Quantum Coherent Atomic Tunneling between Two Trapped Bose-Einstein Condensates

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We study the coherent atomic tunneling between two zero-temperature Bose-Einstein condensates (BEC) confined in a double-well magnetic trap. Two Gross-Pitaevskii equations for the self-interacting BEC amplitudes, coupled by a transfer matrix element, describe the dynamics in terms of the interwell phase difference and population imbalance. In addition to the anharmonic generalization of the familiar ac Josephson effect and plasma oscillations occurring in superconductor junctions, the nonlinear BEC tunneling dynamics sustains a self-maintained population imbalance: a novel "macroscopic quantum self-trapping" effect. [S0031-9007(97)04613-9]

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The recent experimental observation of the Bose-Einstein condensation (BEC) in a dilute gas of trapped atoms [1,2] has generated much interest in the properties of this new state of matter. A fascinating possibility is the observation of new quantum phenomena on macroscopic scales, related with the superfluid nature of the condensate. In fact, broken symmetry arguments show that the condensate atoms can be described by a common, "macroscopic" one-body wave function $\Psi(\vec{r},t) = \sqrt{\rho}\, e^{i\theta}$ (the order parameter), with ρ the condensate density. For a weakly interacting BEC, the order parameter obeys a nonlinear Schrödinger, or Gross-Pitaevskii equation (GPE) [3]:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\Psi + [V_{\rm ext}(\vec{r}) + g_0\|\Psi\|^2]\Psi\,, \quad (1)$$

where $V_{\rm ext}$ is the external potential and $g_0 = \frac{4\pi\hbar^2 a}{m}$ is the interatomic scattering pseudopotential, with a, m the atomic scattering length and mass, respectively.

The GPE has been successfully applied to investigate the collective mode frequencies of a trapped BEC in the linear regime [4], the relaxation times of monopolar oscillations [5], and, because of the nonlinear self-interaction, it could also induce chaotic behavior in dynamical quantum observables [5,6].

The existence of a macroscopic quantum phase (difference) was dramatically demonstrated recently [2]. A far off-resonant intense laser sheet divided a trapped condensate, creating a high barrier in between. Switching off the double-well trap, the two released condensates overlapped, producing a robust "two-slit" atomic interference pattern, clear signature of phase coherence over a macroscopic scale ($\simeq 10^{-2}$ cm) [2,7]. The *non*destructive detection of phase differences between two trapped BEC could be achieved by lowering the intensity of the laser sheet. This allows for atomic tunneling through the barrier, and the detection of Josephson-like current-phase effects [2,8–10]. In superconductor Josephson junctions (SJJ), phase coherence signatures include a dc external voltage producing an ac current, or the "plasma" oscillations of an initial charge imbalance [11,12]. For neutral superfluid He II, voltage drives, tunnel junctions, or capacitive charges are absent. The only accessible Josephson analog [13] involves two He II baths connected by a submicron orifice, at which vortex phase slips [14] support a chemical potential (height) difference, through the Josephson frequency relation.

Although the trapped BEC is also a neutral-atom Bose system, its population can be monitored by phase-contrast microscopy; the double-well curvatures and barrier heights can be tailored by the position and the intensity of the laser sheet partitioning the magnetic trap [2]. We note that the chemical potential between the two condensates depends both on the zero-point energy difference from an asymmetrically positioned laser barrier, that acts like an external "dc" SJJ voltage; and on the nonlinear interaction that, through an initial population imbalance, acts like a capacitive SJJ charging energy. Thus, we propose that the BEC tunnel junction can show the analogs of the familiar Josephson effects in superconductor junctions, with the ability to tailor traps and the atomic interaction compensating for electrical neutrality.

In this Letter we study the atomic tunneling at zero temperature between two nonideal, weakly linked BEC in a (possibly asymmetric) double-well trap. This induces a coherent, oscillating flux of atoms between wells, that is a signature of the *macroscopic superposition of states* in which the condensates evolve. The dynamics is governed by two Gross-Pitaevskii equations for the BEC amplitudes, coupled by a transfer matrix element (Josephson tunneling term). Analogs of the superconductor Josephson effects such as the ac effects and plasma oscillations are predicted. We also find that the nonlinearity of the dynamic tunneling equations produces their anharmonic generalization and a novel self-trapping effect.

