(5.26).

KPZ方程重整化

已讨论：① Brown motion

② Linear E-W model — k-space Brown motion

③ Parisi-Wu $\Leftrightarrow$ ϕ^3 theory 图表示

$$\dot h(k,t) = -[r+\nu k^2]\, h(k,t) + \eta + g\sum_q h(q,t)\, h(k-q,t)$$

④ $\langle h^2(x,t)\rangle \longrightarrow t^\sigma$

今天讲 | Medina - Hwa - Kardar - Zhang 1989

1986年 KPZ eq.

New: Correlated disorder

一般形式：$\langle \eta(x,t)\,\eta(x',t')\rangle = \underline{2D}\,\delta(x-x')\,\delta(t-t')$

$\downarrow$ 可以推广为非白噪声

$2D\, f_1(x-y)\, f_2(t-t')$

or

$[\partial\delta(x-x')]\,\delta(t-t')$

$\downarrow$ k-space.

$$\langle \eta(k,\omega)\,\eta(k',\omega')\rangle = 2D\,\delta(k+k')\,\delta(\omega+\omega')\,\underline{|k|^2}$$

可以改变临界值

eg: Schrödinger eq. dispersion relation $\dfrac{1}{\nu k^2+r} \sim \dfrac{1}{k^2}$

Dirac eq. $\dfrac{1}{\sqrt{\nu k^2+r}} \sim \dfrac{1}{k}$

主要方程：$\dot h(x,t) = \nu\nabla^2 h + \eta + \frac{g}{2}(\nabla h)^2$　　如果是 $\frac{g}{2}h^2 \Leftrightarrow \phi^3$ theory (Parisi-Wu).

动量空间：

$$h(x,t) = \sum_{k\omega} e^{i(\vec k\cdot\vec x - \omega t)}\, h(k,\omega)$$

关心 $e^{i(\vec{k}\cdot\vec{x}-\omega t)}$ 系数.

$$-i\omega h(k,\omega) = -\nu k^2 h(k,\omega) + \eta(k,\omega) + \frac{g}{2}\sum_{q,\Omega}(iq)[i(k-q)]\, h(q,\Omega)\, h(k-q,\omega-\Omega)$$

$$(\nu k^2 - i\omega)\, h(k,\omega) = \eta(k,\omega) - \frac{g}{2}\sum_{q,\Omega} q(k-q)\, h(q,\Omega)\, h(k-q,\omega-\Omega)$$

定义 $G_0(k,\omega) = \dfrac{1}{\nu k^2 - i\omega}$

$$h(k,\omega) = G_0(k,\omega)\,\eta(k,\omega) - \frac{g}{2} G_0(k,\omega)\sum_{q,\Omega} q(k-q)\, h(q,\Omega)\, h(k-q,\omega-\Omega)$$

若多次迭代，表达式复杂

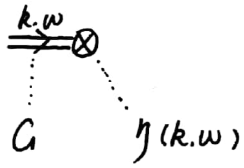
key point: 读懂图

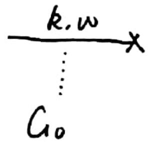
1) IF $g=0$.

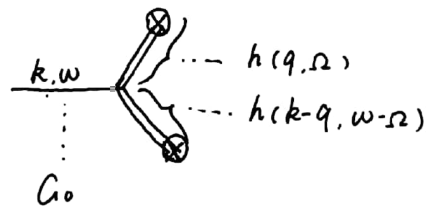
$$h(k,\omega) = G_0(k,\omega)\,\eta(k,\omega)$$

2) IF $g\neq 0$.

$$h(k,\omega) = G(k,\omega)\,\eta(k,\omega)$$ （类似 Dyson eq.）

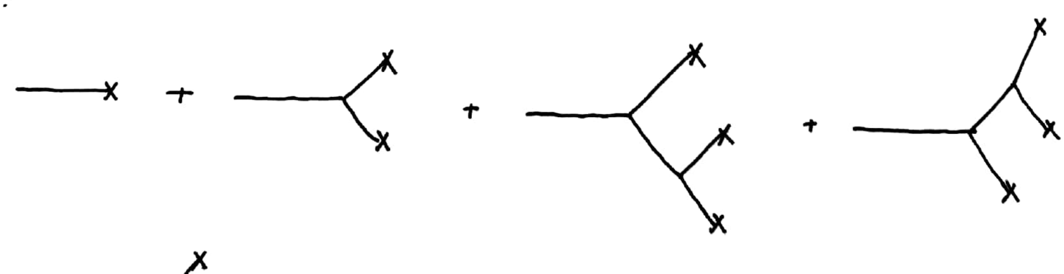
$h(k,\omega) =$ 

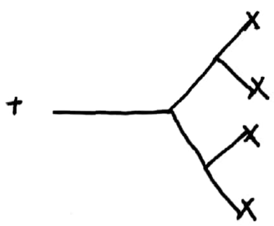
$=$ 

$+$ 

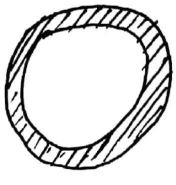
 $= -\frac{g}{2}q(k-q)$

进一步迭代:

$h(k,\omega) =$ 

 $+$ ......

