Computers & Graphics (2018)

Contents lists available at ScienceDirect

Computers & Graphics

journal homepage: www.elsevier.com/locate/cag

Sphere-based cut construction for planar parameterizations

Shuangming Chai, Xiao-Ming Fu*, Xin Hu, Yang Yang, Ligang Liu

University of Science and Technology of China, 230026 Hefei, China

ARTICLE INFO

Article history:
Received May 9, 2018

Keywords: Planar parameterizations, Sphere-based cut construction, Hierarchical clustering, Low isometric distortion

ABSTRACT

We present a novel algorithm to compute high-quality cuts for generating low isometric distortion planar parameterizations. Based on the observation that the conformal spherical and planar parameterizations have similar distortion distributions at the extrusive areas that lead to high isometric distortions, our method utilizes the spherical parameterization of the input mesh to guide the cut construction. After parameterizing the input mesh onto a sphere as conformal as possible, a hierarchical clustering of the divisive type is conducted on the sphere to find the high isometric distortion regions, where high isometric distortion may also be introduced in the planar parameterization and which are connected to define the cuts. Compared with previous methods, this approach can generate better cuts, resulting in lower isometric distortions. We demonstrate the efficacy and practical robustness of our method on a data set of over 5000 meshes, which are parameterized with low isometric distortion by two existing parameterization approaches.

1. Introduction

Computing inversion-free planar parameterizations with low isometric distortion is fundamental in many computer graphics and geometry processing applications, such as texture mapping [2, 3], remeshing [4, 5] and inter-surface mapping [6, 7]. The low isometric distortion property requires that the parameterized mesh should preserve isometry to its original shape as much as possible.

Since good cuts are able to improve the quality of parameterizations, while inappropriate cuts tend to introduce unacceptable effects, cutting closed triangular meshes to disk topology is an important procedure for generating low distortion parameterizations. In the context of this paper, a cut is considered to be *good* when it satisfies the following requirements as much as possible: (1) the resulting parameterizations contain low isometric distortions; (2) the cuts are feature-aligned, which implies visual beauty in terms of high-quality texturing; (3) the cuts are short.

Many attempts have been proposed to construct cuts in a way that satisfies above requirements. Gaussian curvature [8, 9] is often used to detect the potential regions that are connected via a minimal spanning tree (MST) to define the resulting cuts. Since these curvature-based methods do not consider the distortion directly, they may ignore some regions with low average Gaussian curvature but small neighborhoods where the curvature is locally high, which introduces high isometric distortion. Gu et al. [10] iteratively parameterize the surface to the plane and find the shortest cut from the vertex with maximal distortion to the boundary. This alternate algorithm stops if the parameterzation distortion increases or the maximal distortion appears on the boundary. However, they may also ignore some interior high-distortion regions, since the highest distortion appears on the existing cut in the last iteration (see the comparison in Fig. 15). Recently, Poranne et al. [3] proposed a method to simultaneously optimize cut length and distortion. However, their

*Corresponding author
e-mail: `fuxm@ustc.edu.cn` (Xiao-Ming Fu)

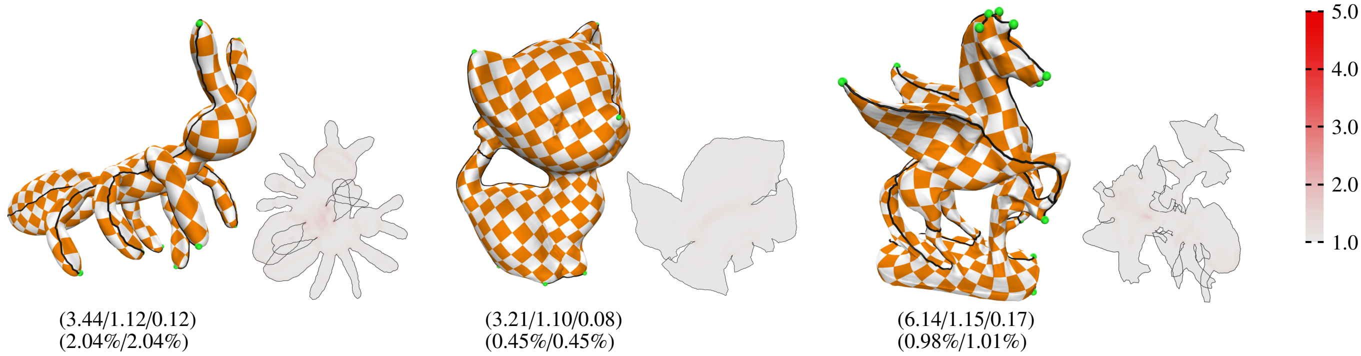


Fig. 1. Planar parameterizations of three models. Our constructed cuts are shown by black lines, and the feature points of our clustered regions are shown by green points. The parameterizations are generated by AQP [1]. The isometric distortion metric (which is defined in Section 4) of each triangle is colored with white being optimal, and the models are textured by a checkerboard image. The first line of the text below the mesh indicates the maximum, average and standard deviation of the isometric distortion over all triangles, and the second line indicates the proportions of edge number and edge length of the cut.

cuts require additional user manipulations to be finalized and are often not feature-aligned (see the comparison in Fig. 16).

In this paper, we propose a sphere-based cut construction method to automatically compute high-quality cuts for the purpose of generating low isometric distortion planar parameterizations. Our idea comes from a simple fact that the high isometric distortion mainly appears at the extrusive regions when a mesh is parameterized onto a constant curvature domain (e.g. a sphere or plane) as conformal as possible. In other words, the high isometric distortion regions from an as-conformal-as-possible spherical parameterization are also the places that may cause high isometric distortion in the planar parameterization. Therefore, we first parameterize an input mesh onto a sphere as conformal as possible, then use a divisive hierarchical clustering algorithm to detect high isometric distortion regions, and finally connect these regions by constructing the MST on that sphere and map back to the input mesh to determine the cut. The MST construction on the sphere results in a good balance between the feature-aligned and short requirements of the good cuts. This is our default choice in our experiments. Besides, we also provide another choice for users to construct cuts. If users want shorter cuts and do not care about the feature-aligned property (i.e., emphasizing requirement (3) and discarding (2)), they could connect the feature regions on the original meshes, which results in a slightly higher distortion but much shorter cuts (see the differences in Fig. 6, 14, 15, 16).

We demonstrate the efficacy of our method on a data set containing more than 5000 complex models, which are parameterized using SA [11] and AQP [1]. Fig. 1 shows planar parameterizations of three models. Compared with state-of-the-art methods, our method constructs better cuts and achieves lower isometric distortion with stronger practical robustness.

2. Related work

Cut construction. There have been many algorithms trying to find optimal cuts in order to parameterize a closed mesh to the plane with low isometric distortion. By using curvature information, some previous methods define the cuts by detecting and connecting regions with high curvature [8, 9, 12, 13], since these regions are often considered as the reason why high isometric distortion of planar parameterizations appears. However, since the curvature information does not directly reveal the distortion distribution, some important places may be ignored, which still causes high isometric distortion parameterizations. Gu et al. [10] alternately parameterize surface meshes onto the plane and find the shortest path from the vertex with maximum distortion to the existing boundary. This alternating method directly uses a measure of distortion to guide the cut construction. However, since in the iterative process, the maximum distortion region may appear on the boundary, the algorithm will stop and some interior high distortion regions will be ignored. The Autocuts method [3] simultaneously optimizes cut length and isometric distortion. Some important parameters in their method should be adjusted by users to fine-tune the cuts. Our method utilizes a sphere to automatically facilitate the cut construction, in which the isometric distortion is directly utilized to find the cuts. We evaluate our constructed cuts via computing low isometric distortion parameterizations for more than 5000 meshes.

