Sir Sam Edwards (1928 - 2015) $\xleftarrow{\text{导师}}$ Schwinger

(Lord Kelvin)
...

① Spin Glass ↗ Parisi 2021 Nober prize.
(with P.W. Anderson)

② Polymer
(with P. de. Gennes)

Edwards model : $\frac{\partial}{\partial t}\phi = D\nabla^2\phi + \xi$ $+B(\nabla^2\phi)^2$

↓

kpz eq $\quad \mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4 + h\phi$

经典

S-K model PRL. 1975
(严格可解)

$H = -\frac{1}{2}\sum_{ij} J_{ij}\sigma_i\sigma_j$. 问题: $S(T=0) < 0$

Parisi : "Replica Symmetry Breaking" (RSB) (1979 - 1983)

推广到量子: Sachdev & Ye, 1993. PRL $\sum_{ij} J_{ij}\vec{S_i}\cdot\vec{S_j}$

$\chi \propto \frac{1}{T\ln^2(T)}$. $\chi \propto \ln\omega$

2015: Kitaev "Report on KITP" $H = -\sum_{ijkl}\underbrace{J_{ijkl}}_{\text{完全随机}}\gamma_i\gamma_j\gamma_k\gamma_l$. $\gamma = c + c^\dagger$

or $\gamma = i(c - c^\dagger)$

2016 PRD: Maldacean, Slanford

Remarks on Sachdev - Ye - Kitaev model

↓

- Strange matter
- Black hole
- Hawking - entropy
- Gravity

关键知识: Replica trick

Replica Symmetry Breaking

中心极限 & 大数定理

无序 ⇒ 平均 ⇒ 平均是什么的平均?

$Z = Tr(e^{-\beta H}) = e^{-\beta F}$

有问题

1) $\bar{Z} = \int Tr(e^{-\beta H})P(\xi)d\xi = \overline{e^{-\beta F}} \simeq e^{-\beta\bar{F}} \Rightarrow$ $C_V = \frac{\partial\bar{F}}{\partial T}$

$S =$

×

计算简单

$\ln z = \lim_{n\to 0}\frac{z^n - 1}{n}$ (Parisi) 深刻含义

2) $F = -\frac{1}{\beta}\ln z$, $\bar{F} = -\frac{1}{\beta}\int \ln(z)P(\xi)d\xi$ × 做不出来.

$$F = -\frac{1}{\beta}\lim_{n\to 0}\int \frac{Z^n-1}{n} P(\xi)\, d\xi \quad , \quad Z = Tr(e^{\beta H})$$

$$H = -\frac{1}{2}\sum_{ij} J_{ij}\,\sigma_i\sigma_j \quad , \quad \sigma_i = \pm 1$$

$$Z = Tr\, e^{\frac{1}{2}\beta\sum_{ij}\sigma_i\sigma_j}$$

$$Z^n = Tr\, e^{\frac{1}{2}\beta\sum_{ij}\sum_{\alpha=1}^{n} J_{ij}\sigma_i^\alpha\sigma_j^\alpha}$$

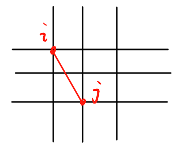
n份 copy
有相同的 J_{ij}

eg: $Z = \int e^{-x^2}dx$, $Z^n = \int e^{-(x_1^2+\cdots+x_n^2)}dx_1\cdots dx_n$

$$F = -\frac{1}{\beta}\lim_{n\to 0}\frac{1}{n}(\overline{Z^n}-1)$$

$$\overline{Z^n} = \int Tr\left(e^{\frac{1}{2}\beta\sum_{ij}\sum_{\alpha=1}^{n}J_{ij}\sigma_i^\alpha\sigma_j^\alpha}\right) P(J_{ij})\, dJ_{ij}$$

假设 $\int Tr = Tr(\int\ )$ (相变点附近可能不对)



$$= Tr\left(\int e^{\frac{1}{2}\beta\sum_{ij}\sum_{\alpha=1}^{n}J_{ij}\sigma_i^\alpha\sigma_j^\alpha} P(J_{ij})\, dJ_{ij}\right)$$

$$= Tr\left[\prod_{ij}\int e^{\frac{1}{2}\beta(\sum_{\alpha=1}^{n}\sigma_i^\alpha\sigma_j^\alpha)J_{ij}} P(J_{ij})\, dJ_{ij}\right] \quad , \quad P(x) = \frac{1}{\sqrt{2\pi J^2}}e^{-\frac{x^2}{2J^2}}$$

$$\int P(J_{ij})\, e^{\frac{1}{2}\beta X_{ij}J_{ij}}\, dJ_{ij} \quad , \quad X_{ij} = \sum_{\alpha=1}^{n}\sigma_i^\alpha\sigma_j^\alpha$$

$$= \int \frac{1}{\sqrt{2\pi J^2}}\, e^{-\frac{J_{ij}^2}{2J^2}+\frac{1}{2}\beta X_{ij}J_{ij}}\, dJ_{ij}$$

$$= \int \frac{1}{\sqrt{2\pi J^2}}\, e^{-\frac{1}{2J^2}(J_{ij}^2-\beta X_{ij}J^2 J_{ij})}\, dJ_{ij}$$

