2022.9.16 第3周第2次课

Integral of motion.

✓ 可积性 $\simeq$ 可解性 $\simeq$ 对称性；

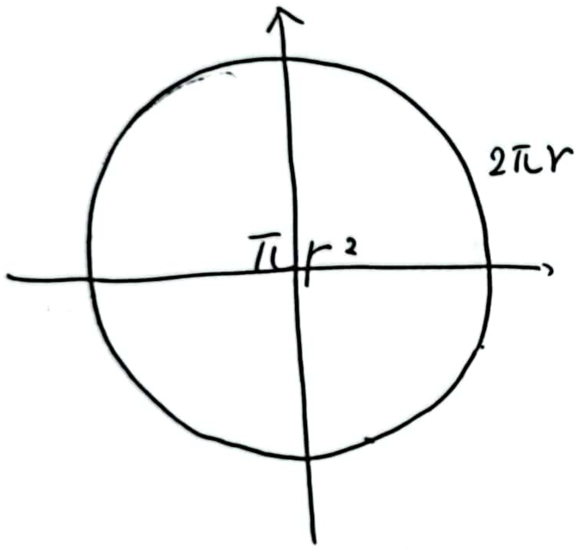
1) 新变量.

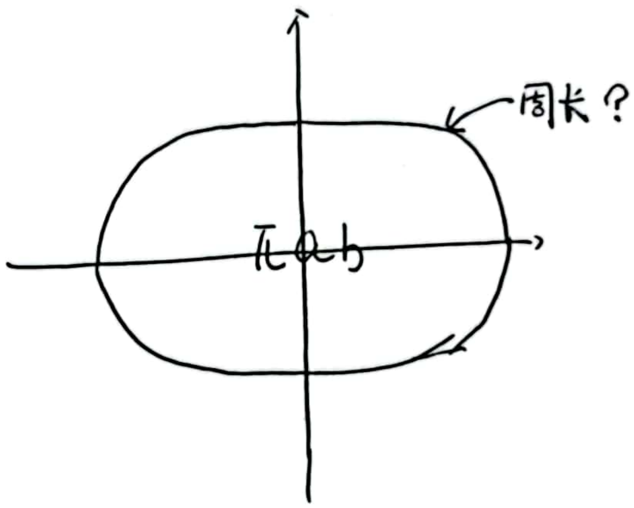
2) 守恒量. $\Rightarrow$ 化简.

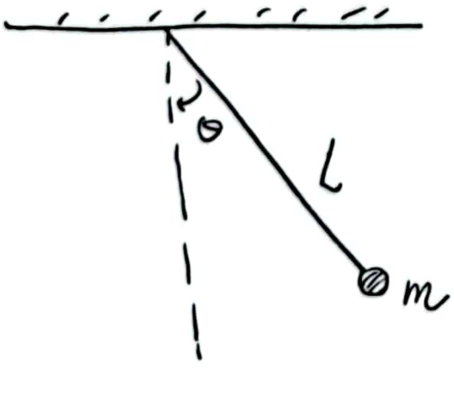
3) 余下少量可解方程；

✓ 椭圆函数与 Euler Beta function.

材料：Ellipic functions and Ellipic integral. by R. Herman.







$$\frac{1}{2}ml^2\dot{\theta}^2 - mgl(1-\cos\theta) = L$$

$$ml^2\ddot{\theta} = -mgl\cos\left(\frac{\pi}{2}-\theta\right)$$

$$= -mgl\sin\theta$$

$T \propto \sqrt{l/g}$. 量纲分析. 与m无关；

$$T = \text{Const} \cdot \int_{-\theta_0}^{\theta_0} \frac{1}{\sqrt{1+\cos\theta}}\, d\theta$$

注: $\cos\theta \xrightarrow{\theta\to 0} 1 - \frac{1}{2}\theta^2 + \frac{1}{4!}\theta^4$

$$\sqrt{1+\cos\theta} \xrightarrow{\theta\to 0} \sqrt{2-\frac{1}{2}\theta^2+\frac{1}{4!}\theta^4}$$

$$\int \frac{1}{\sqrt{1+\cos\theta}}\, d\theta \xrightarrow{\theta\to 0} \int \frac{1/2}{\sqrt{2-\frac{1}{2}\theta^2+\frac{1}{4!}\theta^4}}\, d\theta^2 \sim \int \frac{1}{\sqrt{1+\frac{a}{r}+\frac{b}{r^2}}}\, dr$$

