(5.26)

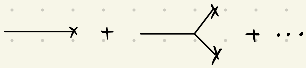
kpz 方程重整化

已讨论：① Brown motion

② Linear E-W model $\Rightarrow$ k-space Brown motion.

③ Parisi-Wu $\to$ ϕ^3 Theory 图表示

$$\dot{h}(k,t) = -[\gamma + \nu k^2]\, h(k,t) + \eta + g \sum_q h(q,t)\, h(k-q,t)$$



④ $\langle h^2(x,t) \rangle \to t^{\sigma}$.

Today: Medina - Hwa - Kardar - Zhang 1989 (kpz eq 1986)

新的东西：correlated disorder

一般来讲：$\langle \eta(x,t)\, \eta(x',t') \rangle = 2D\, \delta(x-x')\, \delta(t-t')$

但也可能是：$2D f_1(x-y)\, f_2(t-t')$ or $(\partial \delta(x-x'))\, \delta(t-t')$

k-space: $\langle \eta(k,\omega)\, \eta(k',\omega') \rangle = 2D\, \delta(k+k')\, \delta(\omega+\omega')\, |k|^{?}$ 改变临界值

dispersion relation: $\frac{1}{\nu k^2 + \gamma} \to \frac{1}{\sqrt{\nu k^2 + \gamma}} \sim \frac{1}{k}$

今天：主要方程：$\dot{h}(x,t) = \nu \nabla^2 h + \eta + \frac{g}{2} (\nabla h)^2 \longleftrightarrow \frac{g}{2} h^2 \to \phi^3$ (Parisi-Wu)

转到动量空间：$h(x,t) = \sum_{k,\omega} e^{i(kx-\omega t)}\, h(k,\omega)$

我们关心的是：$e^{i(kx-\omega t)}$ 的系数.

即：$-i\omega h(k,\omega) = -\nu k^2 h(k,\omega) + \eta(k,\omega) + \sum_{q,\Omega} \frac{g}{2} (iq)\,[i(k-q)]\, h(q,\Omega)\, h(k-q,\omega-\Omega)$

$$\Rightarrow (\nu k^2 - i\omega)\, h(k,\omega) = \eta(k,\omega) - \frac{g}{2} \sum_{q,\Omega} q(k-q)\, h(q,\Omega)\, h(k-q,\omega-\Omega)$$

定义：$G_0(k,\omega) = \frac{1}{\nu k^2 - i\omega}$

$$\Longrightarrow h(k,\omega) = G_0(k,\omega)\, \eta(k,\omega) - \frac{g}{2} G_0(k,\omega) \sum_{q,\Omega} q(k-q)\, h(q,\Omega)\, h(k-q,\omega-\Omega)$$

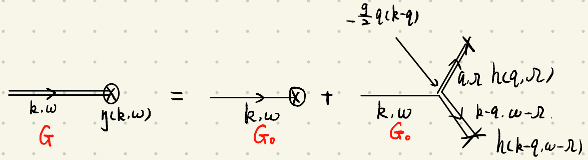
重点：读懂图

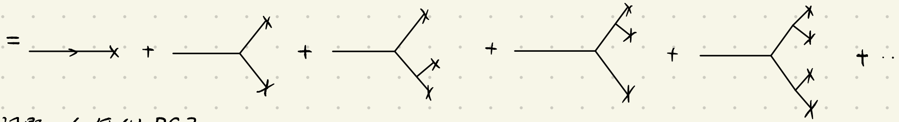
1) $g = 0$

$h(k,\omega) = G_0(k,\omega)\, \eta(k,\omega)$

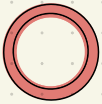
2) $g \neq 0$

$h(k,\omega) = G(k,\omega)\, \eta(k,\omega)$

$h(k,\omega) =$ 



问题：如何做 RG ?



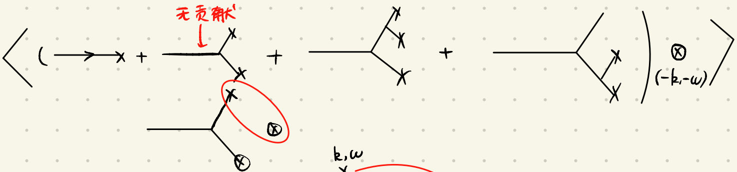
平均：原来 $\langle \phi\phi \rangle$
现在 $\langle \eta\eta \rangle$

