In[1]:=
```
A = {{1, 2}, {2, 3}};
A // MatrixForm
```

Out[2]//MatrixForm=

$$\begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$$

In[4]:=
```
A.A
A * A
A.A // MatrixForm
A * A // MatrixForm
```

Out[4]= $\{\{5, 8\}, \{8, 13\}\}$

Out[5]= $\{\{1, 4\}, \{4, 9\}\}$

Out[6]//MatrixForm=

$$\begin{pmatrix} 5 & 8 \\ 8 & 13 \end{pmatrix}$$

Out[7]//MatrixForm=

$$\begin{pmatrix} 1 & 4 \\ 4 & 9 \end{pmatrix}$$

In[8]:=
```
Exp[A] // MatrixForm
```

Out[8]//MatrixForm=

$$\begin{pmatrix} e & e^2 \\ e^2 & e^3 \end{pmatrix}$$

In[10]:=
```
MatrixExp[A] // MatrixForm
```

Out[10]//MatrixForm=

$$\begin{pmatrix} \frac{e^{2-\sqrt{5}}}{2} + \frac{e^{2-\sqrt{5}}}{2\sqrt{5}} + \frac{e^{2+\sqrt{5}}}{2} - \frac{e^{2+\sqrt{5}}}{2\sqrt{5}} & -\frac{e^{2-\sqrt{5}}}{\sqrt{5}} + \frac{e^{2+\sqrt{5}}}{\sqrt{5}} \\ -\frac{e^{2-\sqrt{5}}}{\sqrt{5}} + \frac{e^{2+\sqrt{5}}}{\sqrt{5}} & \frac{e^{2-\sqrt{5}}}{2} - \frac{e^{2-\sqrt{5}}}{2\sqrt{5}} + \frac{e^{2+\sqrt{5}}}{2} + \frac{e^{2+\sqrt{5}}}{2\sqrt{5}} \end{pmatrix}$$

In[11]:=
```
Table[i^2, {i, 10}]
```

Out[11]= $\{1, 4, 9, 16, 25, 36, 49, 64, 81, 100\}$

In[13]:=
```
Table[i^2 + j^2 * 0.01, {i, 1, 4}, {j, 1, 4}] // MatrixForm
```

Out[13]//MatrixForm=

$$\begin{pmatrix} 1.01 & 1.04 & 1.09 & 1.16 \\ 4.01 & 4.04 & 4.09 & 4.16 \\ 9.01 & 9.04 & 9.09 & 9.16 \\ 16.01 & 16.04 & 16.09 & 16.16 \end{pmatrix}$$

In[18]:=
```
A = Table[i^2 + j^2 * 0.01, {i, 1, 4}, {j, 1, 4}];
A[[1, 1]]
A[[2, 1]]
A[[1, 4]]
```

Out[19]= 1.01

Out[20]= 4.01

Out[21]= 1.16

In[37]:=
```
A = Table[0, {i, 1, 4}, {j, 1, 4}];
Do[{A[[i, j]] = i^2 + j^2 * 0.01; Print[A[[i, j]], "   ", i, "   ", j]},
 {i, 1, 4}, {j, 1, 4}]
```

```
1.01   1   1
1.04   1   2
1.09   1   3
1.16   1   4
4.01   2   1
4.04   2   2
4.09   2   3
4.16   2   4
9.01   3   1
9.04   3   2
9.09   3   3
9.16   3   4
16.01   4   1
16.04   4   2
16.09   4   3
16.16   4   4
```

In[39]:=
```
Eigenvalues[A]
```

Out[39]= $\{30.4694 + 0.\,i,\ -0.169351 + 0.\,i,\ -4.62048 \times 10^{-16} + 1.65779 \times 10^{-15}\,i,\ -4.62048 \times 10^{-16} - 1.65779 \times 10^{-15}\,i\}$

In[40]:=
```
Eigensystem[A]
```

Out[40]=
$$\left\{\left\{30.4694,\ -0.169351,\ -4.62048\times10^{-16}+1.65779\times10^{-15}\,i,\ -4.62048\times10^{-16}-1.65779\times10^{-15}\,i\right\},\right.$$
$$\left\{\left\{-0.0587995+0.\,i,\ -0.216673+0.\,i,\ -0.479797+0.\,i,\ -0.848169+0.\,i\right\},\right.$$
$$\left\{-0.576005+0.\,i,\ -0.311877+0.\,i,\ 0.128336+0.\,i,\ 0.744635+0.\,i\right\},$$
$$\left\{-0.192596-0.290379\,i,\ -0.1997+0.362974\,i,\ 0.755048+0.\,i,\ -0.362752-0.0725948\,i\right\},$$
$$\left.\left.\left\{-0.192596+0.290379\,i,\ -0.1997-0.362974\,i,\ 0.755048+0.\,i,\ -0.362752+0.0725948\,i\right\}\right\}\right\}$$