There have been many other methods that divide the input meshes into multi charts [14, 2, 15, 16, 17, 18]. Although these methods can produce results with very low isometric distortion, the cut lengths are usually very long. Some applications, such as surface correspondence [6] and remeshing [4], prefer short cuts and a single chart. We only consider how to cut surface meshes to one chart using feature-aligned and short cuts in this paper.

Some recent parameterization literature tends to focus on quadrilateral remeshing applications [5, 19]. These methods require extra integer constraints on the cut, which brings higher isometric distortion. We focus on a different goal, that is to find good cuts for generating low isometric distortion planar parameterizations.

Low isometric distortion parameterizations. Numerous methods for mesh parameterization have been developed (cf. the surveys in [20, 21]). To achieve the inversion-free property, many techniques have been proposed, such as Tutte embedding and its variants [22, 23, 24, 25], maintenance of inversion-free property [26, 27, 28, 29, 30, 1, 31, 32, 33, 34, 35], computation of bounded distortion parameterizations [36, 6, 37],

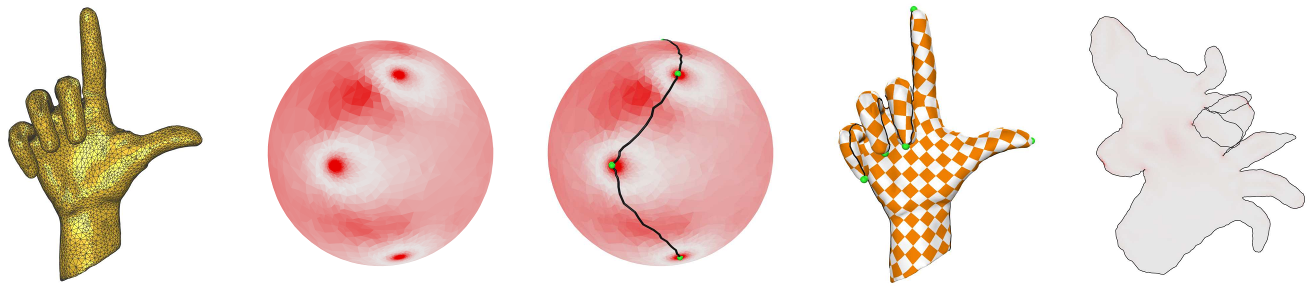


Fig. 2. The pipeline of our method. From an input triangular mesh (a), we first compute an as-conformal-as-possible spherical parameterization (b), then find feature points and a cut path on the sphere (c), finally cut the mesh (d) and parameterize to the plane using AQP [1] (e).

or relying on different representations [38, 11]. The former two kinds of methods guarantee inversion-free parameterizations for arbitrary triangular meshes of disk topology. Methods based on Tutte's embedding have the guarantee of bijective property, but contain very high isometric distortion. Maintenance based methods use them as initializations and minimize the isometric distortion while keeping the inversion-free property by using different techniques, such as barrier functions [27, 28] and explicit checks in combination with line search [30, 29, 1, 31, 32, 33, 34]. In this paper, we utilize Simplex Assembly (SA) [11] and AQP [1] to compute low isometric distortion parameterizations to demonstrate the high quality of our constructed cuts.

3. Method

3.1. Overview

The input closed triangular mesh $\mathcal{M}$ is an oriented 2-manifold that consists of N facets $\mathcal{F} = \{\mathbf{f}_i, i = 1, ..., N\}$ and has no boundary. First, we consider a parameterization f_s that maps the mesh $\mathcal{M}$ to a unit sphere $\mathcal{S}$ as conformal as possible. Then, a hierarchical clustering method is proposed to find a set of vertices $\mathcal{P}$, where high isometric distortion is potential to appear in the subsequent planar parameterization. Next, we connect these vertices to construct the cut path C. Finally, we cut the mesh $\mathcal{M}$ and generate a disk topology mesh $\mathcal{M}^c$, which can be parameterized to the plane using existing parameterization methods. The workflow of our algorithm is shown in Fig. 2.

3.2. Spherical parameterization

Our first step is to compute a spherical parameterization f_s as conformal as possible, which is used to guide the cut construction. The quality of a parameterization is usually evaluated by analyzing the distortion, which can be measured in different forms according to different objectives. In the area of parameterization, the conformal, areal and isometric distortions are commonly considered, and MIPS and its variants [26, 30] are often used to define them. When computing the distortion of the parameterization, we use a planar triangle to approximate a spherical triangle, and measure the distortion of the mapping from the source triangle $\mathbf{f}_i$ on $\mathcal{M}$ to the planar triangle. We define this map as an affine transformation g_i. After defining a local coordinate system, each affine map g_i can be represented by $g_i(\mathbf{x}) = J_i\mathbf{x} + \mathbf{b}_i$ where J_i is the Jacobian of g_i and $\mathbf{b}_i$ is the

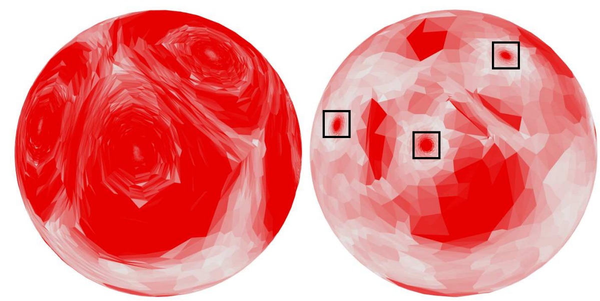


Fig. 3. Comparison between the spherical parameterizations of the Spider model (Fig. 12) using AIAP (a) and ACAP (b). Note that the clustering effect of the ACAP parameterization is more significant (black boxes in (b)) than the AIAP parameterization.

translation vector. For the sake of simplicity, we first assume that $\mathcal{M}$ is a closed and orientable surface having genus zero, while high-genus surfaces are discussed in Section 3.5.

The conformal distortion of an affine map g_i is defined by:

$$d_i^{\mathrm{conf}} = \frac{1}{2}\left(\frac{\sigma_1}{\sigma_2} + \frac{\sigma_2}{\sigma_1}\right) = \frac{1}{2}\frac{\|J_i\|_F^2}{\det J_i} \tag{1}$$

where σ_1 and σ_2 are the singular values of J_i, and $\|\cdot\|_F$ indicates the Frobenius matrix norm. The areal distortion is defined by:

$$d_i^{\mathrm{area}} = \frac{1}{2}\left(\det J_i + (\det J_i)^{-1}\right). \tag{2}$$

Because a map is isometric if and only if it is conformal and equiareal, the isometric distortion can be defined by linearly combining the above two distortions.

$$d_i^{\mathrm{iso}} = \alpha d_i^{\mathrm{conf}} + (1-\alpha) d_i^{\mathrm{area}}. \tag{3}$$

The parameter α is usually set to be 0.5 [30].