Consider a double-well magnetic trap 1,2 as in Fig. 1 with an asymmetrically placed laser barrier producing different well curvatures. This system can be described

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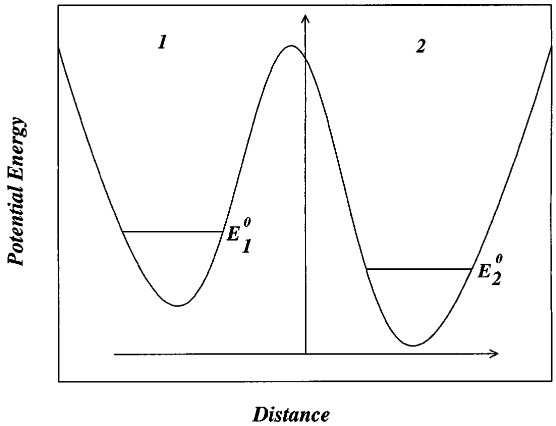

FIG. 1. The double-well trap for two Bose-Einstein condensates with $N_{1,2}$ and $E^0_{1,2}$ the number of particles and the zero-point energies in the trap 1, 2, respectively.

by a two-state model

$$i\hbar \frac{\partial \psi_1}{\partial t} = (E_1^0 + U_1 N_1)\psi_1 - K\psi_2 , \tag{2a}$$

$$i\hbar \frac{\partial \psi_2}{\partial t} = (E_2^0 + U_2 N_2)\psi_2 - K\psi_1 , \tag{2b}$$

with uniform amplitudes $\psi_{1,2} = \sqrt{N_{1,2}}\, e^{i\theta_{1,2}}$, where $N_{1,2}$, $\theta_{1,2}$ are the number of particles and phases in the trap 1, 2, respectively, and K [15] is the coupling matrix element. The parameters $E^0_{1,2}$, $U_{1,2}$, and K can be determined from appropriate overlap integrals of the time-independent GPE eigenfunctions $\Phi_{1,2}$ of the isolated traps, as outlined later. The total number of atoms $N_T = N_1 + N_2$ is constant, but we stress that a coherent phase description, i.e., the existence of definite phases $\theta_{1,2}$, implies that the phase fluctuations ($\simeq 1/\sqrt{N_{1,2}}$ [16]) must be small, giving $N_{1,2} > N_{\min} \simeq 10^3$, say.

We note that the Bose Josephson junction (BJJ) tunneling Eqs. (2) are similar in form to models of single-trap atomic-level transitions [9], or polaron hopping in semiclassical approximation [17], although they describe quite different physics. The U_1N_1, U_2N_2 terms are like bulk on-site charging energies in a bosonic (and "classical") analogy to two-grain mesoscopic systems [12].

In terms of the phase difference $\phi = \theta_2 - \theta_1$ and fractional population difference $-1 < z = \frac{N_1 - N_2}{N_T} < 1$, Eqs. (2) become ($\hbar = 1$)

$$\dot z = -\sqrt{1 - z^2}\,\sin\phi , \tag{3a}$$

$$\dot\phi = \Lambda z + \frac{z}{\sqrt{(1 - z^2)}}\cos\phi + \Delta E , \tag{3b}$$

where the time has been rescaled as $2Kt \rightarrow t$. The dimensionless parameters are

$$\Delta E = (E_1^0 - E_2^0)/(2K) + (U_1 - U_2)N_T/(4K) , \tag{4a}$$

$$\Lambda = (U_1 + U_2)N_T/(4K) . \tag{4b}$$

For two symmetric traps, $E_1^0 = E_2^0$ ($\Delta E = 0$), $U_1 = U_2 = U$, and $\Lambda = UN_T/2K$. In the following we will assume a positive scattering length a ($\Lambda > 0$); note, however, that Eqs. (3) are invariant under the transformation $\Lambda \rightarrow -\Lambda$, $\phi \rightarrow -\phi + \pi$, $\Delta E \rightarrow -\Delta E$.

The z, ϕ variables are canonically conjugate, with $\dot z = -\frac{\partial H}{\partial \phi}$, $\dot\phi = \frac{\partial H}{\partial z}$ and the Hamiltonian is given by

$$H = \frac{\Lambda}{2} z^2 - \sqrt{1 - z^2}\cos\phi + \Delta E z . \tag{5}$$

In a simple mechanical analogy, H describes a *nonrigid* pendulum, of tilt angle ϕ and length proportional to $\sqrt{1 - z^2}$, that decreases with the "angular momentum" z.

