SINGULAR EQUIVALENCES
INDUCED BY HOMOLOGICAL EPIMORPHISMS

XIAO-WU CHEN

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Abstract. We prove that a certain homological epimorphism between two algebras induces a triangle equivalence between their singularity categories. Applying the result to a construction of matrix algebras, we describe the singularity categories of some non-Gorenstein algebras.

1. Introduction

Let A be a finite dimensional algebra over a field k. Denote by A-mod the category of finitely generated left A-modules and by D^b(A-mod) the bounded derived category. Following [20], the singularity category D_{sg}(A) of A is the Verdier quotient triangulated category of D^b(A-mod) with respect to the full subcategory formed by perfect complexes; see also [4,5,11,16,17,23].

The singularity category measures the homological singularity of an algebra: the algebra A has finite global dimension if and only if its singularity category D_{sg}(A) is trivial. Meanwhile, the singularity category captures the stable homological features of an algebra ([6]).

A fundamental result of Buchweitz and Happel states that for a Gorenstein algebra A, the singularity category D_{sg}(A) is triangle equivalent to the stable category of (maximal) Cohen-Macaulay A-modules ([6,14]), where the latter category is related to Tate cohomology theory ([2,6]). This result specializes Rickard’s result ([23]) on self-injective algebras. For non-Gorenstein algebras, not much is known about their singularity categories ([7,9]).

The following concepts might be useful in the study of singularity categories. Two algebras A and B are said to be singularly equivalent provided that there is a triangle equivalence between D_{sg}(A) and D_{sg}(B). Such an equivalence is called a singular equivalence; compare [21]. In this case, if A is non-Gorenstein and B is Gorenstein, then Buchweitz-Happel's theorem applies to give a description of D_{sg}(A) in terms of (maximal) Cohen-Macaulay B-modules. We observe that a derived equivalence of two algebras, that is, a triangle equivalence between their bounded derived categories, naturally induces a singular equivalence. The converse is not true in general.

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Let $A$ be an algebra and let $J \subseteq A$ be a two-sided ideal. Following [22], we call $J$ a homological ideal provided that the canonical map $A \to A/J$ is a homological epimorphism ([12]), meaning that the naturally induced functor $\text{D}^b(A/J\text{-mod}) \to \text{D}^b(A\text{-mod})$ is fully faithful.

The main observation we make is as follows.

**Theorem.** Let $A$ be a finite dimensional $k$-algebra and let $J \subseteq A$ be a homological ideal which has finite projective dimension as an $A$-$A$-bimodule. Then there is a singular equivalence between $A$ and $A/J$.

This paper is structured as follows. In Section 2, we recall some ingredients and then prove the Theorem. In Section 3, we apply the Theorem to a construction of matrix algebras and then describe the singularity categories of some non-Gorenstein algebras. In particular, we give two examples which extend in different manners an example considered by Happel in [14].

2. Proof of the Theorem

We will present the proof of the Theorem in this section. Before that, we recall from [25] and [15] some results on triangulated categories and derived categories.

Let $T$ be a triangulated category. We will denote its translation functor by $[1]$. For a triangulated subcategory $N$, we denote by $T/N$ the Verdier quotient triangulated category. The quotient functor $q: T \to T/N$ has the property that $q(X) \simeq 0$ if and only if $X$ is a direct summand of an object in $N$. In particular, if $N$ is a thick subcategory, that is, it is closed under direct summands, we have that $\text{Ker } q = N$. Here, for a triangle functor $F$, $\text{Ker } F$ denotes its essential kernel, that is, the (thick) triangulated subcategory consisting of objects on which $F$ vanishes.

The following result is well known.

**Lemma 2.1.** Let $F: T \to T'$ be a triangle functor which allows a fully faithful right adjoint $G$. Then $F$ induces uniquely a triangle equivalence $T/\text{Ker } F \simeq T'$.

**Proof.** The existence of the induced functor follows from the universal property of the quotient functor. The result is a triangulated version of [11, Proposition I. 1.3]. For details, see [5, Propositions 1.5 and 1.6].

Let $F: T \to T'$ be a triangle functor. Assume that $N \subseteq T$ and $N' \subseteq T'$ are triangulated subcategories satisfying $FN \subseteq N'$. Then there is a uniquely induced triangle functor $\bar{F}: T/N \to T'/N'$.