To compute an as-conformal-as-possible (ACAP) parameterization, we use a modified AHSP method [39], which has strong practical robustness. The original AHSP method only optimizes the isometric distortion, which leads to an as-isometric-as-possible (AIAP) spherical parameterization. So we modified the AHSP method by using its results as the initialization, and minimzing the conformal distortion $\sum_{\mathbf{f}_i \in \mathcal{F}} d_i^{\mathrm{conf}}$. Since the AIAP parameterization tries to distribute the isometric distortion evenly, the clustering effect is not significant (Fig. 3 (a)). By contrast, an ACAP spherical parameterization exhibits the

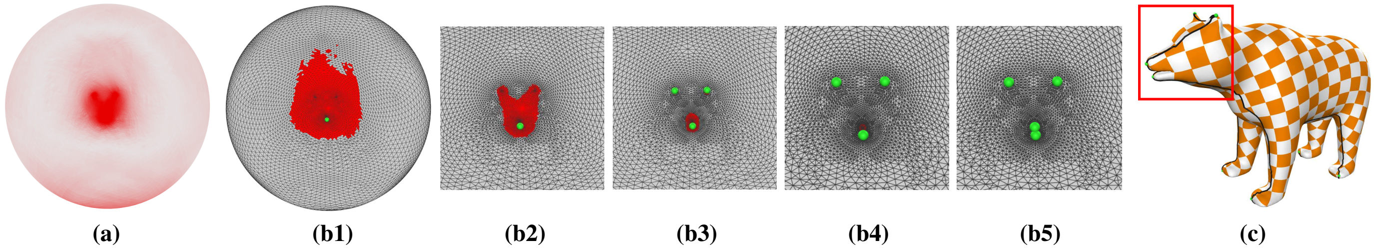


Fig. 4. An example of hierarchical clustering. From an ACAP spherical parameterization, we compute and colorize the isometric distortion (a). After filtering half of the triangles, we get the high distortion region $\mathcal{R}_1$ (red triangles in (b1)). Then one vertex on the triangle that has the largest distortion is marked in green. After 5 iterations (b1) – (b5), we finally obtain the feature points (green points in the red box of (c)).

clustering effect when measuring the isometric distortion, i.e. the clustered locations have high isometric distortion. In fact, after the triangles of extrusive regions in $\mathcal{M}$ are mapped onto the sphere $\mathcal{S}$ as conformal as possible, their angles are usually preserved, and they shrink together due to high areal (or isometric) distortion (Fig. 3 (b)). The shrinking effects on the sphere are very similar with those on the plane when considering ACAP parameterizations. Thus, the clustered regions of the ACAP parameterization f_s are also the potential reasons for introducing high isometric distortion to the planar parameterizations.

3.3. *Cut construction*

After computing an ACAP parameterization, we can construct the cut by first finding a set of points $\mathcal{P}$ and then connecting them to become a cut C. The points in $\mathcal{P}$ can be recognized as the extremal points of the clustering regions in terms of the isometric distortion.

Hierarchical clustering. To find the extremal point set $\mathcal{P}$, we introduce a hierarchical clustering method of the divisive type over $\mathcal{S}$. In this paper, two triangles are said to be connected if they share one common vertex. We denote a set of regions that composed of connected triangles at the kth iteration as $\mathcal{R}_k$ and a set of feature triangles as $\mathcal{P}_t$. Our clustering method first finds the high distortion region set $\mathcal{R}_k$, by which $\mathcal{P}_t$ is then determined, and we select one vertex from each feature triangle as the feature point. Fig. 4 shows the process of our hierarchical clustering, and the algorithm is described as follows:

1. Compute the ACAP f_s and record the isometric distortion (Equation (3)) on each facet.
2. Initialize $k := 0$, the initial region set $\mathcal{R}_0$ only contains one region $\mathcal{F}$, i.e. all of the triangles, and the feature triangle set $\mathcal{P}_t := \emptyset$.
3. In each region in $\mathcal{R}_k$, find a triangle with maximal isometric distortion. If this triangle is not in $\mathcal{P}_t$, then add it to the set $\mathcal{P}_t$.
4. For each region in $\mathcal{R}_k$, first find the median of isometric distortions on its triangles, then filter half of the triangles whose isometric distortions are below the median, and finally group the connected triangles into several new isolated regions. For each new region, if the number of triangles is larger than a threshold N_R then add this region to $\mathcal{R}_{k+1}$.
5. If $\mathcal{R}_{k+1} = \emptyset$, stop the algorithm and output $\mathcal{P}$ by randomly selecting one vertex from each triangle in $\mathcal{P}_t$. Otherwise, let $k := k + 1$ and go to Step 3.

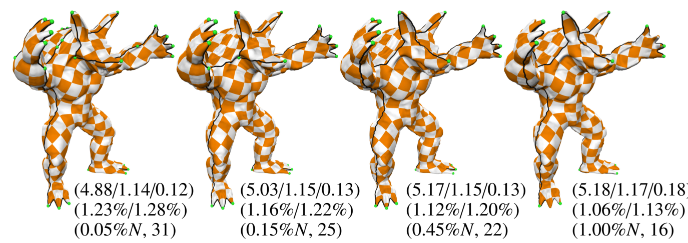


Fig. 5. Cut construction using different N_R. The last line of the text is the value of N_R and the number of feature points. From the comparison, we observe that larger N_R results in less feature points, higher distortion and shorter cut length.

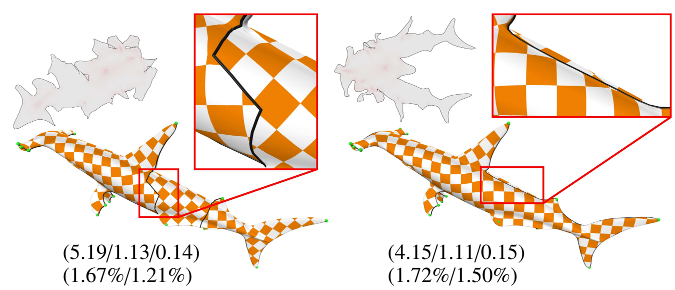


Fig. 6. Construct cuts on the original mesh $\mathcal{M}$ (left) and the sphere $\mathcal{S}$ (right). Note that if we find cuts directly on the original mesh, the cuts do not tend to be along the feature lines and can result in higher distortions. The parameterizations are both generated by AQP [1].

Note that the threshold N_R is important in the above algorithm. Large N_R may make some feature points lost and small one may result in some redundant feature points. Thus, we choose an appropriate value and set $N_R = 0.15\%N$ in all our experiments. For comparison, we also test several different values of N_R and show in Fig. 5. From the results, we can find that although the numbers of feature points are varients, the final distortions are changed slightly. This implies that our method is not sensitive to this parameter.

Minimal spanning tree on $\mathcal{S}$. Next, we connect the obtained feature points $\mathcal{P}$ by constructing a minimal spanning tree on the sphere $\mathcal{S}$ and then map back to the original mesh $\mathcal{M}$ to define the cut. The detailed steps are as follows:

1. For each pair of the feature points, we compute the shortest path on the sphere $\mathcal{S}$ between them.
2. Construct a complete graph $\mathcal{G}$ by treating all of the feature points as nodes, and the edge weight is the length of the shortest path.

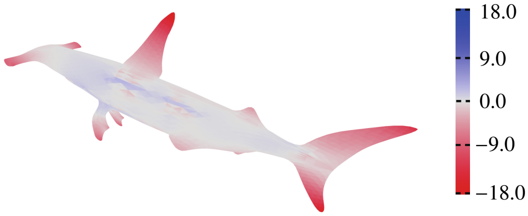


Fig. 7. We color the triangles in terms of $\ln(|\widehat{\mathbf{f}}_i| \cdot |\mathbf{f}_i|^{-1})$, where $\widehat{\mathbf{f}}_i$ refers to the image of a triangle $\mathbf{f}_i$ and $|\cdot|$ denotes the area of a triangle.

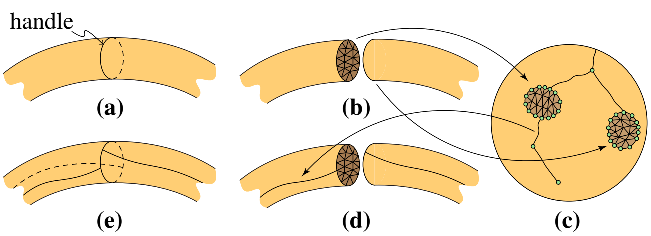


Fig. 8. Cut construction on high genus models. We first find its handles (a), cut the mesh along these handles, and fill the holes (b). After computing a conformal spherical parameterization (c), we can use our method to find feature points (green), and connect the feature points, as well as the handle vertices, by a spanning tree. Then, a cut can be generated by our method (d). The final cut is the union of its handles and the spanning tree (e).

