Color Lie 代数的表示和上圈扭

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文 摘：本文研究 color Lie 代数的有限维表示和上圈扭，以及它们的关系。Color Lie 代数是一般的 Lie 代数和超 Lie 代数的自然推广，所以它的表示理论是 Lie 代数和超 Lie 代数表示的理论的变形. 其独特之处是 color Lie 代数有它的上圈扭。

我们主要利用 color Lie 代数 L 的泛包络代数 U(L)和其增广形式 U˜(L)，以及它们的 FCR-性质，即所有的有限维表示是完全可约的：L 的表示理论和 U(L)的表示理论是等价的； U˜(L)是一个 Hopf 代数且包含 U(L).并且我们利用到 L 和 U(L)的分次性，特别的，U(L)是一个群分次环。具体说来，如果 L 是 G-分次 ε-Lie 代数，其中 G 是一 Abel 群，ε 是其上的双特征，那么 U(L)是 G-分次环。

我们主要证明了，如果 L 是有限维的且群 G 是有限的， 则 U(L)有 FCR-性质当且仅当 U(L^c)有 FCR-性质，其中 L^c 是 L 关于上圈 c 的扭；U(L)有 FCR-性质当且仅当 U˜(L)有 FCR-性质。 作为应用，我们证明一个重要的 color Lie 代数：sl_2^c，它的任何有限维表示完全可约，并且，在假设基域是代数闭的情形下，我们计算初它所有的有限维单模。

其重要结论（或后续结果）是：利用更多的群分次环理论，我们可以确定所有的有限维 color Lie 代数 L 使得其泛包络代数 U(L)(和其增广形式 U˜(L))有 FCR-

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性质：这个结果将联系了 Cartan 的关于半单 Lie 代数的分类和 Kac 的关于半单超 Lie 代数的分类。另外，对于半单 Lie 代数的群分次和相应的上圈扭，以及所得到的 color Lie 代数的表示是很值得研究的。我们期望更多的例子能被完全算出来。

关键词：Color Lie 代数； FCR-性质； 上圈扭。

Representations and Cocycle Twists of Color Lie Algebras

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Abstract: We study finite-dimensional representations and cocycle twists of color Lie algebras, and the relation between them. Color Lie algebras are natural generalizeation of Lie algebras and super Lie algebras, hence their representation theory is a deformation of the one of Lie algebras and super Lie algebras. However their speciality is that there are cocycle twists on color Lie algebras.

The main tools are the universal enveloping algebra U(L) and the augmented one $U^{\sim}$(L) of the color Lie algebra L, and their FCR-property, i.e., all finite-dimensional representations are completely reducible: the representation theory of L and the one of its universal enveloping algebra U(L) are equivalent; $U^{\sim}$(L) is a Hopf algebra, which contains U(L) as a subalgebra. Moreover, we note that both L and U(L) are graded. More precisely, if L is a G-graded ε − Lie algebra, where G is an Abelian group, ε is a bi-character over G, then U(L) is a G-graded algebra.

The main result is that: if G is a fnite group and L is finite-dimensional, then U(L) is FCR if and only if U(L^c) is FCR, where L^c is a cocycle twist of L with respect to cocyle c (over G); U(L) is FCR if and only if $U^{\sim}$(L) is; As an application, we prove that an important color Lie algebra sl_2^c is FCR, i.e., all finite-dimensional representations of sl_2^c are completely reducible. Moreover, if we assume the base field is algebraically closed, we compute all the finite-dimensional representations of sl_2^c up to isomorphism.

The conclusion we can draw now (in some forthcoming paper): using more technique from graded ring theory, we can determine all the finite-dimensional color Lie algebras L such that U(L) (or $U^{\sim}(L)$) are FCR: this result links the classification of semisimple Lie algebras of E.Carten and the classification of semisimple super Lie algebras of V.Kac. Moreover, it is worth studying the group-grading of semisimple Lie algebras and their cocycle twists, and the representation theory of the resulting color Lie algebras. We expect more examples can be computed explicitly.

Key words: Color Lie algebras; FCR-property; cocycle twists.