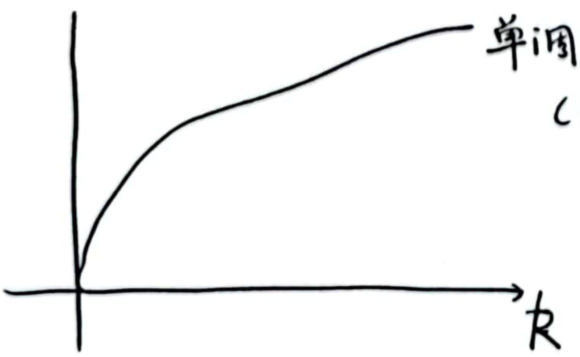
参考: Landau 书. P27

$$F(\phi, k) = \int_0^{\phi} \frac{d\theta}{\sqrt{1-k^2\sin^2\theta}} = \int_0^{\sin\phi} \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}$$

$$K(k) = \int_0^{\frac{\pi}{2}} \frac{dt}{\sqrt{1-k^2\sin^2\theta}} = \int_0^{1} \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}$$

$$E(\phi, k) = \int_0^{\phi} \sqrt{1-k^2\sin^2\theta}\, d\theta = \int_0^{\sin\phi} \frac{\sqrt{1-k^2t^2}}{\sqrt{1-t^2}}\, dt$$

$$E(k) = \int_0^{\cancel{\phi}\,1} \frac{\sqrt{1-k^2t^2}}{\sqrt{1-t^2}}\, dt = \int_0^{\pi/2} \sqrt{1-k^2\sin^2\theta}\, d\theta$$



单调性.
(可以求逆)

$u = F(\phi, k)$

$(\phi, k) = F^{-1}(u)$

例4. 例5.

吸引势中的情况;

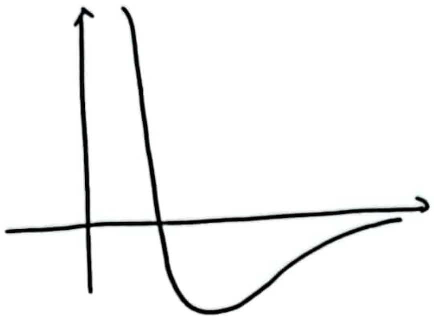
4. 坠落至 $U(r) = -\alpha/r^2$ 中心场的质点的有效截面;

