$$\langle h^2(x,t)\rangle = \sum_{kk'} \langle h(k,t)h(k',t)\rangle = \sum_k \langle h(k,t)h(-k,t)\rangle$$

$k \longrightarrow k'=-k$

$$= \sum_k \int_0^t e^{-\nu k^2(t-t')} e^{-\nu k^2(t-t'')} dt' dt''$$

$h = h_1 + h_2$

确 非确定

$$= 2D\sum_k \int_0^t e^{-2\nu k^2(t-t')} dt'$$

$$\sim \sum_k \frac{D}{2k}(1-e^{-2\nu k^2 t})$$

$$= 2D\sum_k e^{-2\nu k^2 t}\int_0^t e^{2\nu k^2 t'}dt'$$

$$\int_0^t e^{At}dt = \frac{1}{A}e^{At}\Big|_0^t = \frac{1}{A}(e^{At}-1)$$

$$= 2D\sum_k e^{-2\nu k^2 t}\frac{1}{2\nu k^2}(e^{2\nu k^2 t}-1)$$

$$= 2D\sum_k \frac{1}{2\nu k^2}(1-e^{-2\nu k^2 t})$$

$$= 2D\left(\frac{1}{2\pi}\right)^d \int_0^\Lambda \frac{dk\, k^{d-1}}{\nu k^2}(1-e^{-2\nu k^2 t})$$

令 $\nu k^2 t = y^2 \iff y = \sqrt{\nu t}\cdot k$

$$= 2D\left(\frac{1}{2\pi}\right)^d \frac{1}{\nu}(\sqrt{\nu t})^{k-2} \int_0^{+\infty} \frac{y^{d-1}}{y^2}(1-e^{-2y})dy$$

$$\propto t^{\left(\frac{d-2}{2}\right)} \cdot \int_0^\Lambda \frac{y^{d-1}}{y^2}(1-e^{-2y^2})dy.$$

$\nu k^2(t-t') \ \eta(k,t')dt']$. KPZ方程重整化:

上讨论: ① Brown model

② Liner EW model $\Rightarrow$ k-space Brown

③ Parisi-Wu $\Leftarrow$ ϕ^3 Theory 图表示.

$$\dot h(k,t) = -(r+\nu k^2)h(k,t) + \eta + g\sum_q h(q,t)h(k-q,t)$$

$\longrightarrow\!\times \ + \ -\!\!<^{\times}_{\times} \ + \ \cdots$

④ $\langle h^2(x,t)\rangle \to t^6$

今天讲: Medina-Hwa-Kardar-Zhang 1989; 1986 KPZ eq — 读懂 Feynman图.

New: Correlated disorder

$$\langle \eta(x,t)\eta(x',t')\rangle = 2D\,\delta(x-x')\,\delta(t-t')$$

$$\downarrow$$

$$2D f_1(x-y)\, f_2(t-t')$$

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DATE

$\downarrow$ k space

$$\langle \eta(k,\omega)\eta(k',\omega')\rangle = 2D\delta(k+k')\delta(\omega+\omega')|k| \leftarrow \frac{1}{\nu k^2+r}$$

$$\to \frac{1}{\sqrt{\nu k^2+r^2}} \sim \frac{1}{k} \qquad \frac{g}{2}h^2 \Leftrightarrow \phi^3$$

主要方程：$h(x,t) = \nu\nabla^2 h + \eta + \frac{g}{2}(\nabla h)^2$ 转到动量空间 (Parisi-Wu ϕ_3)

$$h(x,t) = \sum_{k\omega} e^{i(\vec{k}\vec{x}-\omega t)} h(k,\omega)$$

关心 $e^{i(\vec{k}\vec{x}-\omega t)}$ 系数

$$-i\omega h(k,\omega) = -\nu k^2 h(k,\omega) + \eta(k,\omega) + \frac{g}{2}(iq)(i(k-q))h(q,\Omega)h(k-q,\omega-\Omega)$$