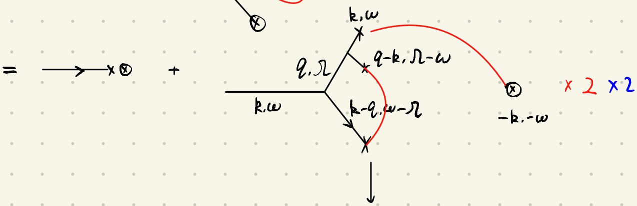
$$G_0(k,0) = \frac{1}{\nu k^2} \longleftrightarrow G(k,0) = \frac{1}{\tilde{\nu} k^2}$$

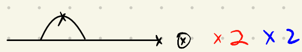
$$h(k,\omega) = \Longrightarrow\!\!\times = G(k,\omega)\,\eta(k,\omega)$$

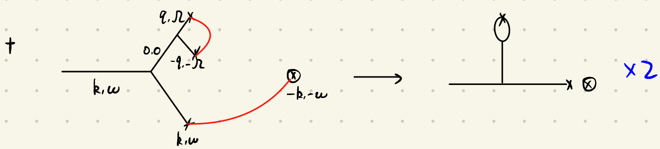
$$\langle h(k,\omega)\,\eta(-k,-\omega)\rangle = G(k,\omega)\langle \underbrace{\eta(k,\omega)\,\eta(-k,-\omega)}_{\times \overset{\text{pairing}}{\longleftrightarrow} \otimes}\rangle = G(k,\omega)\,D(k,\omega)$$

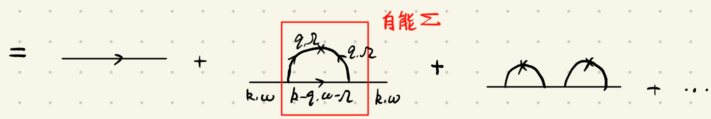
$$\langle \times \otimes \rangle = D(k,\omega)$$











$$G_0 \quad + \quad G_0\,\Sigma\,G_0 \quad + \quad G_0(\Sigma G_0)^2 + G_0(\Sigma G_0)^3 + \cdots$$

考虑简并度：

$$G(k,\omega) = G_0(k,\omega) + 4\left(-\frac{q}{2}\right)^2 G_0^2(k,\omega)\,\Sigma + \cdots$$

$$\text{自能 } \Sigma = \sum_{q,\Omega} q(k-q)\,kq\,G_0(q,\Omega)\,G_0(k-q,\omega-\Omega)\,G_0(-q,-\Omega)\,2D(q,\Omega)$$

讨论g很小的情况.

$$G=\frac{1}{\tilde{\nu}k^2}\ . \quad \text{set } \omega=0 \quad (\text{意义是 } t\to\infty)$$

Taylor expansion:

$$G_0=\frac{1}{\nu k^2} \qquad \tilde{\nu}=\nu+\delta\nu$$

$$\text{IF}\left(\frac{1}{A+x}\right)=\frac{1}{A}-\frac{x}{A^2}$$

$$G=\frac{1}{(\nu+\delta\nu)k^2}$$

$$A=\nu k^2,\ x=-g^2\Sigma$$

$$\frac{1}{(\nu+\delta\nu)k^2}=\frac{1}{\nu k^2}+4\left(-\frac{g}{2}\right)^2\left(\frac{1}{\nu k^2}\right)^2\Sigma$$

$$\Longrightarrow \tilde{\nu}k^2=\nu k^2-g^2\Sigma=\nu k^2-g^2\sum_{q,\Omega} q(k-q)kq\, G_0(q,\Omega)\,G_0(k-q,\omega-\Omega)\,G_0(-q,-\Omega)\,2D(q,\Omega)$$

$$D(q)=D|q|^{-2\rho}$$

$$=\nu k^2-g^2\left(\frac{1}{2\pi}\right)^d\int dq\,d\Omega\,\frac{(kq\cos\theta-q^2)\,kq\cos\theta\ 2D|q|^{-2\rho}}{(\nu q^2-i\Omega)(\nu q^2+i\Omega)(\nu(k-q)^2+i\Omega)}$$

$$\tilde{\nu}k^2=\nu k^2-Ak^2\int_0^\Lambda f(q)\,q^{d-1}dq \ \text{的发散} \begin{cases} q\to\infty \\ q\to 0\ .\ k\text{有限}\end{cases}$$

$$f(q)\sim q^{-2\rho-3}$$

2D δ potential:

$$E=-\frac{\hbar^2\Lambda^2}{2m}e^{\frac{\hbar^2}{m\lambda_\Lambda}}=-\frac{\hbar^2(\Lambda-d\Lambda)^2}{2m}e^{\frac{\hbar^2}{m\lambda_{\Lambda-d\Lambda}}}$$