怎么做RG?

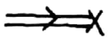


不同的平均　　原来 $\langle \phi\phi \rangle$

现在 $\langle \eta\eta \rangle$

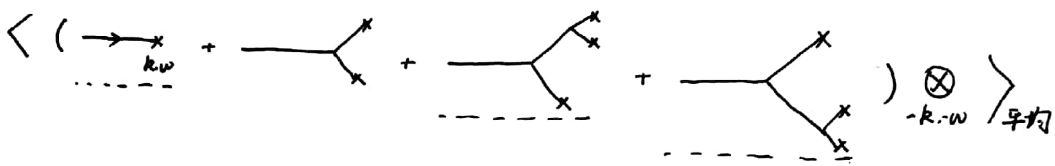
$$G_0(k,0) = \frac{1}{\nu k^2} \quad (\text{真实值})$$

$$G(k,0) = \frac{1}{D k^2} \quad (\text{观测值})$$

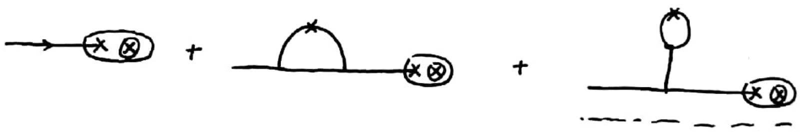
$h(k,\omega) =$  $= G(k,\omega)\,\eta(k,\omega)$

$\langle h(k,\omega)\,\eta(-k,-\omega) \rangle = G(k,\omega)\,\langle \eta(k,\omega)\,\eta(-k,-\omega) \rangle = G(k,\omega)\,D(k,\omega)$

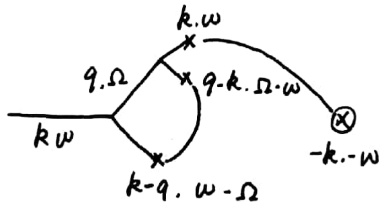
$=$ 

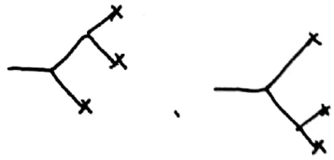
考虑 

只有一三图项能配对有贡献

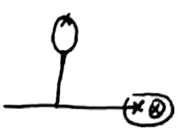
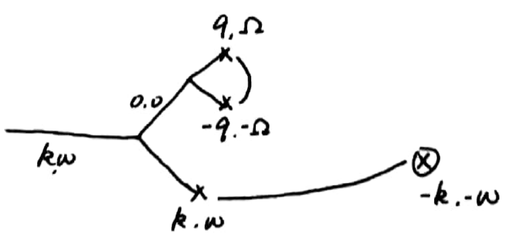
$\langle h(k,\omega)\,\eta(-k,-\omega) \rangle =$ 

(可忽略在动量不为0)

 来源: 

拉直并考虑简并度  都贡献

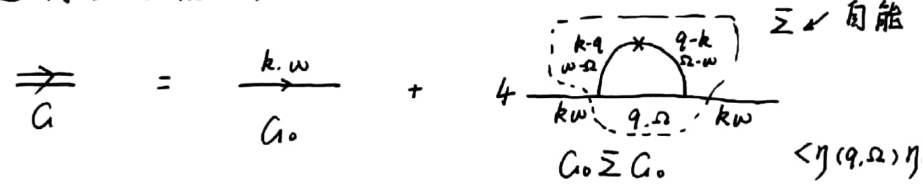
$C_2^1 \times 2$

 来源: 

在动量为0表量图不被允许时可不考虑该图

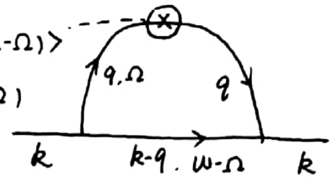
拉直并考虑简并度　2.