3. Compute the minimal spanning tree $\mathcal{T}$ of $\mathcal{G}$, and the resulting edges on $\mathcal{T}$ form the cut C.

Cut construction on $\mathcal{S}$ vs. on $\mathcal{M}$. We construct the cut on the sphere $\mathcal{S}$, but a more intuitive way is to construct the cut on the original mesh $\mathcal{M}$. In Fig. 6, we compare these two ways in the cut construction. From the results, the cut on the sphere $\mathcal{S}$ finally tends to follow the feature regions of $\mathcal{M}$, which results in lower isometric distortion in the final parameterization. Since the feature regions of a mesh (e.g. an sharp edge of a cube) are a kind of bulges or depressions, the ACAP parameterization on the sphere $\mathcal{S}$ produces high area distortion and shortens the edge lengths (see Fig. 7). The shortened edges along the feature lines have high possibility to be added into the shortest paths and the minimal spanning tree $\mathcal{T}$. Thus, the cut path C tend to be along these feature lines. In fact, there is a trade-off between low distortion and short cut. In our experiments, the cuts are constructed on $\mathcal{S}$ by default. Nevertheless, if the user prefers shorter cuts while sacrificing its low distortion and visual effect, we also provide an optional construction on the original surface mesh $\mathcal{M}$.

3.4. High genus meshes

For an input high genus model (Fig. 8), we first use [40] to find its handles, cut the mesh along these handles, and fill the holes to generate a new genus-zero surface. Note that each vertex on a handle have two copies, and both of them have to be connected to the final cut. We then compute a spherical parameterization and find feature points using the hierarchical clustering method. When connecting the feature points, we mark the copied vertices on the handles as feature points, which should be connected to other feature points or handles. Next, the cut $\widehat{C}$ can be generated by our cut construction method. Note that

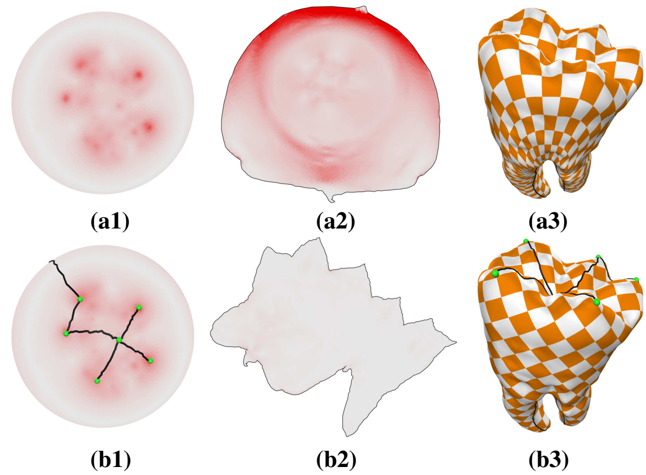


Fig. 9. Comparison of different filter choices. If we filter the triangles by average (top row), some feature points will be missed (a1) and the final parameterization (a2), (a3) has large distortion. However, using the median to filter half of the triangles, we can get six more feature points and a better cut (b1), and the final parameterization has lower distortion (b2), (b3). The parameterizations are generated by AQP [1].

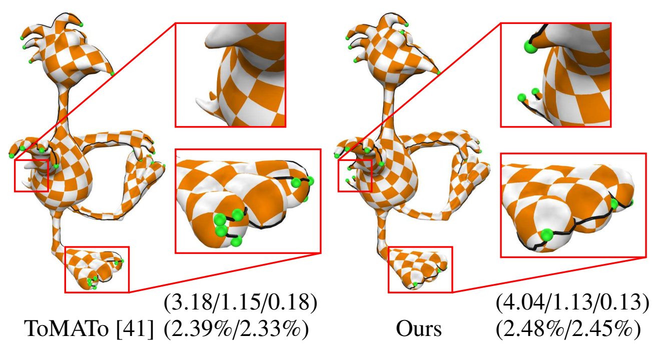


Fig. 10. Comparison with persistence-based clustering [41]. Both of the results have 25 feature points. Note that in their result, some feature points are not found, while some regions contain too many feature points (red box zoomins). Thus, our result has higher maximum distortion and lower average distortion.

these handle points are only marked as feature points used in our algorithm, but not feature points in terms of high distortion. The final cut C of the input model is the union of its handles and $\widehat{C}$. Some results of high genus models are shown in Fig. 1 and 13.

3.5. Discussions

Median of the distortions. We use the median of isometric distortion to filter out half of the triangles in one region, which brings about fast convergence of clustering and insensitivity to the quality of the spherical parameterization f_s. However, there are some other possible choices, such as the average distortion, i.e. in each iteration, the triangles whose distortion measures are less than the average distortion of the region will be filtered out. Fig. 9 shows a comparison with the average distortion. We observe that the average distortion can be easily affected by the maximum and minimum values, so the clustering will be susceptible to the quality of f_s.

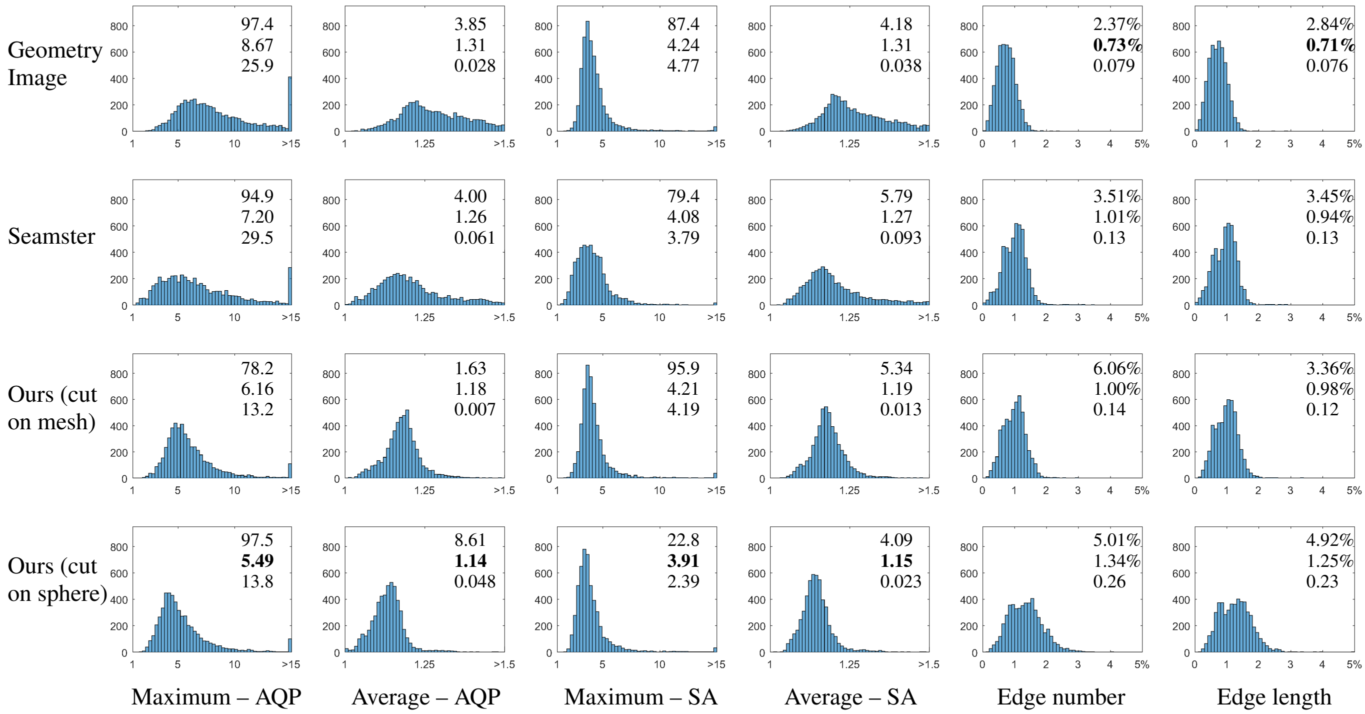


Fig. 11. The histograms on about 5000 models using different cut construction method. We tested four cut construction method from top to bottom: Geometry Image [10], Seamster [9], Ours (cut on mesh), Ours (cut on sphere). We use AQP [1] and SA [11] to compute planar parameterizations and compute maximum and average isometric distortion for each model, and compute the proportions of edge lengths and edge numbers of the cut over the total edge lengths and edge numbers, respectively. In addition, for each histogram, we also compute maximum, average and standard deviation texted at the right top corner on each figure.