**Lemma 2.2 ([20, Lemma 1.2]).** Let $F: T \to T'$ be a triangle functor which has a right adjoint $G$. Assume that $N \subseteq T$ and $N' \subseteq T'$ are triangulated subcategories satisfying the fact that $FN \subseteq N'$ and $GN' \subseteq N$. Then the induced functor $\bar{F}: T/N \to T'/N'$ has a right adjoint $\bar{G}$. Moreover, if $G$ is fully faithful, so is $\bar{G}$.

**Proof.** The unit and counit of $(F,G)$ induce uniquely two natural transformations $\text{Id}_{T/N} \to GF$ and $FG \to \text{Id}_{T'/N'}$, which are the corresponding unit and counit of the adjoint pair $(\bar{F},\bar{G})$; consult [19, Chapter IV, Section 1, Theorem 2(v)]. Note that the fully-faithfulness of $G$ is equivalent to the fact that the counit of $(F,G)$ is an isomorphism. It follows that the counit of $(\bar{F},\bar{G})$ is also an isomorphism, which is equivalent to the fully-faithfulness of $\bar{G}$; consult [19, Chapter IV, Section 3, Theorem 1].

□
Let $k$ be a field and let $A$ be a finite dimensional $k$-algebra. Recall that $A$-$\text{mod}$ is the category of finite dimensional left $A$-modules. We write $\mathcal{A}A$ for the regular left $A$-module. Denote by $\mathcal{D}(A$-$\text{mod})$ (resp. $\mathcal{D}^b(A$-$\text{mod})$) the (resp. bounded) derived category of $A$-$\text{mod}$. We identify $A$-$\text{mod}$ as the full subcategory of $\mathcal{D}^b(A$-$\text{mod})$ consisting of stalk complexes concentrated at degree zero; see [15] Proposition I. 4.3.

A complex of $A$-modules is usually denoted by $X^\bullet = (X^i, d^i)_{i \in \mathbb{Z}}$, where $X^i$ are $A$-modules and the differentials $d^i : X^i \to X^{i+1}$ are homomorphisms of modules satisfying $d^{i+1} \circ d^i = 0$. Recall that a complex in $\mathcal{D}^b(A$-$\text{mod})$ is \textit{perfect} provided that it is isomorphic to a bounded complex consisting of projective modules. The full subcategory consisting of perfect complexes is denoted by perf$(A)$. Recall from [6] Lemma 1.2.1 that a complex $X^\bullet$ in $\mathcal{D}^b(A$-$\text{mod})$ is perfect if and only if there is a natural number $n_0$ such that for each $A$-module $M$, $\text{Hom}_{\mathcal{D}^b(A$-$\text{mod})}(X^\bullet, M[n]) = 0$ for all $n \geq n_0$. It follows that perf$(A)$ is a thick subcategory of $\mathcal{D}^b(A$-$\text{mod})$. Indeed, it is the smallest thick subcategory of $\mathcal{D}^b(A$-$\text{mod})$ containing $\mathcal{A}A$.

Let $\pi : A \to B$ be a homomorphism of algebras. The functor of restricting of scalars $\pi^* : B$-$\text{mod} \to A$-$\text{mod}$ is exact, and it extends to a triangle functor $\mathcal{D}^b(B$-$\text{mod}) \to \mathcal{D}^b(A$-$\text{mod})$, which will still be denoted by $\pi^*$. Following [12], we call the homomorphism $\pi$ a \textit{homological epimorphism} provided that $\pi^* : \mathcal{D}^b(B$-$\text{mod}) \to \mathcal{D}^b(A$-$\text{mod})$ is fully faithful. By [12] Theorem 4.1(1)] this is equivalent to the fact that $\pi \otimes_A^L B : B \simeq A \otimes_A^L B \to B \otimes_A^L B$ is an isomorphism in $\mathcal{D}(A^c$-$\text{mod})$. Here, $A^c = A \otimes_k A^{\text{op}}$ is the enveloping algebra of $A$, and we identify $A^c$-$\text{mod}$ as the category of $A$-$\text{bimodules}$.

