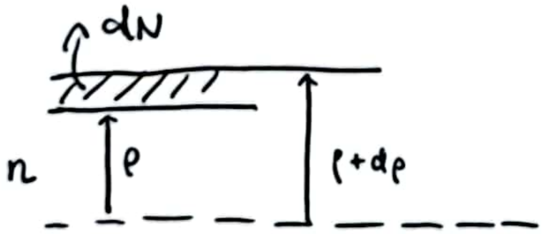
$$\frac{dr}{dt}=\sqrt{v_\infty^2-\frac{2}{m}U-\frac{\rho^2 v_\infty^2}{r^2}}$$

利用角动量守恒. $mr^2\dot{\theta}=m\rho v_\infty$

$$\frac{d\theta}{dt}=\frac{\rho v_\infty}{r^2}$$

得到 $$\frac{d\cancel{\theta}r}{d\theta}=\frac{r^2}{\rho v_\infty}\sqrt{v_\infty^2-\frac{2}{m}U-\frac{\rho^2 v_\infty^2}{r^2}}$$

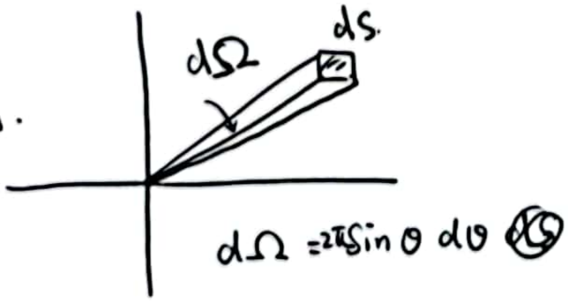
$$\int_{-\infty}^{r_{min}}\frac{\rho v_\infty}{r^2}\sqrt{v_\infty^2-\frac{2}{m}U-\frac{\rho^2 v_\infty^2}{r^2}}\,dr=\int d\theta=\varphi$$



$$dN=\rho\cdot 2\pi\cdot d\rho\cdot n$$

$$=n\,2\pi\rho\left|\frac{d\rho}{d\chi}\right|d\chi$$

立体角.



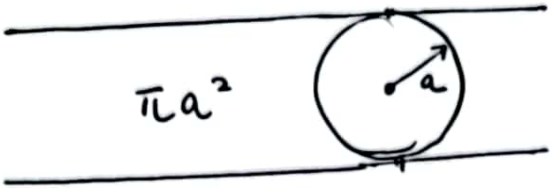
$$d\Omega=\frac{d\chi}{2\pi\sin\theta},\quad d\chi=\frac{d\Omega}{2\pi\sin\theta};$$

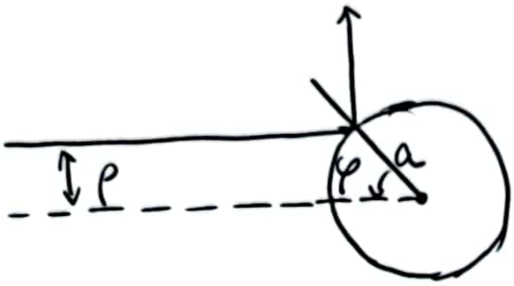
$$dN=n\cdot\cancel{2\pi}\rho\cdot\left|\frac{d\rho}{d\chi}\right|\frac{d\Omega}{\cancel{2\pi}\sin\theta};$$

$$d\sigma=\frac{dN}{n}=\rho\frac{d\Omega}{\sin\theta}\left|\frac{d\rho}{d\chi}\right|;\ \int\frac{d\sigma}{d\Omega}d\Omega=\sigma$$

3)

例子：半径为a的钢性球的散射；





$$a\sin\varphi = \rho = a\cos\frac{\chi}{2}$$

$$\pi - 2\varphi = \chi$$

$$\varphi = \frac{\pi-\chi}{2} = \frac{\pi}{2} - \frac{\chi}{2}$$

$$\frac{d\sigma}{d\Omega} = \frac{\rho}{\sin\chi}\left|\frac{d\rho}{d\chi}\right|$$

$$= \frac{a\cos\frac{\chi}{2}}{\sin\chi}\,a\sin\left(\frac{\chi}{2}\right)\cdot\frac{1}{2}$$

$$= \frac{a^2}{4}$$

$$\sigma = \frac{a^2}{4}\cdot 4\pi$$

$$= \pi a^2$$

物理意义

作业. 2022.9.23

①.

● M

证明. 如果无相互作用. $\frac{d\sigma}{d\Omega}=0$;

$U(r)=0$

②. Landau书 P51 ~ P53

(1~7题;)