The BJJ intertrap tunneling current is given by

$$I = \dot z \frac{N_T}{2} = I_0\sqrt{(1 - z^2)}\,\sin\phi ; \qquad I_0 = KN_T . \tag{6}$$

It differs from Cooper-pair SJJ tunneling current in its nonlinearity in z. The $\ddot\phi$ equations hence also differ.

The detailed analysis of Eqs. (3) with exact analytical solutions in terms of Jacobian and Weierstrassian elliptic functions will be presented elsewhere [18]; here we consider three regimes.

(1) *Noninteracting limit.*—For symmetric wells and negligible interatomic interactions ($\Lambda \rightarrow 0$), Eqs. (2) yield Rabi-like oscillations in the population of each trap with a frequency [8,9]

$$\omega_R = 2K . \tag{7}$$

However, the ideal Bose gas limit is not accessible.

(2) *Linear regime.*—In the linear limit ($|z| \ll 1, |\phi| \ll 1$) Eqs. (3) become

$$\dot z \simeq -\phi , \tag{8}$$

$$\dot\phi \simeq (\Lambda + 1)z . \tag{9}$$

These describe the small amplitude oscillations of the pendulum analog, with a sinusoidal $z(t)$ with a frequency

$$\omega_L = \sqrt{2UN_TK + 4K^2} . \tag{10}$$

The BJJ oscillations of population should show up as temporal oscillations of phase-contrast patterns [2], or other probes of atomic population [1].

Linearizing Eqs. (3) in $z(t)$ only, we obtain

$$\dot z \simeq -\sin\phi , \tag{11a}$$

$$\dot\phi \simeq \Delta E + (\Lambda + \cos\phi)z , \tag{11b}$$

$$I \simeq I_0 \sin\phi . \tag{11c}$$

For large trap asymmetries with $\Delta E \gg [\Lambda + \cos(\phi)]z$, we have $\phi = \phi(0) + \Delta E t$, giving an oscillating $z(t)$ with frequency

$$\omega_{\rm ac} \simeq E_1^0 - E_2^0 , \tag{12}$$

where an "ac" current $I(t)$ is produced by the dc trap asymmetry ΔE. It is simple to show that a small oscillation in the laser position ("voltage" $\Delta E \rightarrow \Delta E[1 + \delta \sin(\omega_0 t)]$), or in its intensity ($K \rightarrow K[1 + \delta \sin(\omega_0 t)]$, $\delta \ll 1$), will result in a dc intertrap current of nonzero time average $\langle I(t)\rangle \simeq \delta\langle \sin(\omega_0 t)\sin(\omega_{\rm ac} t)\rangle \neq 0$, at a resonant match $\omega_0 = \omega_{\rm ac}$. This is the analog of the Shapiro effect [11] with an applied dc voltage in superconductor junctions. (In SJJ, of course, barrier modulation is not possible.) In practice, in the experimental setup, the intensity of laser barrier can have a small (~10%) random noise component $\tilde{K}(t)$. The (dc) Shapiro effect will be unaffected by this noise. The barrier noise will show up as a small additive component $z(t) \sim (K + \tilde{K})$ to the oscillations of Fig. 2 (see below), that could still be detectable. If the barrier noise $\tilde{K}(t)$ is monitored, and combined with a π-shifted $z(t)$ signal, then the oscillations would stand out more clearly.

In SJJ, the current of Cooper pairs $N_{1,2}$ is $I_{\rm SJJ} = -2e(\dot{N}_1 - \dot{N}_2) = 2eE_J \sin\phi$, and the Josephson frequency relation for the relative phase is $\dot{\phi} = \Delta\mu = 2eV$, for a junction voltage $V = (N_1 - N_2)E_c/2e$ and a junction capacitance C, with $E_c = (2e)^2/2C$ [11]. These rigid pendulum SJJ equations can be directly compared with the BJJ Eqs. (3). It is then clear that the ac Josephson frequency $\omega_{\rm ac} = 2eV$ and the Josephson plasma frequency [11] $\omega_p = \sqrt{E_c E_J}$ are the analog of Eqs. (12) and (10), respectively. Note however that, for SJJ, ω_p is independent of the barrier cross section A, since $E_J \sim A$ and $E_c \sim C^{-1} \sim A^{-1}$, while the BJJ has $\omega_L \simeq A^{1/2}$ since $K \sim A$, and the bulk energy UN_T is approximately A independent.