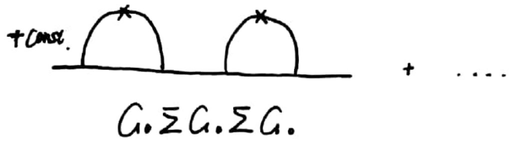
两边约去 $D(k.\omega)$



规定动量方向

$\langle \eta(q.\Omega)\eta(-q.-\Omega)\rangle = 2D(q.\Omega)$





$$G(k.\omega) = G_0(k.\omega) + 4\left(-\frac{g}{2}\right)^2 \underline{G_0^2(k,\omega)\Sigma}$$ 5个传播子

$$\Sigma = \sum_{q.\Omega} q(k-q)kq\, G_0(q.\Omega)\, G_0(k-q.\omega-\Omega)\, G_0(-q.-\Omega)\, 2D(q.\Omega)$$

讨论 g 很小

$$G = \frac{1}{\tilde{\nu}k^2} \quad , \text{ set } \omega = 0 \quad (t\to\infty)$$

$$G_0 = \frac{1}{\nu k^2} \qquad \tilde{\nu} = \nu + \delta\nu$$

$$G = \frac{1}{(\nu+\delta\nu)k^2}$$

$$\frac{1}{(\nu+\delta\nu)k^2} = \frac{1}{\nu k^2} + \underbrace{4\left(-\frac{g}{2}\right)\left(\frac{1}{\nu k^2}\right)\Sigma}_{x}$$

$$\frac{1}{A+x} = \frac{1}{A} - \frac{x}{A^2} \qquad A: \nu k^2 \qquad x = -g^2\Sigma$$

$$\boxed{\tilde{\nu}k^2 = \nu k^2 - g^2\Sigma}$$

$$\tilde{\nu}k^2 = \nu k^2 - g^2 \sum_{q.\Omega} q(k-q)kq\, G_0(q.\Omega)\, G_0(-q.-\Omega)\, G_0(k-q,\omega-\Omega)\, \overset{\langle\eta\eta\rangle}{2D(q.\Omega)}$$

$$= \nu k^2 - g^2\left(\frac{1}{2\pi}\right)^d \int dq \underbrace{d\Omega}_{(-\infty.+\infty)} \frac{(kq\cos\theta - q^2)\, kq\cos\theta\, 2D|q|^{-2\rho}}{(\nu q^2 - i\Omega)(\nu q^2 + i\Omega)(\nu(k-q)^2 \mp i(\omega-\Omega))}$$

一般化: $\tilde{\nu}k^2 = \nu k^2 - Ak^2\int_0^\Lambda f(q)\, dq\, q^{d-1}$ $\qquad f(q) \sim q^{-2\rho-3}$.

回顾 φ^4 theory & $2D$ δ potential.

$$\tilde{\nu}_R = \nu_\Lambda - A_\Lambda \int_0^\Lambda f_\Lambda(q)\, q^{d-1}\, dq$$

$$\tilde{\nu}_R = \nu_\Lambda - A_\Lambda \int_0^\Lambda f_\Lambda(q)\, q^{d-1}\, dq$$

$$= \nu_{\Lambda-d\Lambda} - A_{\Lambda-d\Lambda} \int_0^{\Lambda-d\Lambda} f_{\Lambda-d\Lambda}(q)\, q'\, dq$$

$$= \nu - \frac{\partial \nu}{\partial \Lambda} d\Lambda$$

$\tilde{\nu}_R$：重整化观测值与Λ无关

直接给出RG-flow结果：$\frac{d\nu}{dl} = \nu\left[z-2-K_{d'}\frac{\lambda^2 D}{\nu^3}\frac{d-2+f(\Lambda)}{4d'}\right]$，$d' = d-1$