Persistence-based clustering. There is another well-developed clustering method, called persistence-based clustering [42, 41], which can be used to find feature points. To compare with this method, we use the isometric distortion of the conformal spherical parameterization as the density function, and compute clusters using ToMATo method [41]. For fair comparison, we set the number of clusters to be the same as the number of feature points found by our method. From the results, we can find that in their method some feature regions are lost, while many feature points are found in one feature region (Fig. 10).

One or no feature point. If the vertices of a model are lying on a sphere, which has no distortion after parameterizing to the sphere, then there is no feature point found. We simply select the two vertices farthest away from each other, and find a shortest path as the cut. If only one vertex is not lying on a sphere, our method will find only one feature point. For this case, we also find a vertex farthest away from this feature point, and construct the cut by connecting them.

4. Experiments and comparisons

We have applied our sphere-based cut construction method to various complex meshes. In this section, we first introduce the constructed data set, and then show comparisons with three previous methods.

Isometric distortion metric. We utilize the isometric distortion metric in [30] to measure the quality of planar parameterizations. For each triangle $\mathbf{f}_i$, its isometric distortion metric is defined as $\delta_i^{iso} := \max\{\sigma_1^p, 1/\sigma_1^p, \sigma_2^p, 1/\sigma_2^p\}$ where σ_1^p and σ_2^p are the singular values of J_i^p that is the Jacobian of the planar parameterization at $\mathbf{f}_i$. We report the maximum, average and standard deviation of the isometric distortion metrics over all the triangles, which are shown in the first line below the mesh. The color bar used for coloring the isometric distortion is the same with the one in Fig. 1.

Data set. We test the practical robustness and effectiveness of our method on a data set containing more than 5000 models, and we show the distribution of the maximun and average isometric distortions of the parameterizations generated by AQP [1] and SA [11] in histograms (last row in Fig. 11). The average distortions vary from 1.0 to 1.5 and concentrate on around 1.15 for both methods, meaning that our generated cuts have very high quality. We also compute the number and length of cut edges over those of all edges for each model, plot the proportion of the edge number and length of cuts in the last row of Fig. 11, and show in the second line below all results. The average proportion of the edge number and length of cuts are 1.34% and 1.26% respectively. We show 13 genus zero meshes in Fig. 12 and 8 high genus meshes in Fig. 13. The rendered cuts in these figures indicate the approximately feature-aligned property of our method. We will release the complete data set in future.

Timings. Our experiments were conducted on a desktop computer with Intel Core i7-4790 processor and 16GB memory. For the ant model in Fig. 1 having 9501 vertices, it takes 12.49s to

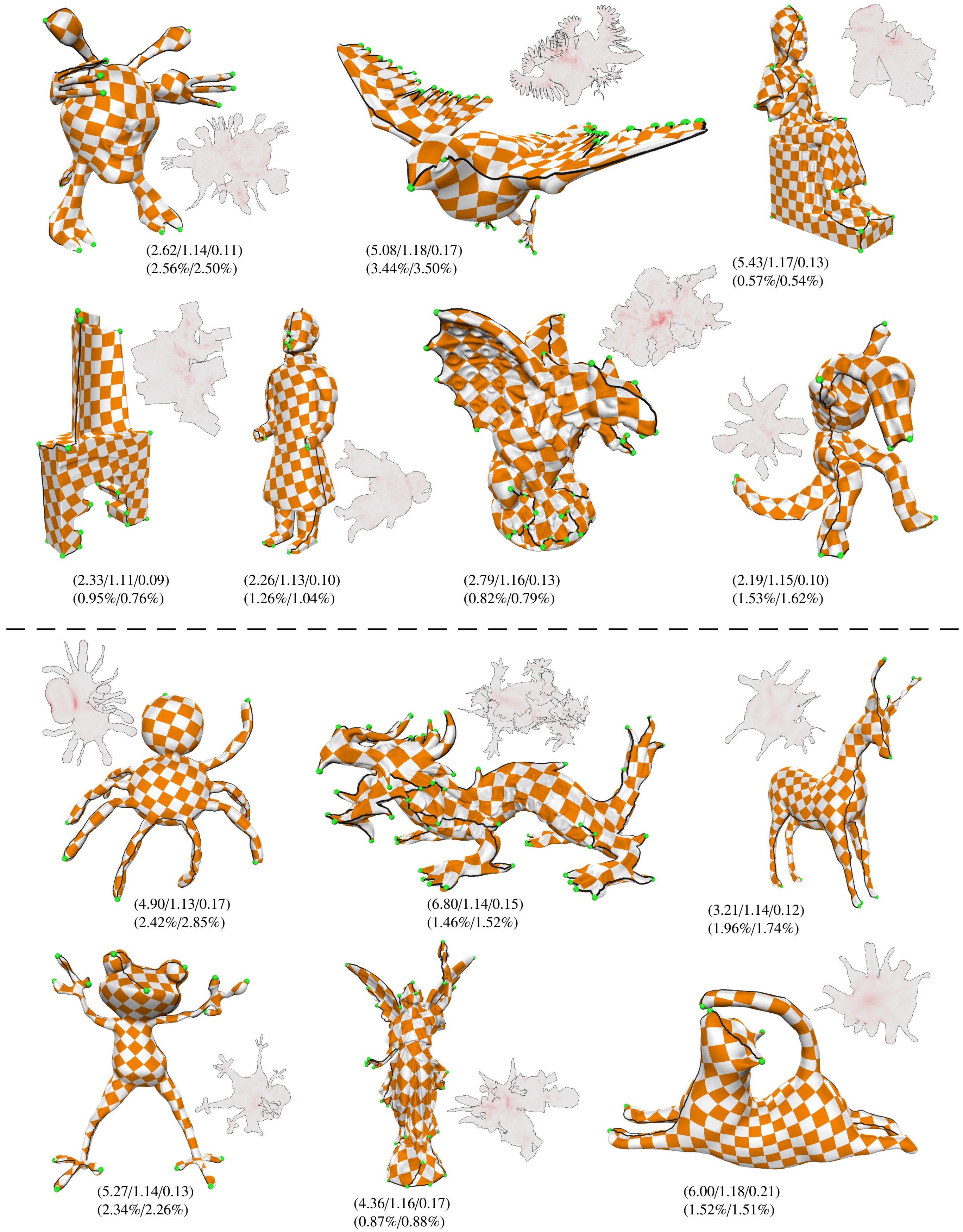


Fig. 12. Gallery of our cut construction results. The parameterization results above and below the dashed line are generated by SA [11] and AQP [1], respectively.