In[41]:=
```
QRDecomposition[A]
```

Out[41]=
$$\left\{\left\{\left\{-0.0536354,\ -0.212949,\ -0.478471,\ -0.850201\right\},\right.\right.$$
$$\left\{-0.758052,\ -0.54737,\ -0.196234,\ 0.295356\right\},$$
$$\left\{0.638721,\ -0.732461,\ -0.113042,\ 0.206781\right\},$$
$$\left.\left\{0.120482,\ 0.344297,\ -0.848398,\ 0.38362\right\}\right\},$$
$$\left\{\left\{-18.8308,\ -18.8787,\ -18.9585,\ -19.0701\right\},\ \left\{0.,\ -0.036189,\ -0.096504,\ -0.180945\right\},\right.$$
$$\left.\left\{0.,\ 0.,\ 3.42734\times10^{-15},\ 1.13034\times10^{-14}\right\},\ \left\{0.,\ 0.,\ 0.,\ 1.17538\times10^{-15}\right\}\right\}\right\}$$

In[44]:=
```
L = QRDecomposition[A];
Q = L[[1]];
R = L[[2]];
Q // MatrixForm
R // MatrixForm
```

Out[47]//MatrixForm=
$$\begin{pmatrix} -0.0536354 & -0.212949 & -0.478471 & -0.850201 \\ -0.758052 & -0.54737 & -0.196234 & 0.295356 \\ 0.638721 & -0.732461 & -0.113042 & 0.206781 \\ 0.120482 & 0.344297 & -0.848398 & 0.38362 \end{pmatrix}$$

Out[48]//MatrixForm=
$$\begin{pmatrix} -18.8308 & -18.8787 & -18.9585 & -19.0701 \\ 0. & -0.036189 & -0.096504 & -0.180945 \\ 0. & 0. & 3.42734\times10^{-15} & 1.13034\times10^{-14} \\ 0. & 0. & 0. & 1.17538\times10^{-15} \end{pmatrix}$$

In[65]:=
```
Q.ConjugateTranspose[Q] // MatrixForm
```

Out[65]//MatrixForm=
$$\begin{pmatrix} 1. & -5.55112\times10^{-17} & 1.11022\times10^{-16} & -5.55112\times10^{-17} \\ -5.55112\times10^{-17} & 1. & -8.32667\times10^{-17} & 8.32667\times10^{-17} \\ 1.11022\times10^{-16} & -8.32667\times10^{-17} & 1. & 8.32667\times10^{-17} \\ -5.55112\times10^{-17} & 8.32667\times10^{-17} & 8.32667\times10^{-17} & 1. \end{pmatrix}$$

In[67]:=
```
L2 = SingularValueDecomposition[A]
```

Out[67]=
```
{{{-0.0567872, 0.757822, -0.28743, -0.582979},
  {-0.215223, 0.54648, -0.135158, 0.797979},
  {-0.479282, 0.194243, 0.847622, -0.118724},
  {-0.848966, -0.298889, -0.425033, -0.0962763}},
 {{37.8698, 0., 0., 0.}, {0., 0.136256, 0., 0.}, {0., 0., 0., 0.}, {0., 0., 0., 0.}},
 {{-0.497248, -0.574686, 0.649672, 0.0201858},
  {-0.498515, -0.310554, -0.670355, 0.4535}, {-0.500628, 0.129666,
   -0.242973, -0.820684}, {-0.503586, 0.745975, 0.263657, 0.346998}}}
```

In[72]:=
```
V = L2[[1]];
Lambda = L2[[2]];
U = L2[[3]];
V // MatrixForm
Lambda // MatrixForm
U // MatrixForm
```

Out[75]//MatrixForm=

$$\begin{pmatrix} -0.0567872 & 0.757822 & -0.28743 & -0.582979 \\ -0.215223 & 0.54648 & -0.135158 & 0.797979 \\ -0.479282 & 0.194243 & 0.847622 & -0.118724 \\ -0.848966 & -0.298889 & -0.425033 & -0.0962763 \end{pmatrix}$$

Out[76]//MatrixForm=

$$\begin{pmatrix} 37.8698 & 0. & 0. & 0. \\ 0. & 0.136256 & 0. & 0. \\ 0. & 0. & 0. & 0. \\ 0. & 0. & 0. & 0. \end{pmatrix}$$

Out[77]//MatrixForm=

$$\begin{pmatrix} -0.497248 & -0.574686 & 0.649672 & 0.0201858 \\ -0.498515 & -0.310554 & -0.670355 & 0.4535 \\ -0.500628 & 0.129666 & -0.242973 & -0.820684 \\ -0.503586 & 0.745975 & 0.263657 & 0.346998 \end{pmatrix}$$