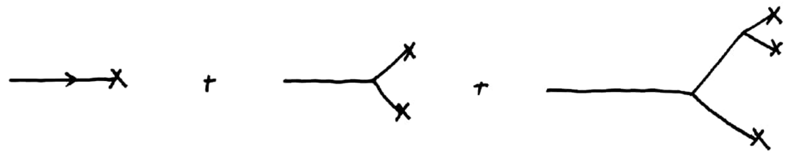
$$= \alpha\nu$$

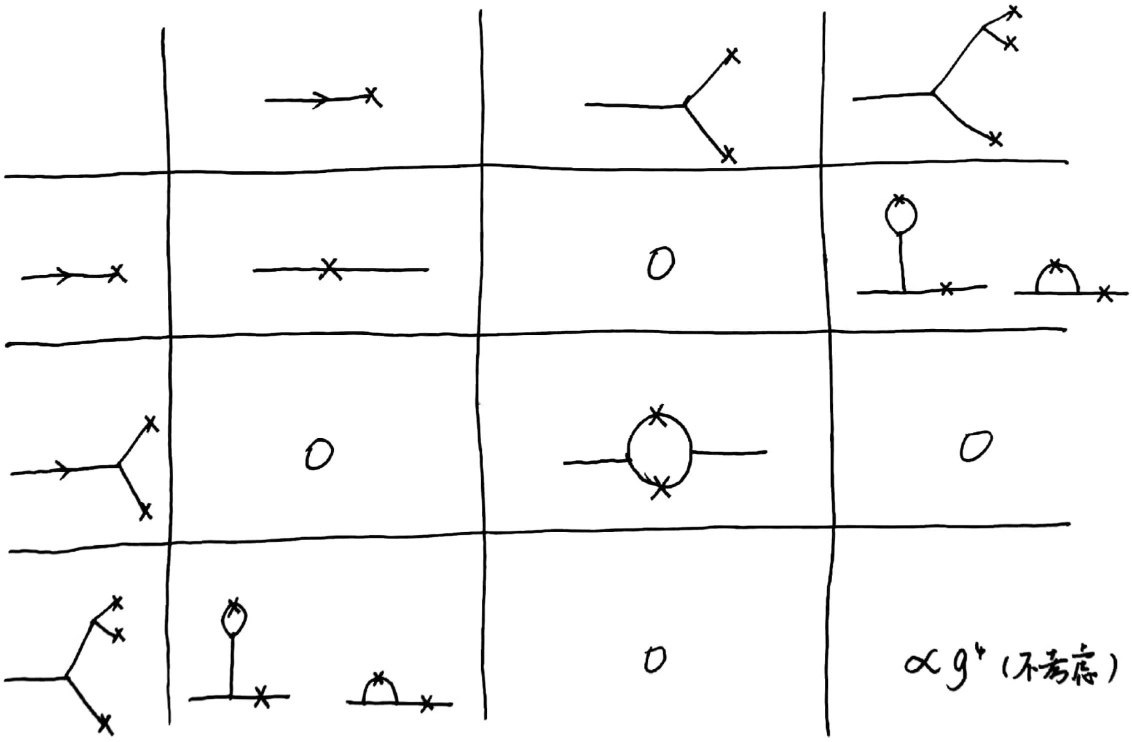
$$\nu \sim e^{\alpha l}$$

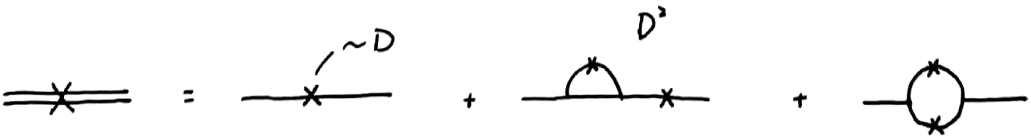
讨论 $\langle h(k,\omega)\, h(-k,-\omega)\rangle = G(k,\omega)\, G(-k,-\omega)\, \langle \eta(k,\omega)\, \eta(-k,-\omega)\rangle$

$$= 2D(k,\omega)\, G(k,\omega) G(-k,-\omega)$$

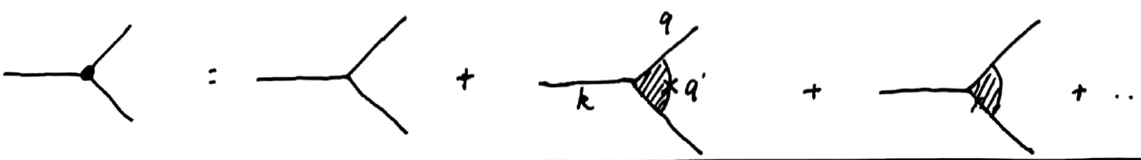
h :







顶点 :



$$\tilde{g} = g_\Lambda + g_\Lambda^3 \int^\Lambda dq\, dq'\, d\Omega\, d\Omega' (\cdots)$$

不变

Summary: KPZ eq

$$\left| \begin{array}{l} \frac{d\nu}{dl} \\ \frac{dD}{dl} \quad \cdots \quad \langle \eta\eta \rangle \\ \frac{dg}{dl} \end{array} \right.$$

⇒ 随机微分方程

场论

E-W model (Kardar 书) ⇒ k-space Brown motion

图表示 Parisi 工作 RSB

spin glass 无序+多体 $Z = \int DJ e^{-S}$

Replica trick 长程相互作用 排序

Fermion:

- SC Fermi Liquid.
- Gross-Neveu model.
- GN-Yukawa model
- Kondo effect.

SYK model $J_{ijkl} \gamma_i \gamma_j \gamma_k \gamma_l$.