$$E = \frac{1}{2} m v_\infty^2$$

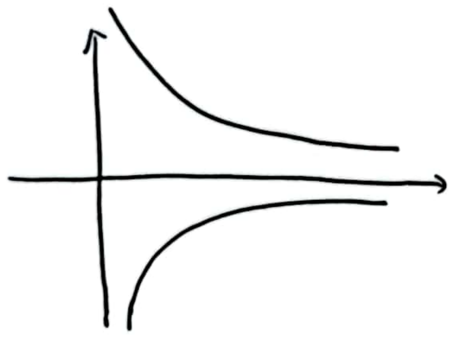
$$= \frac{1}{2} m \dot{r}^2 + U_{eff}$$

$$U_{eff} = \frac{m \rho^2 v_\infty^2}{2r^2} - \frac{\alpha}{r^n}$$

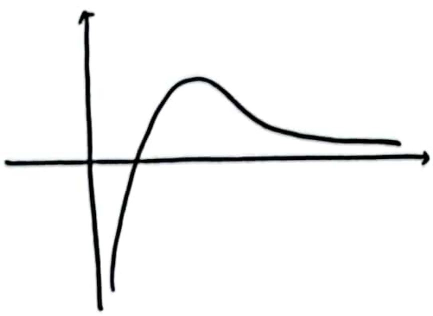
$n=1$



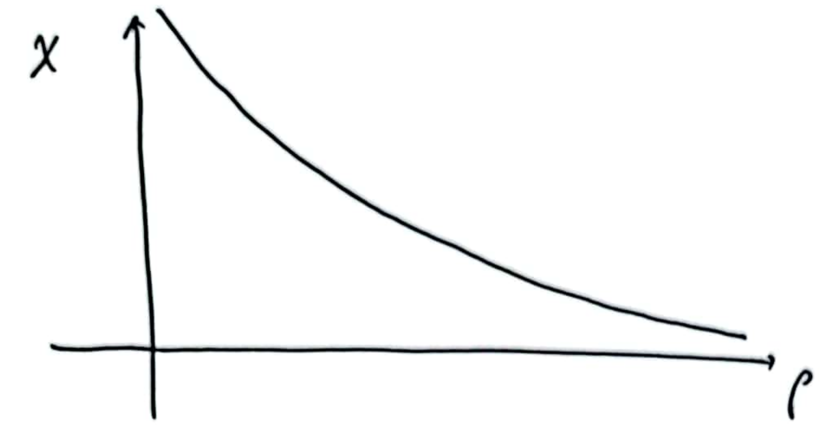
$n=2$



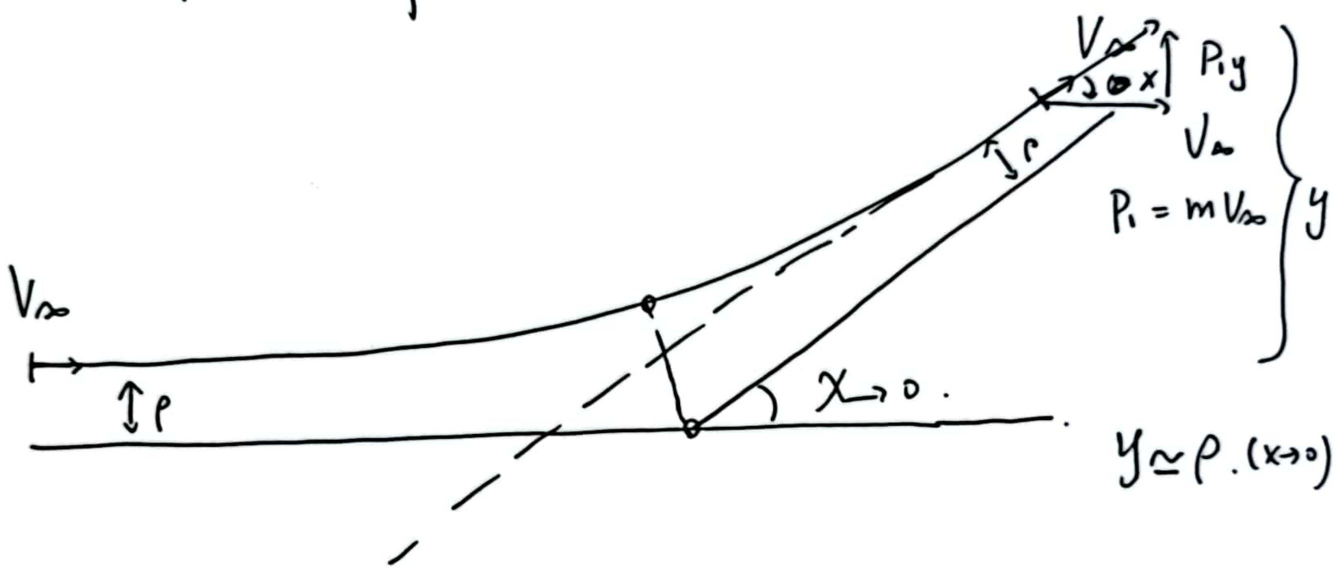
$n=3$



小角度散射.



$$\frac{\rho}{\sin(\chi)}\left|\frac{d\rho}{d\sin(\chi)}\right| \xrightarrow{\chi\to 0\ (\rho\to\frac{1}{\chi})} \frac{1}{\chi^4}$$



$y \simeq \rho.\ (\chi\to 0)$

$$\sin\chi = \frac{P'_{1y}}{P'_1} \simeq \frac{P'_{1y}}{mV_\infty} \simeq \chi$$

$$\dot{P}_{1y} = F_y \Rightarrow P'_{1y} = \int_{-\infty}^{+\infty} F_y \, d\tau$$

$$F_y = -\frac{\partial U}{\partial y} = -\frac{\partial U}{\partial r}\frac{\partial r}{\partial y} = -\frac{\partial U}{\partial r}\left(\frac{y}{r}\right)$$

$$\sim -\frac{\partial U}{\partial r}\left(\frac{\rho}{y}\right)$$

χ很小时. $y \sim \rho$. (几乎不变)

$$P_{iy} = \int_{-\infty}^{+\infty} F_y \, dt$$

$$= \int_{-\infty}^{+\infty} - \frac{\partial U}{\partial r} \left(\frac{y}{r} \right) \frac{dx}{V_\infty}$$

$$= -\frac{\rho}{V_\infty} \int_{\rho}^{+\infty} \left(\frac{\partial U}{\partial r} \frac{dx}{\partial r} \right)$$

$$= -\frac{2\rho}{mV_\infty^2} \int_{0}^{+\infty} \frac{dU}{dr} \frac{dr}{\sqrt{r^2 - \rho^2}}$$

(算出来放主页上).

$$U(r) = \frac{a}{\sqrt{y^2 + x^2}} = \frac{a}{\sqrt{\rho^2 + x^2}}$$

$$r = \sqrt{x^2 + \rho^2}$$

$$U(r) = \begin{cases} 0 . & |r| > R \\ U . & |r| < R \end{cases}$$

与光学折射相似：

2020年9月16日. 第3周第2次课;

作业: 9.3.37

9.3.51