$$\tilde{\nu}_k=\nu_\Lambda-A_\Lambda\int_0^\Lambda f_\Lambda(q)\,q^{d-1}dq$$

$$=\nu_{\Lambda-d\Lambda}-A_{\Lambda-d\Lambda}\int_0^{\Lambda-d\Lambda}f_{\Lambda-d\Lambda}(q)\,q^2dq$$

$$=\nu-\frac{\partial\nu}{\partial\Lambda}d\Lambda$$

直接给出RG-flow结果: $\frac{d\nu}{dl}=\nu\left[z-2-K_{d'}\frac{\lambda^2D}{\nu^3}\,\frac{d-2+f(\rho)}{4d'}\right]$, $d'=d-1$

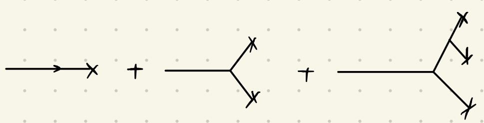
$$=a\nu$$

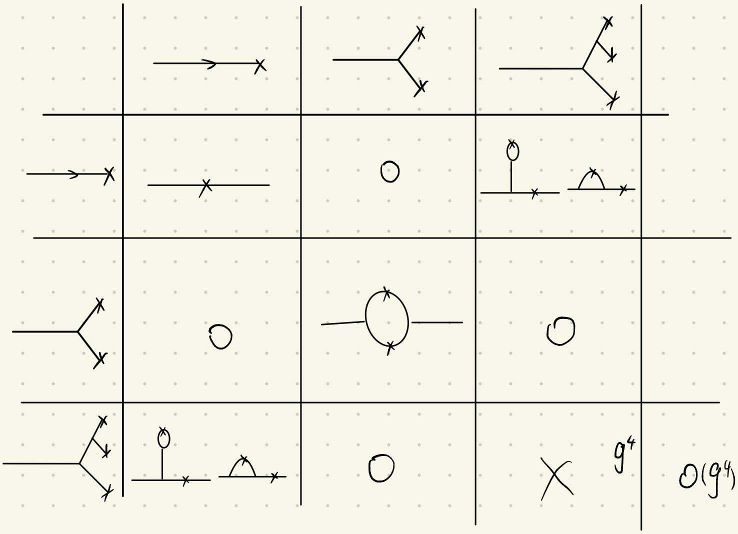
$$\Rightarrow \nu\sim e^{al}$$

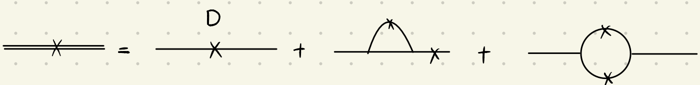
下面讨论: $\langle h(k,\omega)\,h(-k,-\omega)\rangle$

$$=G(k,\omega)\,G(-k,-\omega)\langle\eta(k,\omega)\,\eta(-k,-\omega)\rangle$$

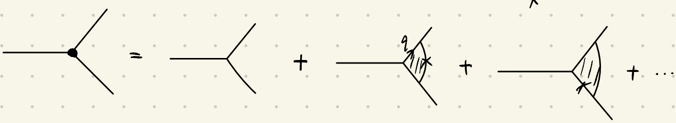
$$=2D(k,\omega)\,G(k,\omega)\,G(-k,-\omega)$$

$h=$ 





顶点：



$$\tilde{g} = g_{\wedge} + \cdots g^3_{\wedge} \int dq\, dq'\, d\Omega\, d\Omega' (\cdots)$$

不变

Summary: kpz eq　除了临界指数：$\begin{cases} \frac{d\nu}{dl} \\ \frac{dD}{dl} \\ \frac{dg}{dl} \end{cases}$　★ 没结束，好的理论有不同证明方法.

讲了 ① 随机微分方程.
② 场论
③ E-W (Kardar 书中的部分) $\longrightarrow$ 动量空间的 Brown motion.
Spin-Glass　无序 + 多体　$Z = \int D\phi\, e^{-S}$?
图表示，Parisi 具体工作.
Replica Trick $\Longrightarrow$ 长程(排斥)作用.　<u>RSB</u>

今年：ϕ^4, Bosonization, SG, kpz

2024年：Fermion　SC, Fermi Liquid. Gross-Neveu model. GN-Yukawa model. Kondo effect.

SYK model
Sachdev-Ye-Kitaev　$J_{ijkl}\chi_i\chi_j\chi_k\chi_l$