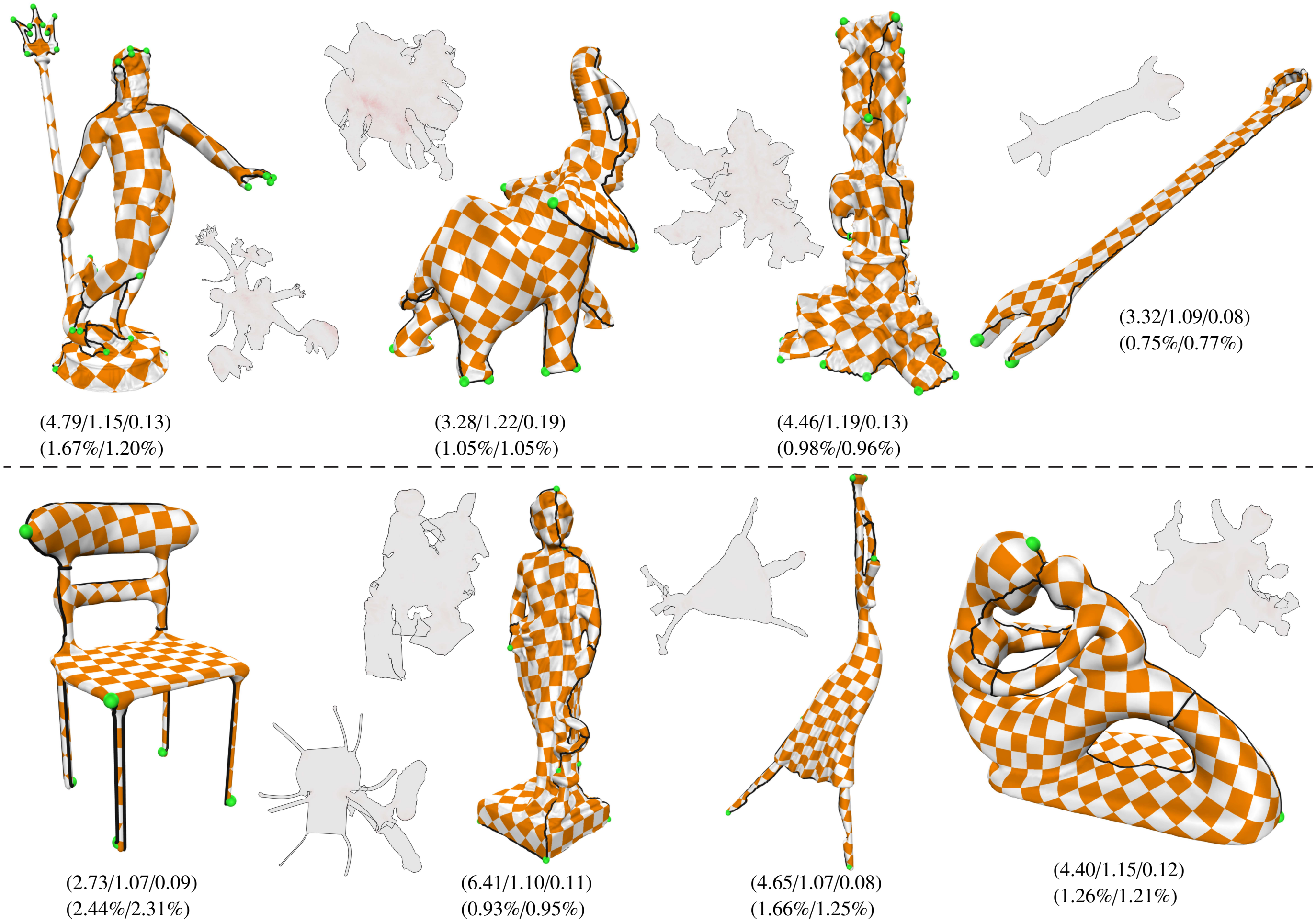


Fig. 13. Results of high genus models. The parameterization results above and below the dashed line are generated by SA [11] and AQP [1], respectively.

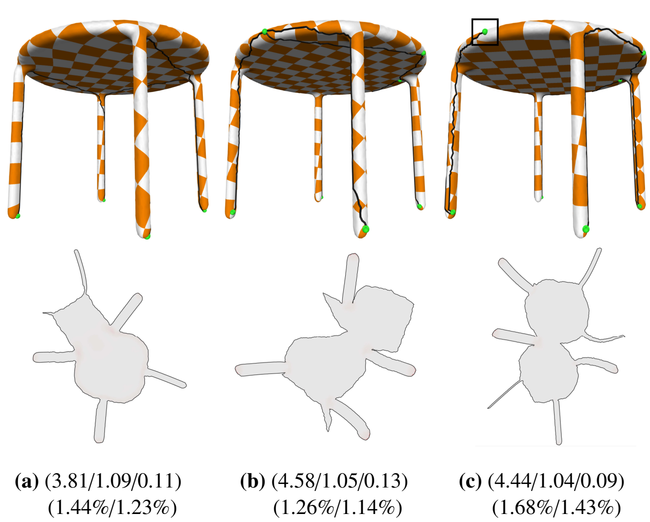


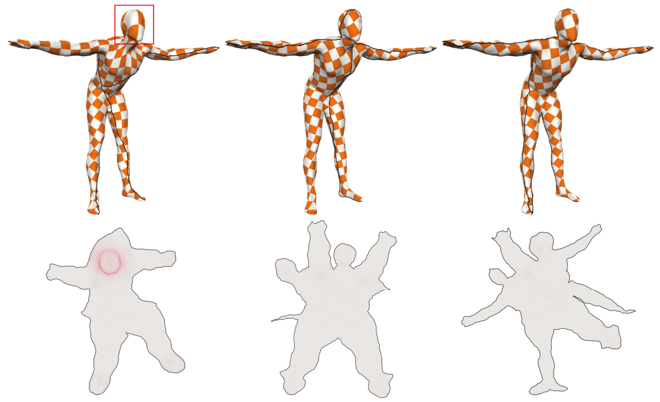
Fig. 14. Comparison with [9]. We use [9] (a) and our method (b & c) to generate feature points (green). In our method, we can construct the cut on the original mesh (b), or on the sphere (c). The parameterizations are all generated by [1]. From the results, we observe that the method [9] do not find some feature points at the edge of the desk (black box), therefore producing higher distortion.

generate the conformal spherical parameterzation and 0.03s to construct the cut. For the dragon model in the center of Fig. 12 having 52513 vertices, it takes 94.97s to generate the spherical parameterization and 0.20s to construct the cut.

4.1. Comparisons

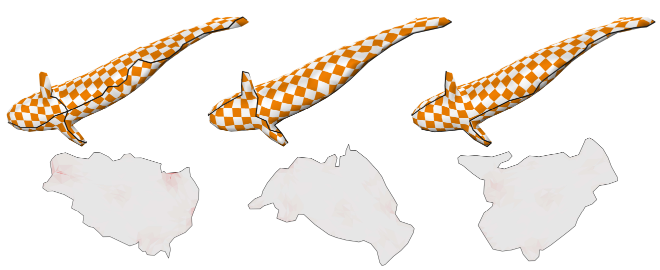
We compare our method with three existing methods: Seamster [9], Geometry Image [10] and Autocuts [3]. We also test over 5000 models using Seamster and Geometry Image, and show the histograms in Fig. 11. As an optional choice, we can directly find cut on the original mesh (the third row in Fig. 11), which lead to higher distortion and shorter cut length than our cuts. The histograms indicate that our method with cutting on the sphere performs the best with respect to the maximum and average isometric distortion, but has the longest cut length and number of cut edges. Comparing with Geometry Image, our cut length is longer than theirs, but their distortion is too high to be accepted by subsequent applications. Comparing with Seamster, our method with cutting on the mesh (the third row) generates cuts with comparable cut length but much lower distortion. Since Autocuts need user interactions to avoid severe global overlap, we only give one example for comparison.

