Homotopy equivalences and recollements induced by cotorsion triples

Ren Wei

Let $(\mathcal{X}, \mathcal{Z}, \mathcal{Y})$ be a cotorsion triple in an abelian category $\mathcal{A}$. By studying abelian models structures on the category of complexes $\mathrm{Ch}(\mathcal{A})$ induced by $(\mathcal{X}, \mathcal{Z}, \mathcal{Y})$, we obtain equivalences and recollements of homotopy categories. As an application, we prove that for a left-Gorenstein ring, there exist equivalences $\mathrm{D}^{sing}_{\mathcal{G}(\mathcal{PI})}(R) := \mathrm{K}_{ex}(\mathcal{GP}) \cong \mathrm{K}_{ex}(\mathcal{GI})$ and $\mathrm{D}_{\mathcal{G}(\mathcal{PI})}(R) := \mathrm{K}(\mathcal{GP}) \cong \mathrm{K}(\mathcal{GI})$, between the homotopy categories of (exact) complexes of Gorenstein projective and Gorenstein injective modules, and then there is a recollement

$$\mathrm{D}^{sing}_{\mathcal{G}(\mathcal{PI})}(R) \rightleftharpoons \mathrm{D}_{\mathcal{G}(\mathcal{PI})}(R) \rightleftharpoons \mathrm{D}(R).$$

Moreover, these restrict to equivalences and a recollement for homotopy categories with respect to projective and injective modules.