A Crash Course on Leavitt path algebras

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ABSTRACT

Leavitt path algebras can be regarded as the algebraic counterparts of the graph C^*-algebras, the descendants from the algebras investigated by J. Cuntz in [11], which have been the focus of much attention from the analysts in the last two decades (see [13] for an overview of the subject). Moreover, Leavitt path algebras can also be viewed as a broad generalization of the algebras constructed by W. G. Leavitt in [12] to produce rings without the Invariant Basis Number property (i.e., whose modules have bases with different cardinals).

The Leavitt path algebra $L_K(E)$ was introduced in 2004 in the papers [1] and [4]. $L_K(E)$ was first defined for a row-finite graph E (countable graph such that every vertex emits only a finite number of edges) and a field K. Although their history is very recent, a flurry of activity has followed since then. The main directions of research include: characterization of algebraic properties of a Leavitt path algebra $L_K(E)$ in terms of graph-theoretic properties of E; study of the modules over $L_K(E)$; computation of various substructures (such as the Jacobson radical, the center, the socle and the singular ideal); investigation of the relationship and connections with $C^*(E)$ and general C^*-algebras; classification programs; study of the K-theory; and generalization of the constructions and results first from row-finite to countable graphs and finally, from countable to completely arbitrary graphs. For examples of each of these directions see for instance [10] and the references therein.

In this Crash Course on Leavitt path algebras we will make an introduction to this subject and will focus on several papers ranging from basic to more advanced ones. Concretely, we will start with the definition of the Leavitt path algebra of a row-finite graph E, pointing out that several well-known algebras (such as matrices $\mathbb{M}_n(K)$, Laurent polynomials $K[x, x^{-1}]$, classical Leavitt algebras $L(1, n)$, the Toeplitz algebra $\mathcal{T}$, and combinations of all those) can be realized as the Leavitt path algebra of a graph. Then we will state their basic properties and mention the main lines of research in this subject.

We will also cover, in more or less detail, some of the results and papers in this theory. Concretely, as the first step, we will consider the characterization of simple Leavitt path algebras [1] as well as the purely infinite simple ones [2]. Those were two of the first papers appearing in the literature. We will also give the theorem characterizing the finite-dimensional $L_K(E)$, by giving their ring-theoretic structure [3].

Other topics that we will cover are the computation of the socle [7], the study of the prime spectrum and the primitive ideals as done in [8], as well as the characterization of the weakly-regular and self-injective Leavitt path algebras performed in [9]. If time

allows, we will also make a quick overview of other works such as [6], where a computation of the center of $L_K(E)$ is given, or [5], where the Kumjian-Pask algebras of higher rank graphs (a generalization of Leavitt path algebras for higher rank graphs) is introduced.

This 10-hours course can be reasonably followed by graduate students who have taken at least a course on noncommutative ring theory.

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