Comparison with Seamster [9]. The method [9] is based on the fact that high curvature regions are the potential places that may introduce high isometric distortion to the final planar parameterizations. However, some flat areas can still cause high isometric



(a) (4.07/1.28/0.37) (0.97%/1.09%) **(b)** (4.78/1.13/0.12) (1.51%/1.40%) **(c)** (4.03/1.11/0.11) (1.89%/1.74%)

Fig. 15. Comparison with [10]. We use [10] (a) and our method (b & c) to find the cuts (black lines). We provide both our resulting cuts constructed on the original mesh (b) and on the sphere (c). The parameterizations are all generated by AQP [1]. Note that in the result of [10] some important regions (like the head in the red box) cannot be found, while our method can find such regions and reduce the isometric distortion.



(a) (7.71/1.27/0.52) (4.20%/4.06%) **(b)** (1.95/1.16/0.14) (3.85%/4.01%) **(c)** (1.95/1.15/0.14) (4.46%/4.38%)

Fig. 16. Comparison with [3] (a). In our method, we construct the cut on either the original mesh (b) or the sphere (c). Note that our method can generate high quality cuts automatically, while their method need additional manipulations.

distortion. (Fig. 14). Furthermore, how to choose appropriate thresholds of the required parameters for all models is also a challenging problem. If the parameters are not well tuned, some redundant feature points can be found or some important ones can be ignored. If we construct the cut on the sphere (Fig. 14 (c)), the cut will be a little longer. However, we provide an optional method, i.e., constructing the cut on the original mesh, which generates shorter cuts and comparable isometric distortion (Fig. 14 (b)).

Comparison with Geometry Image [10]. We show a comparison to [10] in Fig. 15. From the comparisons, we observe that their cuts are not along the feature line because of the alternate scheme, and some high distortion area is not found (the head of the man), leading to high distortion results. This is actually because in the last iteration step, the highest distortion region is on the boundary so that the algorithm ignores some interior high distortion regions. Although the cuts generated by our method are longer and contains more edges, our method generates more reasonable cuts and lower isometric distortion parameterization.

Comparison with Autocuts [3]. The most recent method Autocuts [3] optimizes cuts and distortion simultaneously, but since the cuts is not along the feature line and the parameterizations often have self-overlaps, this method requires additional user manipulations to finalize the results. Moreover, this method becomes slow if the input mesh has a high resolution and easily gets stuck into a local minimum without user interactive operations. Compared to their method, our method is fully automatic and more efficient for the high resolution input mesh. For a low resolution mesh (252 vertices and 500 triangles) as shown in Fig. 16, they generate reasonable results, but their parameters need to be adjusted carefully.

5. Conclusion and discussions

We present a sphere-based method to construct high-quality paths used to cut the input meshes to be disk topology ones so that they can be parameterized to the plane with low isometric distortions. Our method exhibits better quality and stronger practical robustness than previously existing methods. However, a few limitations do exist, and we would like to address them in future work.

Coupled planar parameterizations. Currently, we only consider how to construct a cut, meaning that the two stages of the planar parameterization, cut construction and parameterization computation, are separate. However, they are coupled in fact. In our method, we use a spherical domain to realize a weak version of the coupling, which produces satisfactory results for all of the testing examples, but we still believe that direct combination of the two stages in a global optimization can generate better results. Recently, the Autocuts method [3] gives a first step of optimizing cut and distortion simultaneously. In future, we would like to extend our method by taking the balance between cut length and distortion into consideration.

Domains other than the sphere. There is another bunch of methods [24, 25] that can generate seamless parameterizations. However, these seamless domains are not suitable for our problem because they require some given landmarks and cuts for computing the embeddings. The different landmarks induce that the generated embeddings have various isometric distortions, leading to inconspicuous clustering effect. To construct cuts for high genus meshes, other specific seamless domains can be utilized, e.g., the complex plane for genus-1 surfaces and hyperbolic disks for higher-genus models [43]. But, we observe that these parameterizations do not exhibit clear clustering phenomena to guide the feature detection and cut construction. Thus, we have to design other customized algorithms for each kind of these special parameterizations, which is beyond the scope of this article. We would like to explore more flexible seamless domains in future.

Conformal spherical parameterizations. The conformal parameterizations from $\mathcal{M}$ to the unit sphere are not unique. Different ACAP spherical parameterizations induce different feature points and cuts. Thus, our method has no theoretical guarantee to generate high-quality cuts. However, the high-quality

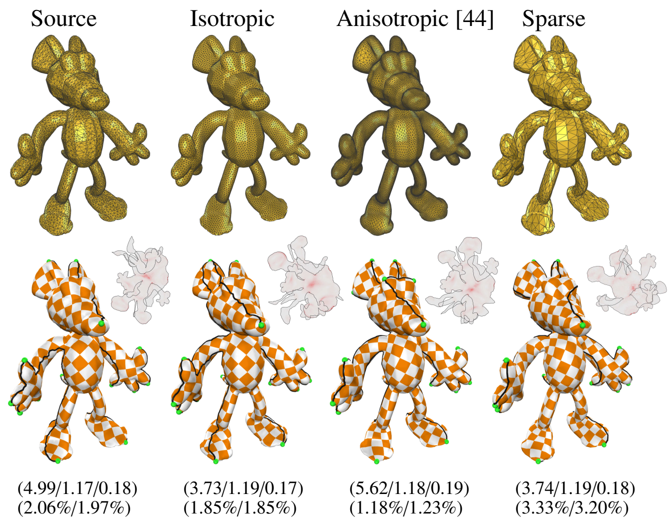


Fig. 17. Parameterization results for one shape with different tessellations. The parameterizations are generated by AQP [1].

cuts generated for over 5000 models, which have been parameterized with low isometric distortions, indicates that our method has strong practical robustness. In future, we want to design theoretical guaranteed conformal parameterizations that exhibit clustering effect.

Tessellations. We get different cuts for different tessellations of one shape, because different ACAP spherical parameterizations are generated and our cut paths are formed by the mesh edges (see Fig. 17). Nevertheless, the final parameterizations of different tessellations still have low distortion. This demonstrates that the cuts generated by our method are effective and reasonable.

Symmetric objects. Symmetry is also important for good cuts and fast processing. Since it is nontrivial to detect symmetry for general models, we do not consider symmetric models, so the cuts generated by our method are not symmtric in general. However, taking the symmetry into account is a meaningful problem, and we leave this as future work.

Acknowledgments

The authors would like to thank Shahar Z. Kovalsky for sharing their code and the anonymous reviewers for their constructive suggestions and comments. This work is supported by the National Natural Science Foundation of China (61672482, 61672481), the One Hundred Talent Project of the Chinese Academy of Sciences, the Fundamental Research Funds for the Central Universities (WK0010460006), and the Anhui Provincial Natural Science Foundation (1808085QF208).

