Workshop on Weighted Projective Spaces and Representation Theory

March 4-10, 2016, USTC, Hefei

**March 5 (Saturday)**

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| **Time** | **Talk** |
| 9:20-9:30 | Welcome Speech (by the dean, Professor Jiayu Li) |
| 9:30-10:30 | Helmut Lenzing (I) |
| 10:30-10:50 | Coffee Break  |
| 10:50-11:50 | Osamu Iyama (I) |
| 11:50-14:00 | Lunch Break |
| 14:00-15:00 | David Ploog (I) |
| 15:00-15:20 | Coffee Break  |
| 15:20-16:10 | Hiroyuki Minamoto |

**March 6 (Sunday)**

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| **Time** | **Talk** |
| 9:30-10:30 | Osamu Iyama (II) |
| 10:30-10:50 | Coffee Break  |
| 10:50-11:50 | Helmut Lenzing (II) |
| 11:50-14:00 | Lunch Break |
| 14:00-15:00 | David Ploog (II) |
| 15:00-15:20 | Coffee Break  |
| 15:20-16:20 | Osamu Iyama (III) |

**March7 (Monday)**

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| **Time** | **Talk** |
| 9:30-10:30 | David Ploog (III) |
| 10:30-10:50 | Coffee Break  |
| 10:50-11:50 | Helmut Lenzing (III) |
| 11:50-14:00 | Lunch Break |
|  | Free Afternoon |

**March 8 (Tuesday)**

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| **Time** | **Chair** | **Talk** |
| 9:00-9:50 | Claus M. Ringel | Bangming Deng |
| 10:00-10:50 |  | Joseph Karmazyn |
| 10:50-11:10 |  | Coffee Break  |
| 11:10-12:00 |  | Lutz Hille |
| 12:00-14:00 |  | Lunch Break |
| 14:00-14:50 | Henning Krause | Ryo Takahashi |
| 15:00-15:50 |  | Tokuji Araya |
| 15:50-16:10 |  | Coffee Break  |
| 16:10-17:00 |  | Dirk Kussin |

**March 9 (Wednesday)**

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| **Time** | **Chair** | **Talk** |
| 9:00-9:50 |  Bangming Deng | Hagen Meltzer |
| 10:00-10:50 |  | Shiquan Ruan |
| 10:50-11:10 |  | Coffee Break  |
| 11:10-12:00 |  | Henning Krause |
| 12:00-14:00 |  | Lunch Break |
|  |  | Informal discussion |

Lecture Room: 1611, the math building

Contact：Ms. Yafei Liu, Office 1512

**Abstracts**

**Osamu Iyama Series**

Weighted projective lines introduced by Geigle-Lenzing in 1987 is one of the fundamental objects in representation theory. The aim of my lecture series is to introduce Geigle-Lenzing complete intersection rings as a higher dimensional generalization of weighted projective lines, based on a joint work with Herschend, Minamoto and Oppermann. They are a class of commutative Gorenstein rings R graded by abelian groups L of rank one. We will study the stable category of L-graded Cohen-Macaulay R-modules as well as the derived category of coherent sheaves, and we show that these triangulated categories have tilting objects. As an application we discuss when these categories are d-Cohen-Macaulay finite as well as d-vector bundle finite in the sense of higher dimensional Auslander-Reiten theory.

**Helmut Lenzing Series**

Lecture 1. Main properties of weighted projective lines

The base field in this talk is algebraically closed of arbitrary characteristic. I am going to introduce weighted projective lines by means of the root construction, starting from the ordinary projective line. (This is with an eye to generalizations to higher dimensions.) I will then review (mostly without proofs) the main features of weighted projective lines, as there are:

1. Graded factoriality of the graded coordinate algebra

2. Serre duality with an explicitly known dualizing sheaf

3. Canonical tilting sheaf with endomorphism ring a canonical algebra of Ringel

4. The link to singularity theory (Kleinian, Fuchsian and triangle singularities)

I will further discuss the roles three closely related invariants for a weighted projective line:

1. Euler characteristic

2. dualizing element and its degree

3. Gorenstein number.

Though being closely related, the mathematical function of these numbers turns out to be quite different.

Lecture 2. Weighted projective lines and Riemann surfaces

The base field in this talk is the field of complex numbers. We are reviewing the relationship between weighted projective lines and smooth projective curves, equivalently, compact Riemann surfaces. There are three cases to consider:

***1. Euler characteristic >0.*** Here, each weighted projective line with three weights is isomorphic to a quotient of the ordinary projective line (or Riemann sphere) by a polyhedral group, i.e. a finite subgroup of PSL(2,C).