(3) *Nonlinear regime.*—A numerical solution of Eqs. (3) yields nonsinusoidal oscillations, that are the anharmonic *generalization* of the sinusoidal Josephson effects. Moreover, an additional novel nonlinear effect occurs in the BJJ: a self-locked population imbalance.

Figure 2 shows solutions of Eqs. (3) with initial conditions $z(0) = 0.6$, $\phi(0) = 0$ and illustrative parameters $\Lambda = 1$, 8, 9.99, 10, and 11, respectively. The sinusoidal oscillations around $z = 0$ became anharmonic as Λ increases, Figs. 2(a), 2(b), and 2(c). With a precursor slowing down, Fig. 2(c), there is a critical transition for $\Lambda = \Lambda_c = 10$, dashed line in Fig. 2(d). Then for $\Lambda = 11$ the population in each trap oscillates around a nonzero time averaged $\langle z(t)\rangle \neq 0$, solid line in Fig. 2(d). In the nonrigid pendulum analogy, this corresponds to an initial angular momentum $z(0)$ sufficiently large to swing the pendulum bob over the $\phi = \pi$ vertical orientation, with a nonzero $\langle z(t)\rangle$ average angular momentum corresponding to the rotatory motion. This critical behavior depends on $\Lambda_c = \Lambda_c[z(0), \phi(0)]$, as can be easily found from the energy conservation constraint and the boundness of the tunneling energy in Eq. (5). In fact, the value $z(t) = 0$ is inaccessible at any time if

$$\Lambda > \Lambda_c = 2\left(\frac{\sqrt{1 - z(0)^2}\,\cos[\phi(0)] + 1}{z(0)^2}\right). \qquad (13)$$

The full dynamical behavior of Eqs. (3) is summarized in Fig. 3, that shows the z-ϕ phase portrait with constant energy lines. There are energy minima along $z = 0$ at $2n\pi$, and "running" solutions $\langle\dot{\phi}(t)\rangle \neq 0$ with $\langle z(t)\rangle \neq 0$, moving along the sides of these wells. The vertical points $\phi = (2n + 1)\pi$, that would be isolated unstable points for the rigid pendulum, now support oscillations of restricted range, as a consequence of nonrigidity, i.e., nonlinearity.

The self-trapping of an initial BEC population imbalance, seen in Figs. 2(d) and 3, arises because of the interatomic interaction in the Bose gas (nonlinear self-interaction in GPE). It has a quantum nature, involving the coherence of a macroscopic number of atoms. It differs from single polaron trapping of an electron in a medium [17] and from external gravitational effects on He II baths

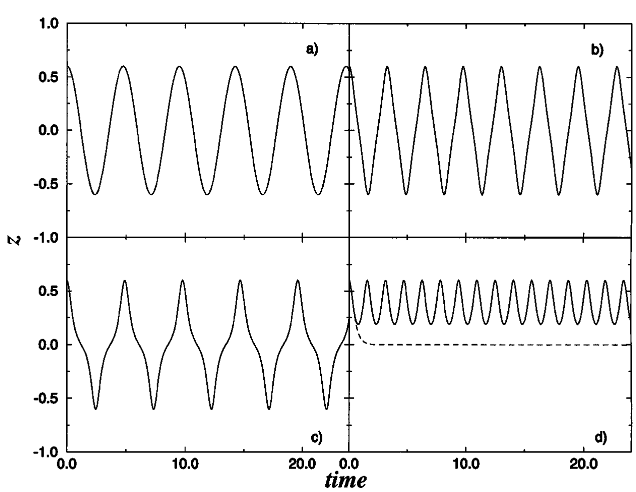

FIG. 2. Fractional population imbalance $z(t)$ versus rescaled time, with initial conditions $z(0) = 0.6$, phase difference $\phi(0) = 0$, and $\Lambda = 1$ (a), $\Lambda = 8$ (b), $\Lambda = 9.99$ (c), $\Lambda = 10$ (dashed line, d), $\Lambda = 11$ (solid line, d).