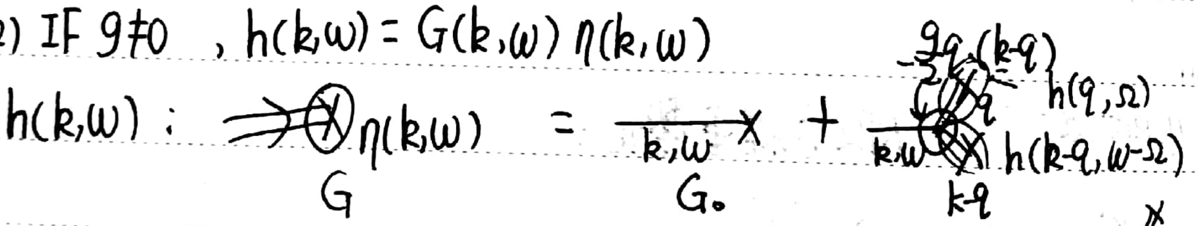
$$(\nu k^2 - i\omega)h(k,\omega) = \eta(k,\omega) - \frac{g}{2}\sum_{q\Omega} q(k-q)h(q,\Omega)h(k-q,\omega-\Omega)$$

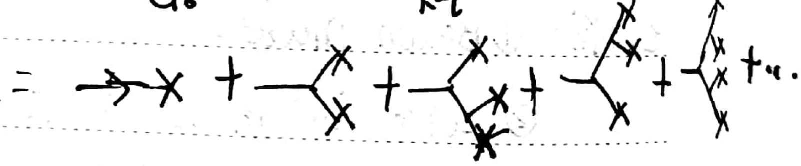
定义 $G_0(k,\omega) = \frac{1}{\nu k^2 - i\omega}$

$$h(k,\omega) = G_0(k,\omega)\eta(k,\omega) - \frac{g}{2}G_0(k,\omega)\sum_{q,\Omega} q\cdot(k-q)h(q,\Omega)h(k-q,\omega-\Omega)$$

1) IF $g=0$, $h(k,\omega) = G_0(k,\omega)\eta(k,\omega)$

2) IF $g\neq 0$, $h(k,\omega) = G(k,\omega)\eta(k,\omega)$





怎么做RG? $\leftrightarrow$ $\langle\phi\phi\rangle$

$\langle\eta\eta\rangle$

$G_0(k,0) = \frac{1}{\nu k^2}$

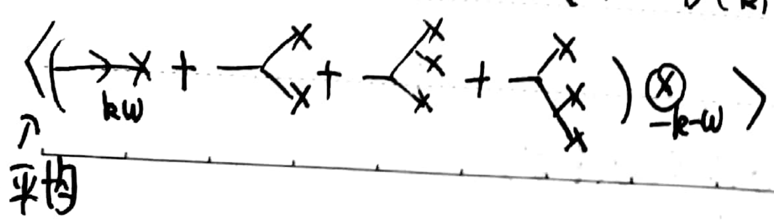
$G(k,0) = \frac{1}{\tilde{\nu} k^2}$

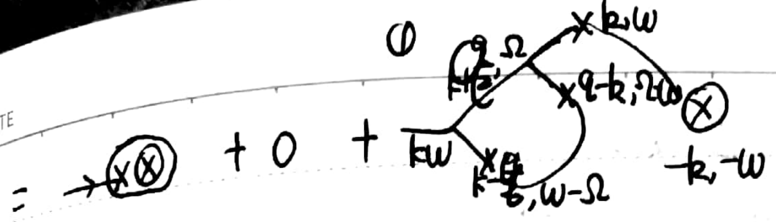
$h(k,\omega) = \Rightarrow\!\!\times = G(k,\omega)\eta(k,\omega)$

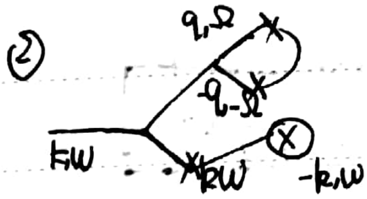
$\langle h(k,\omega)\eta(-k,-\omega)\rangle = G(k,\omega)\langle\eta(k,\omega)\eta(-k,-\omega)\rangle$