In[79]:=
```
V.ConjugateTranspose[V] // MatrixForm
```

Out[79]//MatrixForm=

$$\begin{pmatrix} 1. & 0. & -1.38778 \times 10^{-17} & 7.63278 \times 10^{-17} \\ 0. & 1. & 2.498 \times 10^{-16} & 6.93889 \times 10^{-17} \\ -1.38778 \times 10^{-17} & 2.498 \times 10^{-16} & 1. & 1.56125 \times 10^{-17} \\ 7.63278 \times 10^{-17} & 6.93889 \times 10^{-17} & 1.56125 \times 10^{-17} & 1. \end{pmatrix}$$

In[80]:=
```
U.ConjugateTranspose[U] // MatrixForm
```

Out[80]//MatrixForm=

$$\begin{pmatrix} 1. & 3.31332 \times 10^{-16} & 1.73472 \times 10^{-16} & -2.60209 \times 10^{-17} \\ 3.31332 \times 10^{-16} & 1. & 5.55112 \times 10^{-17} & -8.32667 \times 10^{-17} \\ 1.73472 \times 10^{-16} & 5.55112 \times 10^{-17} & 1. & 1.11022 \times 10^{-16} \\ -2.60209 \times 10^{-17} & -8.32667 \times 10^{-17} & 1.11022 \times 10^{-16} & 1. \end{pmatrix}$$

In[81]:=
```
Eigenvalues[A]
```

Out[81]=
$$\{30.4694 + 0.\,i,\ -0.169351 + 0.\,i,\ -4.62048 \times 10^{-16} + 1.65779 \times 10^{-15}\,i,\ -4.62048 \times 10^{-16} - 1.65779 \times 10^{-15}\,i\}$$

In[83]:=
```
RandomReal[]
RandomReal[]
RandomReal[]
RandomReal[]
```

Out[83]= 0.965249

Out[84]= 0.0465626

Out[85]= 0.778405

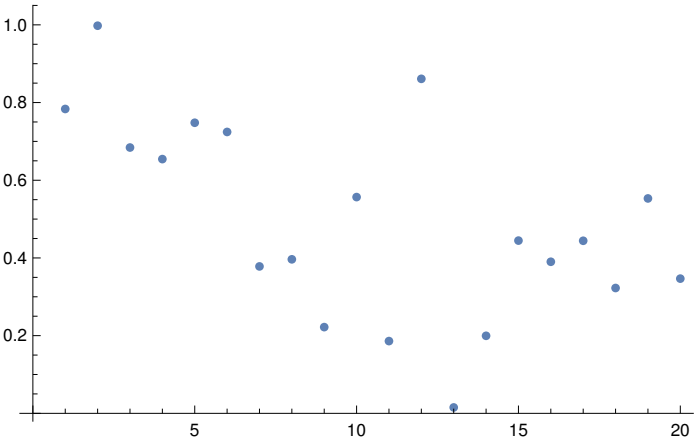
Out[86]= 0.133948

In[87]:=
```
B = Table[{i, RandomReal[]}, {i, 1, 20}]
```

Out[87]=
$$\{\{1, 0.783448\}, \{2, 0.997787\}, \{3, 0.684217\}, \{4, 0.654444\}, \{5, 0.747984\}, \{6, 0.724305\}, \{7, 0.378167\}, \{8, 0.396443\}, \{9, 0.221934\}, \{10, 0.556643\}, \{11, 0.18598\}, \{12, 0.861034\}, \{13, 0.0151922\}, \{14, 0.199585\}, \{15, 0.444647\}, \{16, 0.390158\}, \{17, 0.444204\}, \{18, 0.322614\}, \{19, 0.553169\}, \{20, 0.34676\}\}$$