References

[1] Kovalsky, SZ, Galun, M, Lipman, Y. Accelerated Quadratic Proxy for Geometric Optimization. ACM Trans Graph (SIGGRAPH) 2016;35(4):134:1–134:11.
[2] Lévy, B, Petitjean, S, Ray, N, Maillot, J. Least squares conformal maps for automatic texture atlas generation. ACM Trans Graph (SIGGRAPH) 2002;21(3):362–371.
[3] Poranne, R, Tarini, M, Huber, S, Panozzo, D, Sorkine-Hornung, O. Autocuts: Simultaneous Distortion and Cut Optimization for UV Mapping. ACM Trans Graph (SIGGRAPH ASIA) 2017;36(6).
[4] Alliez, P, Meyer, M, Desbrun, M. Interactive Geometry Remeshing. ACM Trans Graph (SIGGRAPH) 2002;21(3):347–354.
[5] Bommes, D, Zimmer, H, Kobbelt, L. Mixed-integer quadrangulation. ACM Trans Graph (SIGGRAPH) 2009;28(3):77:1–77:10.
[6] Aigerman, N, Poranne, R, Lipman, Y. Lifted bijections for low distortion surface mappings. ACM Trans Graph (SIGGRAPH) 2014;33(4):69:1–69:12.
[7] Aigerman, N, Poranne, R, Lipman, Y. Seamless Surface Mappings. ACM Trans Graph (SIGGRAPH) 2015;34(4):72:1–72:13.
[8] Sheffer, A. Spanning tree seams for reducing parameterization distortion of triangulated surfaces. In: Shape Modeling International. 2002, p. 61–66.
[9] Sheffer, A, Hart, JC. Seamster: inconspicuous low-distortion texture seam layout. In: Proceedings of the conference on Visualization'02. 2002, p. 291–298.
[10] Gu, X, Gortler, SJ, Hoppe, H. Geometry Images. ACM Trans Graph (SIGGRAPH) 2002;21(3):355–361.
[11] Fu, XM, Liu, Y. Computing Inversion-Free Mappings by Simplex Assembly. ACM Trans Graph (SIGGRAPH ASIA) 2016;35(6).
[12] Ben-Chen, M, Gotsman, C, Bunin, G. Conformal flattening by curvature prescription and metric scaling. In: Comput. Graph. Forum; vol. 27. 2008, p. 449–458.
[13] Myles, A, Zorin, D. Global Parametrization by Incremental Flattening. ACM Trans Graph (SIGGRAPH) 2012;31(4):109:1–109:11.
[14] Sorkine, O, Cohen-Or, D, Goldenthal, R, Lischinski, D. Bounded-distortion piecewise mesh parameterization. In: Proceedings of the Conference on Visualization '02. 2002, p. 355–362.
[15] Sander, PV, Wood, ZJ, Gortler, SJ, Snyder, J, Hoppe, H. Multi-chart Geometry Images. In: Proceedings of the 2003 Eurographics/ACM SIGGRAPH Symposium on Geometry Processing. 2003, p. 146–155.
[16] Zhou, K, Synder, J, Guo, B, Shum, HY. Iso-charts: Stretch-driven Mesh Parameterization Using Spectral Analysis. In: Proceedings of the 2004 Eurographics/ACM SIGGRAPH Symposium on Geometry Processing. 2004, p. 45–54.
[17] Julius, D, Kraevoy, V, Sheffer, A. D-Charts: Quasi-Developable Mesh Segmentation. In: Comput. Graph. Forum; vol. 24. 2005, p. 581–590.
[18] Zhang, E, Mischaikow, K, Turk, G. Feature-based surface parameterization and texture mapping. ACM Trans Graph 2005;24(1):1–27.
[19] Bommes, D, Lévy, B, Pietroni, N, Puppo, E, Silva, C, Tarini, M, et al. Quad-mesh generation and processing: a survey. Comput Graph Forum 2013;32(6):51–76.
[20] Floater, MS, Hormann, K. Surface parameterization: a tutorial and survey. In: In Advances in Multiresolution for Geometric Modelling. Springer; 2005, p. 157–186.
[21] Sheffer, A, Praun, E, Rose, K. Mesh parameterization methods and their applications. Found Trends Comput Graph Vis 2006;2(2):105–171.
[22] Tutte, WT. How to draw a graph. In: Proceedings of the London Mathematical Society; vol. 13. 1963, p. 747–767.
[23] Floater, MS. One-to-one piecewise linear mappings over triangulations. Math Comput 2003;72:685–696.
[24] Aigerman, N, Lipman, Y. Orbifold Tutte Embeddings. ACM Trans Graph (SIGGRAPH ASIA) 2015;34(6):190:1–190:12.
[25] Aigerman, N, Lipman, Y. Hyperbolic Orbifold Tutte Embeddings. ACM Trans Graph (SIGGRAPH ASIA) 2016;35(6):190:1–190:12.
[26] Hormann, K, Greiner, G. MIPS: An efficient global parametrization method. In: Curve and Surface Design: Saint-Malo 1999. Vanderbilt University Press; 2000, p. 153–162.
[27] Schüller, C, Kavan, L, Panozzo, D, Sorkine-Hornung, O. Locally injective mappings. Comput Graph Forum (SGP) 2013;32(5):125–135.
[28] Jin, Y, Huang, J, Tong, R. Remeshing-assisted optimization for locally injective mappings. Comput Graph Forum (SGP) 2014;33(5):269–279.
[29] Smith, J, Schaefer, S. Bijective Parameterization with Free Boundaries. ACM Trans Graph (SIGGRAPH) 2015;34(4):70:1–70:9.
[30] Fu, XM, Liu, Y, Guo, B. Computing locally injective mappings by advanced MIPS. ACM Trans Graph (SIGGRAPH) 2015;34(4):71:1–71:12.
[31] Rabinovich, M, Poranne, R, Panozzo, D, Sorkine-Hornung, O. Scalable Locally Injective Mappings. ACM Trans Graph 2017;36(2):16:1–16:16.
[32] Shtengel, A, Poranne, R, Sorkine-Hornung, O, Kovalsky, S, Lipman, Y. Geometric Optimization via Composite Majorization. ACM Trans

Graph (SIGGRAPH) 2017;36(4):38:1–38:11.
[33] Claici, S, Bessmeltsev, M, Schaefer, S, Solomon, J. Isometry-Aware Preconditioning for Mesh Parameterization. In: Comput. Graph. Forum (SGP); vol. 36. 2017, p. 37–47.
[34] Jiang, Z, Schaefer, S, Panozzo, D. Simplicial Complex Augmentation Framework for Bijective Maps. ACM Trans Graph (SIGGRAPH ASIA) 2017;36(6):186:1–186:9.
[35] Liu, L, Ye, C, Ni, R, Fu, XM. Progressive Parameterizations. ACM Trans Graph (SIGGRAPH) 2018;37(4).
[36] Lipman, Y. Bounded distortion mapping spaces for triangular meshes. ACM Trans Graph (SIGGRAPH) 2012;31(4):108:1–108:13.
[37] Kovalsky, SZ, Aigerman, N, Basri, R, Lipman, Y. Large-scale bounded distortion mappings. ACM Trans Graph (SIGGRAPH ASIA) 2015;34(6):191:1–191:10.
[38] Sheffer, A, Lévy, B, Mogilnitsky, M, Bogomyakov, A. ABF++: fast and robust angle based flattening. ACM Trans Graph 2005;24(2):311–330.
[39] Hu, X, Fu, XM, Liu, L. Advanced Hierarchical Spherical Parameterizations. IEEE T Vis Comput Gr 2017;PP.
[40] Dey, TK, Fan, F, Wang, Y. An Efficient Computation of Handle and Tunnel Loops via Reeb Graphs. ACM Trans Graph (SIGGRAPH) 2013;32(4):32:1–32:10.
[41] Chazal, F, Guibas, LJ, Oudot, SY, Skraba, P. Persistence-based clustering in riemannian manifolds. Journal of the ACM (JACM) 2013;60(6):41.
[42] Edelsbrunner, H, Letscher, D, Zomorodian, A. Topological persistence and simplification. In: Foundations of Computer Science, 2000. Proceedings. 41st Annual Symposium on. IEEE; 2000, p. 454–463.
[43] Li, X, Bao, Y, Guo, X, Jin, M, Gu, X, Qin, H. Globally optimal surface mapping for surfaces with arbitrary topology. IEEE T Vis Comput Gr 2008;14(4):805–819.
[44] Fu, XM, Liu, Y, Snyder, J, Guo, B. Anisotropic simplicial meshing using local convex functions. ACM Trans Graph (SIGGRAPH ASIA) 2014;33(6):182:1–182:11.