**Lemma 2.3** ([22] Proposition 2.2(a))]. Let $J \subseteq A$ be an ideal and let $\pi : A \to A/J$ be the canonical projection. Then $\pi$ is a homological epimorphism if and only if $J^2 = J$ and $\text{Tor}^i_A(J, A/J) = 0$ for all $i \geq 1$.

In the situation of the lemma, the ideal $J$ is called a \textit{homological ideal} in [22]. As a special case, we call an ideal $J$ a \textit{hereditary ideal} provided that $J^2 = J$ and $J$ is a projective $A$-$\text{bimodule}$; compare [22] Lemma 3.4.

**Proof.** The natural exact sequence $0 \to J \to A \to A/J \to 0$ of $A$-$\text{bimodules}$ induces a triangle $J \to A \to A/J \to [1]$ in $\mathcal{D}^b(A^c$-$\text{mod})$. Applying the functor $- \otimes_A^L A/J$, we get a triangle $J \otimes_A^L A/J \to A/J \to A/J \otimes_A^L A/J \to A/J$ in $\mathcal{D}^b(A$-$\text{mod})$. Then $\pi$ is a homological epimorphism if, equivalently, $\pi \otimes_A^L A/J$ is an isomorphism if and only if $J \otimes_A^L A/J = 0$; see [13] Lemma I.1.7]. This is equivalent to the fact that $\text{Tor}^i_A(J, A/J) = 0$ for all $i \geq 0$. We note that $\text{Tor}^j_A(J, A/J) \simeq J \otimes_A A/J \simeq J/J^2$. \qed

Now we are in the position to prove the Theorem. Recall that for an algebra $A$, its singularity category $\mathcal{D}_{\text{sg}}(A) = \mathcal{D}^b(A$-$\text{mod})/$\text{perf}(A)$. Moreover, a complex $X^\bullet$ becomes zero in $\mathcal{D}_{\text{sg}}(A)$ if and only if it is perfect. Here, we use the fact that $\text{perf}(A) \subseteq \mathcal{D}^b(A$-$\text{mod})$ is a thick subcategory.

**Proof of the Theorem.** Write $B = A/J$. Since $J$, as an $A$-$\text{bimodule}$, has finite projective dimension, so it has finite projective dimension both as a left and right $A$-module. Consider the natural exact sequence $0 \to J \to A \to B \to 0$. It follows that $B$, both as a left and right $A$-module, has finite projective dimension. Moreover, for a complex $X^\bullet$ in $\mathcal{D}^b(A$-$\text{mod})$, $J \otimes_A^L X^\bullet$ is perfect. Indeed, take a bounded projective resolution $P^\bullet \to J$ as an $A^c$-$\text{module}$. Then $J \otimes_A^L X^\bullet \simeq P^\bullet \otimes_A X^\bullet$. This is a perfect complex, since each left $A$-module $P^i \otimes_A X^j$ is projective.
Denote by \( \pi : A \to B \) the canonical projection. By the assumption, the functor \( \pi^* : D^b(B\text{-}mod) \to D^b(A\text{-}mod) \) is fully faithful. Since \( \pi^*(B) = AB \) is perfect, the functor \( \pi^* \) sends perfect complexes to perfect complexes. Then it induces a triangle functor \( \pi^* : D_{\text{sg}}(B) \to D_{\text{sg}}(A) \). We will show that \( \pi^* \) is an equivalence.

The functor \( \pi^* : D^b(B\text{-}mod) \to D^b(A\text{-}mod) \) has a left adjoint \( F = B \otimes^L_A \) and a right adjoint \( B \otimes^L_A \). Here we use the fact that the right \( A\)-module \( B_A \) has finite projective dimension. Since \( F \) sends perfect complexes to perfect complexes, we have the induced triangle functor \( \bar{F} : D_{\text{sg}}(A) \to D_{\text{sg}}(B) \). By Lemma 2.2 we have the adjoint pair \( (\bar{F}, \pi^*) \); moreover, the functor \( \pi^* \) is fully faithful. By Lemma 2.1 there is a triangle equivalence \( D_{\text{sg}}(A)/\text{Ker} \bar{F} \simeq D_{\text{sg}}(B) \).

