

# SINGULARITY CATEGORIES OF THE CLUSTER-TILTED ALGEBRAS OF FINITE TYPE

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joint work with Xinhong Chen and Shengfei Geng

ABSTRACT. We use the stable categories of some selfinjective algebras to describe the singularity categories of the cluster-tilted algebras of Dynkin type. Furthermore, in this way, we settle the problem of singularity equivalence classification of the cluster-tilted algebra of type  $A$ ,  $D$  and  $E$  respectively.

As a generalization of the structure of the cluster-tilted algebra of type  $A$ , we define the *gluing Nakayama algebra*, and use technique of recollement to describe its singularity category clearly.

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