$$= \int \frac{1}{\sqrt{2\pi J^2}}\, \exp\left[-\frac{(J_{ij}-\frac{1}{2}\beta J^2 X_{ij})^2}{2J^2}+\frac{1}{2J^2}\cdot\frac{1}{4}\beta^2 J^2 X_{ij}^2\right] dJ_{ij}$$

$$= e^{\frac{1}{8}\beta^2J^2X_{ij}^2} = e^{\frac{1}{8}\beta^2J^2(\sum_\alpha\sigma_i^\alpha\sigma_j^\alpha)^2}$$

$$\overline{Z^n} = Tr\prod_{ij} e^{\frac{1}{8}\beta^2J^2(\sum_\alpha\sigma_i^\alpha\sigma_j^\alpha)^2}$$

$$= Tr\, e^{\frac{1}{8}\beta^2J^2\sum_{ij}(\sum_\alpha\sigma_i^\alpha\sigma_j^\alpha)^2}$$

$$= Tr\, e^{\frac{1}{8}\beta^2J^2\sum_{ij}\sum_\alpha\sum_\beta\sigma_i^\alpha\sigma_j^\alpha\sigma_i^\beta\sigma_j^\beta}$$

$$= Tr\, e^{\frac{1}{8}\beta^2J^2\sum_{\alpha\beta}(\sum_i\sigma_i^\alpha\sigma_i^\beta)(\sum_j\sigma_j^\alpha\sigma_j^\beta)}$$

$$= Tr\, e^{\frac{1}{8}\beta^2J^2\sum_{\alpha\beta}(\sum_i\sigma_i^\alpha\sigma_i^\beta)^2}$$

↑ $2^{N^2/2}$维 ⟶ 1维

$\sum_\alpha\sigma_i^\alpha\sigma_j^\alpha$: same bar

$\sum_i\sigma_i^\alpha\sigma_i^\beta$:
① 只和 i 格点有关
② $\langle X\rangle \simeq \frac{1}{N}\sum_{i=1}^{N}X_i$ (大数定理)

~ 0

$$H = -\frac{J}{2N}\sum_{ij}\sigma_i\sigma_j - h\sum_i \sigma_i$$

$$\mathrm{Tr}(e^{-\beta H}) = \mathrm{Tr}\left[e^{\frac{1}{2}\frac{J\beta}{N}(\sum_i\sigma_i)^2+\beta h(\sum_i\sigma_i)}\right]$$

平均 $\sigma_i \longrightarrow \int P(m)\, e^{\frac{1}{2}\frac{J\beta}{N}m^2+\beta h m}\, dm$ $\quad m = \sum_{i=1}^{N}\sigma_i$

$m = \mathcal{N}(0, N\sigma^2)$

定义：$\sum_{i=1}^{n}\sigma_i^{\alpha}\sigma_i^{\beta} = m^{\alpha\beta} \simeq N$

$$\int P(m)\, e^{\frac{1}{2}\frac{J\beta}{N}m^2}\, dm$$

$$= \int \frac{1}{\sqrt{2\pi A}}\, e^{-\frac{m^2}{2A}+\frac{1}{2}\frac{J\beta}{N}m^2}\, dm \quad \begin{cases} \text{有限：} \int e^{-(\ )m^2}\, dm \\ \text{发散：} \int e^{+(\ )m^2}\, dm \end{cases}$$

变换：
$$\begin{cases} ① \ \ln z = \lim_{n\to 0}\left(\frac{z^n-1}{n}\right) \\ ② \int \mathrm{Tr}(\ )\, P(J)\, dJ_{ij} = \mathrm{Tr}\left[\int P(J)(\ )\, dJ\right] \\ ③ \sum_{ij}\sum_{\alpha} \Leftrightarrow \sum_{\alpha}\left(\sum_i \sum_j\right) \longrightarrow \text{简化 Hilbert 空间} \end{cases}$$

问题：不一定成立.

Next：

$\begin{cases} \text{S-K model} \\ \text{Parisi} \end{cases}$

Quenched disorder

Anealed disorder

(5.9)

无序 —— 概率论、随机过程 ⇒ 核心：$\begin{cases} \text{中心极限} \\ \text{大数定理} \end{cases}$

Brown 运动 ⟶ Black-sholes eq ⟶ Anderson localization 单体

Ito：严格数学化

$\sum\int \overset{?}{=} \int\sum$

相互作用 Edwards-Anderson . Shrington-kirkpartrick eq (SK)

经典多体

Sachdev, Ye 1992：Kitaev 2015 ⇒ SYK model

$\chi \propto \frac{1}{T-\theta}$

Heisenberg：

$$H = \sum_{ij} J_{ij}\, \vec{S_i}\cdot\vec{S_j}$$

$$H = \sum_{ijkl} J_{ijkl}\, \gamma_i\gamma_j\gamma_k\gamma_l$$

$\Rightarrow \langle J\rangle = 0$

$\langle J_\alpha J_\beta\rangle \propto \delta_{\alpha\beta}\left(\frac{1}{N}\right)$