It remains to show that the essential kernel \( \text{Ker} \bar{F} \) is trivial. For this, we assume that a complex \( X^* \) lies in \( \text{Ker} \bar{F} \). This means that the complex \( F(X^*) \) in \( D^b(B\text{-}mod) \) is perfect. Since \( \pi^* \) preserves perfect complexes, it follows that \( \pi^* F(X^*) \) is also perfect. The natural exact sequence \( 0 \to J \to A \to B \to 0 \) induces a triangle \( J \otimes^L_A X^* \to X^* \to \pi^* F(X^*) \to J \otimes^L_A X^*[1] \) in \( D^b(A\text{-}mod) \). Recall that \( J \otimes^L_A X^* \) is perfect. It follows that \( X^* \) is perfect, since \( \text{perf}(A) \subseteq D^b(A\text{-}mod) \) is a triangulated subcategory. The proves that \( X^* \) is zero in \( D_{\text{sg}}(A) \).

The following special case of the Theorem is of interest.

**Corollary 2.4.** Let \( A \) be a finite dimensional algebra and \( J \subseteq A \) a hereditary ideal. Then we have a triangle equivalence \( D_{\text{sg}}(A) \simeq D_{\text{sg}}(A/J) \).

**Proof.** It suffices to observe by Lemma 2.3 that \( J \) is a homological ideal. \( \square \)

### 3. Examples

In this section, we will describe a construction of matrix algebras to illustrate Corollary 2.4. In particular, the singularity categories of some non-Gorenstein algebras are described.

The following construction is similar to [18, Section 4]. Let \( A \) be a finite dimensional algebra over a field \( k \). Let \( A_M \) and \( N_A \) be a left and right \( A \)-module, respectively. Then \( M \otimes_k N \) becomes an \( A\)-bimodule. Consider an \( A\)-bimodule monomorphism \( \phi : M \otimes_k N \to A \). Then \( \text{Im} \phi \) is a two-sided ideal of \( A \). We require further that \( (\text{Im} \phi)M = 0 \) and \( N(\text{Im} \phi) = 0 \). The matrix \( \Gamma = \begin{pmatrix} A & M \\ N & k \end{pmatrix} \) becomes an associative algebra via the following multiplication:

\[
\begin{pmatrix} a & m \\ n & \lambda \end{pmatrix} \begin{pmatrix} a' & m' \\ n' & \lambda' \end{pmatrix} = \begin{pmatrix} aa' + \phi(m \otimes n') & am' + \lambda m' \\ na' + \lambda n' & \lambda \lambda' \end{pmatrix}.
\]

For the associativity, we need the above requirement on \( \text{Im} \phi \).

**Proposition 3.1.** Keep the notation and assumption as above. Then there is a triangle equivalence \( D_{\text{sg}}(\Gamma) \simeq D_{\text{sg}}(A/\text{Im} \phi) \).

**Proof.** Set \( J = \Gamma e \Gamma \) with \( e = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \). Observe that \( \Gamma/J = A/\text{Im} \phi \). The ideal \( J \) is hereditary: \( J^2 = J \) is clear, while the natural map \( \Gamma e \otimes_k e \Gamma \to J \) is an isomorphism of \( \Gamma\Gamma\)-bimodules and then \( J \) is a projective \( \Gamma\Gamma\)-bimodule. The isomorphism uses the assumption that \( \phi \) is mono. Then we apply Corollary 2.4. \( \square \)
Remark 3.2. The above construction contains the one-point extension and coextension of algebras, where $M$ or $N$ is zero. Hence Proposition 3.1 contains the results in [9, Section 4].

We will illustrate Proposition 3.1 by three examples. Two of these examples extend an example considered by Happel in [14]. In particular, based on results in [9], we obtain descriptions of the singularity categories of some non-Gorenstein algebras.

Recall from [14] that an algebra $A$ is Gorenstein provided that both as a left and right module, the regular module $A$ has finite injective dimension. It follows from [6, Theorem 4.4.1] and [14, Theorem 4.6] that in the Gorenstein case, the singularity category $D_{sg}(A)$ is Hom-finite. This means that all Hom spaces in $D_{sg}(A)$ are finite dimensional over $k$.

For algebras given by quivers and relations, we refer to [11, Chapter III].