***2. Euler characteristic =0.*** Here, each weighted projective line (then of tubular type) arises as the quotient of a smooth elliptic curve by a cyclic group of order 2, 3, 4, or 6. This uses unpublished work with Meltzer from 2004, alternatively the detailed account by Chen-Chen-Zhou (2015).

***3. Euler characteristic <0.***  I will present the Bundgaard-Nielsen-Fox-Chau theorem giving a positive answer to the Nielsen conjecture, which can be expressed as follows: Each weighted projective line (actually each weighted smooth projective curve) arises as a quotient $M/G$, where $M$ is a compact Riemann surface and $G$ is a finite subgroup of $Aut(M)$. I will explain the strategy of proof, and present a couple of examples, some of them being infinite families.

Lecture 3. Weighted projective spaces

The base field in this talk is again the field of complex numbers, though most results will not depend on this restriction. While there is universal agreement what a weighted projective line is, there are many - different - competing concepts for the concept of a weighted projective space in dimension greater or equal two. The aim of this talk is to give a short history -  starting in 1975 -  of the development of these concepts and to show how they compare (mostly by means of group actions yielding branched coverings). The talk will include at least one application of weighted projective planes.

**David Ploog Series**

First talk: We consider finite group actions on topological spaces, manifolds and varieties. If the action is free, then the naive quotient is good enough; otherwise, various notions have been introduced to obtain good notions ("V-manifold", "orbifold", "(algebraic/topological/etc.) stack").

Second talk: We introduce G-linearisations and use them to get a different description of sheaves on a global quotient stack. As examples, we will look at the "points" of stacky curves.

Third talk: We will see that there is a natural, geometric semi-orthogonal decomposition of the derived category of a stacky curve. If the underlying curve has genus 0, then we obtain a full exceptional collection.

**Tokuji Araya, Thick subcategories over graded simple singularities of type A.**

Abstract: Takahashi classified the thick subcategories of the stable category of maximal Cohen-Macaulay modules over a hypersurface local ring. By his classification, we can see that if the base ring has a simple singularity, then the thick subcategories are trivial. On the other hand, if the base ring is graded, then there exist non-trivial thick subcategories. In this talk, we will classify the thick subcategories of the stable category of graded maximal Cohen-Macaulay modules over a graded hypersurface which has a simple singularity of type A.

**Bangming Deng: Hall algebras of cyclic quivers and $q$-deformed Fock spaces**

Abstract:  By extending a construction of Varagnolo and Vasserot, we define a module structure on the $q$-deformed Fock space $\mathcal F$ over the double Ringel-Hall algebra $D(Q)$ of a cyclic quiver $Q$ and then show that $\mathcal F$ is isomorphic to the basic representation of $D(Q)$. We also interpret the canonical basis of $\mathcal F$ defined by Leclerc and Thibon in terms of certain monomial basis elements in $D(Q)$. This is joint work with Jie Xiao.

**Lutz Hille: Tilting line bundles on certain toric varieties associated to some Fano weighted projective spaces**

Abstract：We consider a toric variety X associated to a reflexive simplex. Such a toric variety admits a categorical resolution by a certain stack, that is a weighted projective space P in the sense of Baer. It also admits an ordinary resolution as an algebraic variety Y (in this case even a toric one) under some additional conditions.

The variety X can also be defined in terms of a weight vector q and a finite group H acting. The stack P always admits a tilting bundle consisting of line bundles. Using the morphisms P ---> X and Y ---> X we can define two associated tilting bundles on Y. We claim, both of these tilting bundles have the same endomorphism algebra, which is the weighted Beilinson algebra associated to the weight vector q. As a consequence, we get varieties Y that has (in general) several tilting bundles with the same endomorphism algebra A(q), that is also the endomorphism algebra of the tilting bundle on the corresponding stack P.

In this talk we illustrate the construction with several examples and discuss also further consequences. Moreover, the result is motivated by several recent conjectures about the global dimension of the endomorphism algebra of a tilting bundle.

**Joseph Karmazyn: Total spaces of line bundles and deformations of surface singularities.**

Abstract:  The total space of the canonical bundle on a weighted projective line of Dynkin type provides a categorical resolution of the corresponding Kleinian surface singularity. This method of resolving the singularity has the dual advantages that the method can be extended to resolve more general rational surface singularities by swapping the canonical bundle for another line bundle and that in type A the tilting bundle on the weighted projective line lifts to a tilting bundle on the total space.

I will discuss how these advantages can be extended from type A surface singularities to their deformations. I will show that a simultaneous resolution of any type A rational surface singularity can be realised as a total space associated to a type A configuration of projective lines and that a tilting bundle on the projective lines lifts to a tilting bundle on the total space.