$= G(k,\omega)D(k,\omega)$

$\to \langle \times \otimes \rangle = D(k,\omega)$



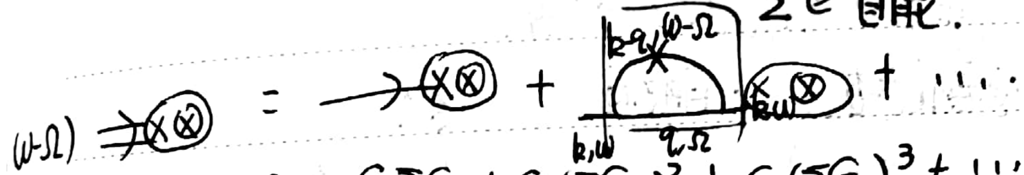


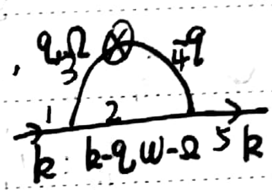


$= 2D(q,\Omega)$

$\langle \eta(q,\Omega)\eta(-q,-\Omega)\rangle$

$\Sigma \leftarrow$ 自能.





$G = G_0 + G_0\Sigma G_0 + G_0(\Sigma G_0)^2 + G_0(\Sigma G_0)^3 + \dots,$

$G(k,\omega) = G_0(k,\omega) + 4\left(-\frac{g}{2}\right)^2 G_0^2(k,\omega)\Sigma$

$\Sigma = \sum_{q,\omega} q(k-q)\cdot k\cdot q\, G_0(q,\Omega)\, G_0(k-q,\omega-\Omega)\, G_0(-q,-\Omega)\, 2D(q,\Omega)$

讨论 g 很小，

$G = \frac{1}{\tilde{\nu}k^2}$ set $\omega = 0$ $(t\to\infty)$

$G_0 = \frac{1}{\nu\cdot k^2}$ $\tilde{\nu} = \nu + \delta\nu$

$G = \frac{1}{(\nu+\delta\nu)k^2}$

$\frac{1}{(\nu+\delta\nu)k^2} = \frac{1}{\nu k^2} + 4\left(-\frac{g}{2}\right)^2\left(\frac{1}{\nu k^2}\right)^2\Sigma$

IF $\left(\frac{1}{A+x}\right) = \frac{1}{A} - \frac{x}{A^2}$, $A = \nu k^2$, $\boxed{x = -g^2\Sigma}$

$$\boxed{\tilde{\nu}k^2 = \nu k^2 - g^2\Sigma}$$

$$\tilde{\nu}k^2 = \nu k^2 - g^2\sum_{q\Omega} q(k-q)kq$$
$$G_0(q,\Omega)\, G_0(-q,-\Omega)$$
$$G_0(k-q,\Omega)\, 2D(q,\Omega)$$

$D(q,\Omega) = D(q) - 2\rho$

$\vec{k}\cdot\vec{q} = kq\cdot\cos\theta$.

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$$= \nu k^2 - g^2\left(\frac{1}{2\pi}\right)^d \int dq\, d\Omega \frac{(kq\cos\theta - q^2)\, kq\cos\theta\; 2D|q|^{-2\rho}}{(\nu q^2 - i\Omega)(\nu q^2 + i\Omega)(\nu(k-q)^2 + i\Omega)}$$

$$\nu(k-q)^2 = \nu(k^2 + q^2 - 2kq\cos\theta)$$

$$\boxed{\tilde{\nu} k^2 = \nu k^2 - A k^2 \int_0^{\Lambda} f(q)\, dq \,.\, q^{d-1}}$$

~~$q \to \infty$~~. ~~$q \to 0$~~ $\qquad \tilde{\nu}_k = \nu - A\int_0^{\Lambda} f(q)\, q^{d-1} dq.$

重整化参数. 有限值.

ϕ^4 Theory $\qquad \Sigma = \Omega \qquad m^2 + \Omega = m_R^2 \qquad \leq \Lambda$

$$\tilde{\nu}_R = \nu_{\Lambda-d\Lambda} A_{\Lambda-d\Lambda} \int_0^{\Lambda-d\Lambda} f_{\Lambda-d\Lambda}(q)\, q^2 dq$$

$$= \nu - \frac{\partial \nu}{\partial \Lambda} d\Lambda \qquad \text{(重整化, 观测结果与}\Lambda\text{无关)}$$