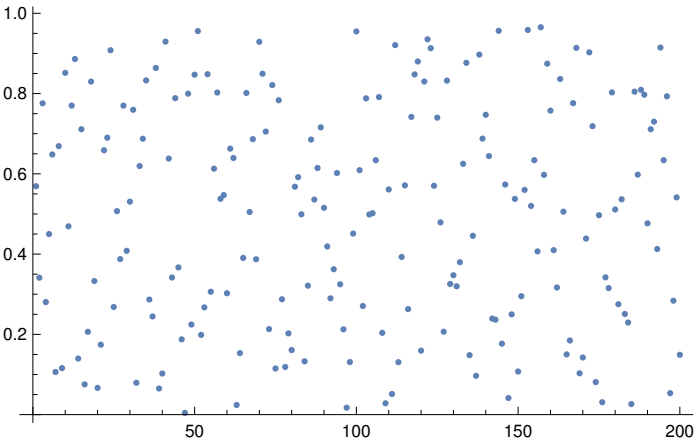
In[88]:=
```
ListPlot[B]
```

Out[88]=



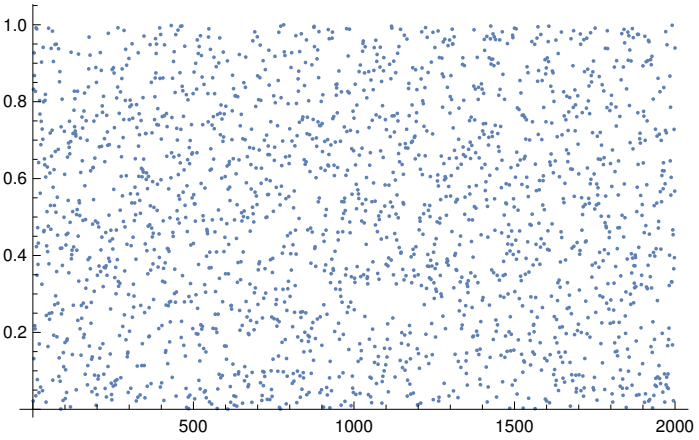
In[89]:=
```
B = Table[{i, RandomReal[]}, {i, 1, 200}];
ListPlot[B]
```

Out[90]=



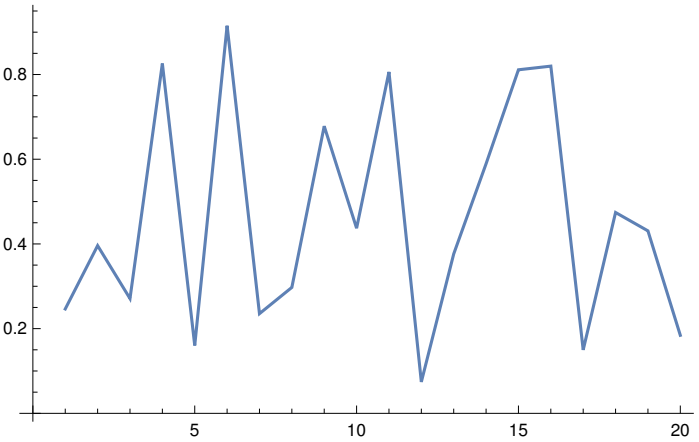
In[91]:=
```
B = Table[{i, RandomReal[]}, {i, 1, 2000}];
ListPlot[B]
```

Out[92]=



In[93]:=
```
B = Table[{i, RandomReal[]}, {i, 1, 20}];
ListPlot[B, Joined → True]
```

Out[94]=



In[121]:=
```
A = Table[i^2 + 0.2 * i + 50 * Sin[j^2 * 1.25 + i], {i, 1, 4}, {j, 1, 4}];
A // MatrixForm
```

Out[122]//MatrixForm=

$$\begin{pmatrix} 40.1037 & -12.7708 & -14.356 & 43.0328 \\ -1.00976 & 37.2493 & 35.9805 & 3.95743 \\ -35.1495 & 59.0679 & 59.2821 & -32.711 \\ -26.1467 & 37.4059 & 38.9061 & -28.4789 \end{pmatrix}$$

In[123]:=
```
Tr[A]
Det[A]
```

Out[123]= 108.156

Out[124]= 3.29655×10^{-11}

In[125]:=
```
Eigenvalues[A]
```

Out[125]= $\{85.6921, 27.2285, -4.76452, -3.18012 \times 10^{-16}\}$

In[126]:=
```
Inverse[A]
```

... Inverse: Result for Inverse of badly conditioned matrix {{40.1037, –12.7708, –14.356, 43.0328}, {–1.00976, 37.2493, 35.9805, 3.95743}, {–35.1495, 59.0679, ≪18≫, –32.711}, {–26.1467, 37.4059, 38.9061, –28.4789}} may contain significant numerical errors.

Out[126]= $\{\{2.3758 \times 10^{13}, -3.89395 \times 10^{13}, 3.80938 \times 10^{13}, -1.32664 \times 10^{13}\},$
$\{7.28372 \times 10^{13}, -1.1938 \times 10^{14}, 1.16788 \times 10^{14}, -4.06722 \times 10^{13}\},$
$\{-7.20379 \times 10^{13}, 1.1807 \times 10^{14}, -1.15506 \times 10^{14}, 4.02259 \times 10^{13}\},$
$\{-2.45573 \times 10^{13}, 4.02495 \times 10^{13}, -3.93754 \times 10^{13}, 1.37128 \times 10^{13}\}\}$

In[128]:=
```
Transpose[A] // MatrixForm
```

Out[128]//MatrixForm=

$$\begin{pmatrix} 40.1037 & -1.00976 & -35.1495 & -26.1467 \\ -12.7708 & 37.2493 & 59.0679 & 37.4059 \\ -14.356 & 35.9805 & 59.2821 & 38.9061 \\ 43.0328 & 3.95743 & -32.711 & -28.4789 \end{pmatrix}$$