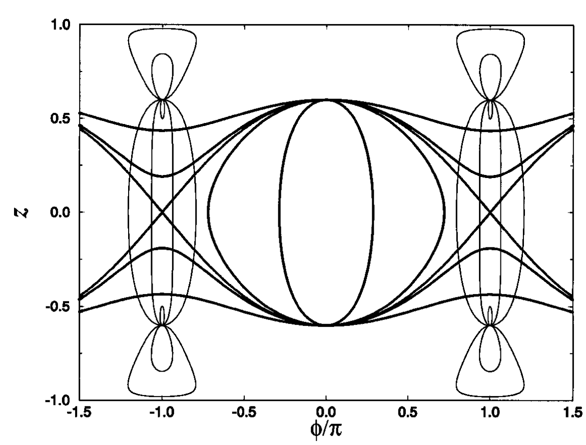

FIG. 3. Constant energy lines in a phase-space plot of population imbalance z versus phase difference ϕ. Bold solid line: $z(0) = 0.6$, $\phi(0) = 0$, $\Lambda = 1$, 8, 10, 11, and 20. Solid line: $z(0) = 0.6$, $\phi(0) = \pi$, $\Lambda = 0$, 1, 1.2, 1.5, and 2.

[13,14]. It can be considered as a novel "macroscopic quantum self-trapping" (MQST).

Nonlinear effects like MQST are unobservable in SJJ where the external circuit suppresses charge imbalances. For isolated coupled superconductor grains [12], the requirement that the chemical potential difference $\mu_1 - \mu_2 \simeq (N_1 - N_2)E_c$ must be less than the quasiparticle gap $2\Delta_{\rm qp}$ (to avoid excitations) implies very small fractional imbalances: $|z| < (2\Delta_{\rm qp}/E_c N_T) < 10^{-9}$ for typical parameters. For the BEC, the requirement that tunneling does not access excitation energies is much less restrictive. As an example, let us consider two weakly linked condensates of $N_T \simeq 10^4$ atoms, confined in two symmetric spherical traps with frequency $\omega_0 \simeq 100$ Hz. Evaluating Eqs. (17) below with a simple variational wave function [19], we have $E^0 \simeq 0.5$ nK, $UN_T \simeq 3$ nK, and from an estimation of the excitation gap $\sim$1 nK, we obtain the constraint $|z| \leq 0.4$. Taking $K \simeq 0.1$ nK, we have $\Lambda \simeq 10$, close to the onset of the critical behavior. [We note that from Eq. (13) for any Λ there is a critical $z(0)$.] Increasing the number of particles or the trap frequencies, the value of Λ can increase, making MQST observable. Typical frequency oscillations are $\omega_L \simeq$ KHz for $N_T \simeq 10^6$, that should be compared with the plasma frequency of SJJ that are [11] of the order of $\omega_p \simeq$ GHz.

We now outline the derivation of the parameters E_0, U, and K. To this purpose we note that in the barrier region the modulus of the order parameter in the GPE is exponentially small. This allows us to look for a solution of GPE of the form

$$\Psi = \psi_1(t)\Phi_1(x) + \psi_2(t)\Phi_2(x), \tag{14}$$

where Φ_1, Φ_2 are the ground state solutions for isolated traps with $N_1 = N_2 = N_T/2$.

Replacing Eq. (14) in the GPE Eq. (1) with the conditions

$$\int \Phi_1 \Phi_2 \, d\vec{r} \simeq 0 \tag{15}$$

and

$$\int \|\Phi_1\|^2 \, d\vec{r} = \int \|\Phi_2\|^2 \, d\vec{r} = 1, \tag{16}$$

we obtain

$$E^0_{1,2} = \int \left[\frac{\hbar^2}{2m} |\nabla \Phi_{1,2}|^2 + \Phi^2_{1,2} V_{\rm ext} \right] d\vec{r}, \tag{17a}$$

$$U_{1,2} = g_0 \int \Phi^4_{1,2} \, d\vec{r}, \tag{17b}$$

$$K = -\int \left[\frac{\hbar^2}{2m} (\nabla \Phi_1 \nabla \Phi_2) + \Phi_1 \Phi_2 V_{\rm ext} \right] d\vec{r}. \tag{17c}$$

At finite temperature the interaction between the normal component of the Bose gas with the condensate should be included, and the parameters become temperature dependent. Such corrections are small for temperatures smaller than excitation energies. High density BEC could induce quasiparticle–collective-mode scattering with finite lifetime of the coherent oscillations [5]; phase diffusion could induce phase coherence collapse and revival [20]. These effects deserve further studies.

In conclusion, the BEC coherent atomic tunneling in a double-well trap induces nonlinear population oscillations that are a generalization of the sinusoidal Josephson effects familiar in superconductors. A novel population imbalance occurs for parameters beyond critical values: a macroscopic quantum self-trapping effect.

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