Example 3.3. Let $\Gamma$ be the $k$-algebra given by the following quiver $Q$ with relations $\{x^2, \delta x, \beta x, x\gamma, x\alpha, \beta\gamma, \delta\alpha, \beta\alpha, \delta\gamma, \alpha\beta - \gamma\delta\}$. We write the concatenation of paths from right to left.

\[
\begin{array}{c}
\circlearrowleft\\1 \\
\circlearrowright\end{array}
\]

We have in $\Gamma$ that $1 = e_1 + e_2 + e_2$, where the $e_i$’s are the primitive idempotents corresponding to the vertices. Set $\Gamma' = \Gamma/e_1\Gamma$. It is an algebra with radical square zero, whose quiver is obtained from $Q$ by removing the vertex 1 and the adjacent arrows.

We identify $\Gamma$ with $\left(\begin{array}{cc}
A & k\alpha \\
\alpha & k\beta \\
\end{array}\right)$, where the $k$ in the southeast corner is identified with $e_1e_1\Gamma$, and $A = (1 - e_1)\Gamma(1 - e_1)$. The corresponding $\text{Im } \phi$ equals $k\alpha\beta$, and we have $A/\text{Im } \phi = \Gamma'$; consult the proof of Proposition 3.1. Then Proposition 3.1 yields a triangle equivalence $D_{sg}(\Gamma) \simeq D_{sg}(\Gamma')$.

The triangulated category $D_{sg}(\Gamma')$ is completely described in [9] (see also [24]); in particular, it is not Hom-finite. More precisely, it is equivalent to the category of finitely generated projective modules on a von Neumann regular algebra. The algebra $\Gamma'$, or rather its Koszul dual, is related to the noncommutative space of Penrose tilings via the work of Smith; see [24, Theorem 7.2 and Example]. We point out that the algebra $\Gamma$ is non-Gorenstein, since $D_{sg}(\Gamma)$ is not Hom-finite.

Example 3.4. Let $\Gamma$ be the $k$-algebra given by the following quiver $Q$ with relations $\{x_1^2, x_2^2, x_1\alpha_1, x_2\alpha_1, \beta_2\alpha_1, \beta_2\alpha_1, x_1\alpha_2, x_2\alpha_2, \beta_1\alpha_2, \beta_2\alpha_2, \alpha_1\beta_1 - x_1x_2, \alpha_2\beta_2 - x_2x_1\}:

\[
\begin{array}{c}
\circlearrowleft\\1 \\
\circlearrowright\end{array}
\]

We claim that there is a triangle equivalence $D_{sg}(\Gamma) \simeq D_{sg}(k(x_1, x_2)/(x_1, x_2)^2)$. Here, $k(x_1, x_2)$ is the free algebra with two variables.

We point out that the triangulated category $D_{sg}(k(x_1, x_2)/(x_1, x_2)^2)$ is described completely in [9, Example 3.11], where related results are contained in [3, Section 10]. Similar to the example above, this algebra $\Gamma$ is non-Gorenstein.
To see the claim, we observe that the quiver $Q$ has two loops and two 2-cycles. The proof is done by “removing the 2-cycles”. We have a natural isomorphism

$$\Gamma = \begin{pmatrix} A & k\alpha_1 \\ k\beta_1 & k \end{pmatrix},$$

where $k = e_1\Gamma e_1$ and $A = (1 - e_1)\Gamma(1 - e_1)$. We observe that Proposition 3.4 applies with the corresponding $\text{Im }\phi = k\alpha_1\beta_1$. Set $A/\text{Im }\phi = \Gamma'$. So $\mathcal{D}_{\text{sg}}(\Gamma) \simeq \mathcal{D}_{\text{sg}}(\Gamma')$. The quiver of $\Gamma'$ is obtained from $Q$ by removing the vertex 1 and the adjacent arrows, while its relations are obtained from the ones of $\Gamma$ by replacing $\alpha_1\beta_1 - x_1x_2$ with $x_1x_2$. Similarly, $\Gamma' = \begin{pmatrix} A' & k\alpha_2 \\ k\beta_2 & k \end{pmatrix}$ with $k = e_2\Gamma'e_2$ and $A' = e_\ast\Gamma'e_\ast$. Then Proposition 3.1 applies again, and we get the equivalence $\mathcal{D}_{\text{sg}}(\Gamma') \simeq \mathcal{D}_{\text{sg}}(k(x_1, x_2)/(x_1, x_2)^2)$.