$2d\,\delta(\vec{x})$: $\quad E = -\frac{\hbar^2\Lambda^2}{2m} e^{\left(\frac{\hbar^2}{m\lambda\Lambda}\right)} = \frac{-\hbar^2(\Lambda - d\Lambda)^2}{2m} e^{\frac{\hbar^2}{m(\Lambda-d\Lambda)}}$

$$\frac{d\nu}{dl} = \nu \underbrace{\left[z - 2 - K_{d'} \frac{\lambda^2 D}{\nu^3} \cdot \frac{d-2+f(\Lambda)}{4d'}\right]}_{a}$$

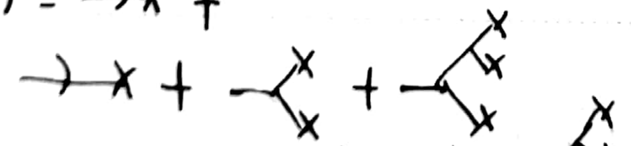
$d' = d-1$

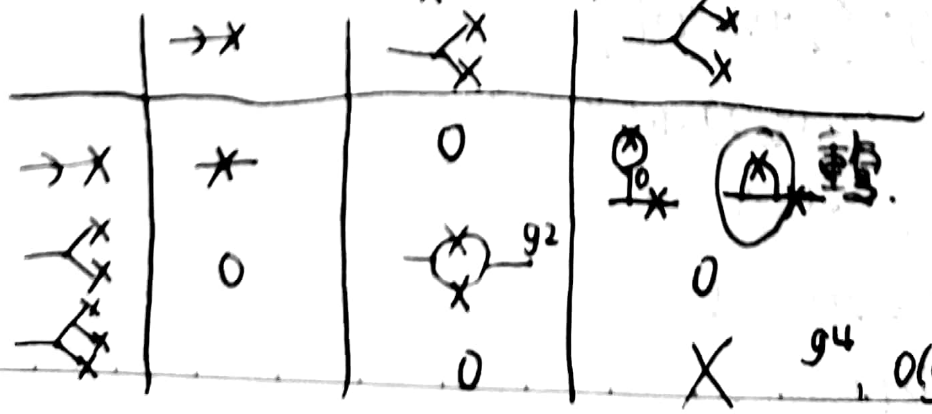
$= a\nu \quad \Longrightarrow \quad \nu = e^{al}$

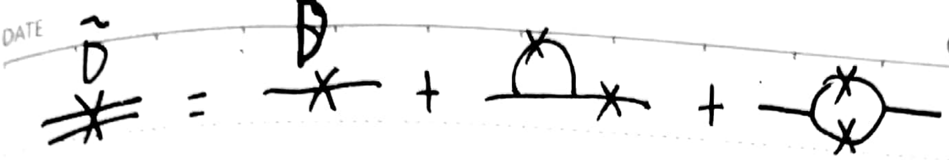
$$\langle h(k,\omega), h(-k,-\omega)\rangle = G(k\omega)\, G(-k,-\omega)\langle \eta(k\omega)\, \eta(-k,-\omega)\rangle$$

$$= 2D(k,\omega)\, G(k,\omega)\, G(-k,-\omega)$$

$h(k,\omega) = \rightarrow\!\times +$

$h =$ 





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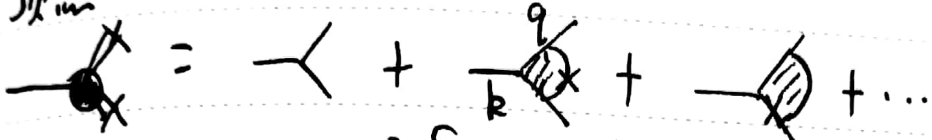

$$\tilde{g}_{修} = g + g^3 \int dq\, dq'\, d\Omega\, d\Omega' \left(\qquad \right)$$

kpz eq

$\frac{d\nu}{dl}$

$\frac{dD}{dl}$

$\frac{dg}{dl}$

SC Fermi Liquid

Gross-Neveu model

GN-Yukawa model

kondo effect

二) 随机微分方程与场论结合 E-W (kardar书)

spin glass: 无序+多体 $\longleftrightarrow$ SYK model.

$\Gamma_{ijkl}\,\gamma_i \gamma_j \gamma_k \gamma_l$.

(图表示)

Replica trick

长程相互作用 (排斥) RSB.

RG

多重尺度分析

边界层

WKB

物质楼 C812.

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