**Henning Krause, Auslander-Reiten duality revisited**

Abstract: The cornerstones of Auslander-Reiten duality are the Auslander-Reiten formulas which involve the dual of the transpose of a finitely presented module. We generalise this in several directions by giving a construction of the Auslander-Reiten translate for Grothendieck abelian categories. It is somewhat surprising that the general approach sheds new light on the dual of transpose for finitely presented modules over artin algebras.

**Dirk Kussin, Real tubular and elliptic curves**

Abstract: We present general Riemann-Hurwitz formulae for weighted noncommutative smooth projective curves and two-dimensional noncommutative compact orbifolds. These formulae express how the orbifold Euler characteristic can be computed from the underlying nonweighted curve (resp. manifold) and additional numerical data like the weights and the so-called tau-multiplicities, which is a local

datum of the Auslander-Reiten translation. These results lead to (resp. shed new light on) the classification of real elliptic (resp. tubular) curves. For example, we compute in each case the precise value of the fractional Calabi-Yau dimension. We also present

a not-commutative Fourier-Mukai partner of the Klein bottle.

**Hiroyuki Minamoto: Tilting bundles on (Anti-)Fano algebras.**

Abstract: This talk is based on a joint work with Osamu Iyama. We will introduce and discuss tilting bundles on Fano algebras. An algebra which is derived equivalent to a Fano algebra is not necessarily Fano. So we would like to know that under what kind of derived equivalence Fano-ness is preserved. One answer is that of induced by tilting bundles. More precisely, we show that the endomorphism algebra of a tilting bundle on a Fano algebra is always Fano. Herschend-Iyama-Oppermann showed that n-APR tilting preserves n-representation infiniteness. Recently, Mizuno-Yamaura generalize this result. They showed that for 0 < m < n+1, m-APR tilting preserves n-represntation infiniteness. In case of n-APR tilting modules, it is easy to see that the noncommutative projective scheme of the n+1 preprojective algebra is preserved under derived equivalence induced by n-APR tilting module. It was not verified that m-APR tilting preserves the noncommutative projective scheme for 0 < m < n. Since m-APR tilting module is an example of a tilting bundle. One of our aim to study tilting bundles on Fano algebras was to solve this problem. We can prove that the derived equivalence induced by a tilting bundle on a Fano algebra preserves the noncommutative projective scheme. We see by example that derived equivalence tilting module which is not tilting bundle does not necessarily preserve Fano-ness. A celebrated result by Bondal-Orlov states that (anti-)Fano variety determined by its derived category. Our example shows that Bondal-Orlov theorem is not true for non-commutative projective schemes.

**Hagen Meltzer, Exceptional objects for nilpotent operators with invariant subspaces**

Abstract: This is a report on joint work with Piotr Dowbor and Markus Schmidmeier. We study the categories of ﬁnite dimensional vector spaces endowed with subspaces and linear operators of nilpotency degree n leaving the subspaces invariant. For n = 6 this category is of tubular type. We show that all exceptional objects in this situation can be described by matrices having as coefficients only 0 and 1. The problem is related to that of vector bundles on a weighted projective line and modules over a certain tubular algebra. One of the tools to prove the main result is the fact that each exceptional vector bundle of higher rank on a tubular weighted projective line is isomorphic to the middle term of an exact sequence where the starting and the end term form an exceptional pair of bundles of smaller rank. We further use the decomposition of modules over linear quivers in order to study the so called ”0-1 interval property”.

**Shiquan Ruan: Generic Ringel--Hall algebras and Hall polynomials for tame type**

Abstract: This is joint work with Bangming Deng. In the talk we will show that Hall polynomial exists for each triple of decomposition sequences which parameterize isocalsses of coherent sheaves of a domestic weighted projective line X over finite fields. These polynomials are then used to define the generic Ringel—Hall algebra of X and its composition subalgebra as well as their Drinfeld doubles. Combining this construction with a result of Cramer, we show that Hall polynomials exist for tame quivers, which not only refines a result of Hubery, but also confirm a conjecture of Berenstein and Greenstein.

**Ryo Takahashi, Thick subcategories of derived categories of isolated singularities**

Abstract: A thick subcategory of a triangulated category is by definition a full triangulated subcategory closed under direct summands. Classifying thick subcategories of triangulated categories is one of the most important subjects shared by ring theory, algebraic topology, algebraic geometry and representation theory. In this talk, we study classifying thick subcategories of the bounded derived category of finitely generated modules over a commutative noetherian local ring with an isolated singularity.