This example generalizes directly to a quiver with $n$ loops and $n$ 2-cycles with similar relations. The corresponding statement for the case $n = 1$ is implicitly contained in [14] 2.3 and 4.8.

The last example is a Gorenstein algebra.

**Example 3.5.** Let $r \geq 2$. Consider the following quiver $Q$ consisting of three 2-cycles and a central 3-cycle $Z_3$. We identify $\gamma_3$ with $\gamma_0$ and denote by $p_i$ the path in the central cycle starting at vertex $i$ of length 3.

\[
\begin{array}{c}
1' \\
\downarrow \quad \beta_3 \quad \downarrow \\
2' \\
\downarrow \quad \beta_2 \quad \downarrow \\
\downarrow \quad \beta_1 \\
1 \\
\end{array}
\]

\[
\begin{array}{c}
1' \\
\downarrow \quad \beta_3 \quad \downarrow \\
2' \\
\downarrow \quad \beta_2 \quad \downarrow \\
\downarrow \quad \beta_1 \\
1 \\
\end{array}
\]

Let $\Gamma$ be the $k$-algebra given by the quiver $Q$ with relations $\{\beta_i\alpha_i, \gamma_i\alpha_i, \beta_i\gamma_{i-1}, \alpha_i\beta_i - p_i^2 | i = 1, 2, 3\}$. We point out that in $\Gamma$ all paths in the central cycle of length strictly larger than $3r + 1$ vanish.

Set $A = kZ_3/(\gamma_1, \gamma_2, \gamma_3)^{3r}$, where $kZ_3$ is the path algebra of the central 3-cycle $Z_3$. The algebra $A$ is self-injective and Nakayama ([11] p.111]). Denote by $A^{\text{-mod}}$ the stable category of $A$-modules; it is naturally a triangulated category (see [13] Theorem 1.2.6).

We claim that there is a triangle equivalence $\mathcal{D}_{\text{sg}}(\Gamma) \simeq A^{\text{-mod}}$.

For the claim, we observe an isomorphism $A = \Gamma/\Gamma(e_1' + e_2' + e_3')\Gamma$. We argue as in Example 3.4 by removing the three 2-cycles and applying Proposition 3.4 repeatedly. Then we get a triangle equivalence $\mathcal{D}_{\text{sg}}(\Gamma) \simeq \mathcal{D}_{\text{sg}}(A)$. Finally, by [23] Theorem 2.1] we have a triangle equivalence $\mathcal{D}_{\text{sg}}(A) \simeq A^{\text{-mod}}$. Then we are done.

We point out that the algebra $\Gamma$ is Gorenstein with self-injective dimension two. Hence by [6] Theorem 4.4.1] and [14] Theorem 4.6 there is a triangle equivalence $\mathcal{D}_{\text{sg}}(\Gamma) \simeq \text{MCM}(\Gamma)$, where $\text{MCM}(\Gamma)$ denotes the stable category of (maximal) Cohen-Macaulay $\Gamma$-modules. Then we have a triangle equivalence

$$\text{MCM}(\Gamma) \simeq A^{\text{-mod}}.$$

We mention that $\Gamma$ is a special biserial algebra of finite representation type (by [10] Lemma II.8.1]). It would be interesting to identify (maximal) Cohen-Macaulay $\Gamma$-modules in the Auslander-Reiten quiver of $\Gamma$. 

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This example generalizes directly to a quiver with $n$ 2-cycles and a central $n$-cycle with similar relations. The case where $n = 1$ and $r = 2$ coincides with the examples considered in [14] 2.3 and 4.8.

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REFERENCES


WU WEN-TSUN KEY LABORATORY OF MATHEMATICS, UNIVERSITY OF SCIENCE AND TECHNOLOGY OF CHINA, CHINESE ACADEMY OF SCIENCES, HEFEI 230026, ANHUI, PEOPLE’S REPUBLIC OF CHINA

E-mail address: xwchen@mail.ustc.edu.cn
URL: http://home.ustc.edu.cn/~xwchen