无序 → 平均　相似点，$\langle x\rangle = \int D\phi\, x e^{-S} / \int D\phi\, e^{-S}$

$\updownarrow$

量 → 平均

无序 $\Leftrightarrow$ $\langle \eta\eta\rangle \sim \delta(x-x')\delta(t-t')$

parisi 工作（不用固定规范的微扰论）

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{g}{3!}\phi^3 - \frac{m^2}{2}\phi^2 \qquad \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu\phi)} = \frac{\partial \mathcal{L}}{\partial\phi}$$

$$\ddot{\phi} = \partial^2\phi - m^2\phi - \frac{g}{2}\phi^2 - \frac{1}{\mu}\dot{\phi} + \xi$$

$\ddot{\phi} \sim 0$　过阻尼

$$\dot{\phi} = \partial^2\phi - m^2\phi - \cancel{g\phi^2} + \eta \qquad g=0$$

$$\frac{\partial\phi(k,t)}{\partial t} = -\mu_k\phi(k,t) + \eta(k,t) \qquad \mu_k = k^2+m^2$$

$$\phi(k,t) = e^{-\mu_k t}\phi(k,0) + \int_0^t e^{-\mu_k(t-t')}\eta(k,t')dt'$$

$t\to\infty$，t足够大时

$$\phi(k,t) = \int_0^t e^{-\mu_k(t-t')}\eta(k,t')dt' \qquad \langle\phi(-k,t)\phi(k,t)\rangle = \int_0^t e^{-\mu_k(t-t_1)}$$

$$e^{-\mu_k(t-t_2)}\langle\eta(k,t_1)\eta(-k,t_2)\rangle = 2D\int_0^t e^{-2\mu_k(t-t')}dt'$$

$$\overset{t\to\infty}{=} \frac{2D}{2\mu_k} = \frac{D}{k^2+m^2}$$

非平衡系统.（Dynamical）

平均下来. propagator 一样. 都是 $G_0 = \frac{1}{k^2+m^2}$

经典场论 $\Leftrightarrow$ ~

$$\dot{\phi} = \partial^2\phi - m^2\phi - g\phi^2 + \eta \Rightarrow \dot{\phi}(k,t) = -(k^2+m^2)\phi(k,t) + g\sum_q \phi(q,t)\phi(k-q,t)$$

$$+\eta(k,t)$$

$t\to\infty$，对应是 frequency ? $\omega\to 0$ 低能

重点. 1) Feyman 图　2) 如何做平均

$$H = H_0 + V(\vec{x}) \qquad \langle V_i\rangle = 0$$

$$V(\vec{x}) = \sum_i V_i\,\delta(\vec{x}-\vec{x_i}) \qquad \langle V_i V_j\rangle = 2D\delta_{ij}$$

E-W model

$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \eta$ } 动量空间 Brown motion

$$\frac{\partial h(k,t)}{\partial t} = -\nu k^2 h(k,t) + \eta(k,t) \longleftrightarrow \langle \eta(k,t)\eta(-k,t)\rangle = 2D(k)\,\delta(t-t')$$

KPZ 方程：1) KPZ, 1986　2) Medina, kardar, Zhang, 1989, PRA

3) parisi, 吴咏时. 不用固定规范的微扰论, 1980, 中国科学

$$\dot{\phi} = \partial^2\phi - m^2\phi + g\phi^2 + \eta$$

4) Toner, Yuhai Tu.

$$\partial_t \vec{v} + \lambda_1(\vec{v}\cdot\nabla)\vec{v} + \lambda_2(\nabla\cdot\vec{v})\vec{v} + \lambda_3\nabla\vec{v}^2 = \alpha\vec{v} - \beta|\vec{v}|^2\vec{v} - \nabla p + \cdots + \vec{\eta}$$

关键：　$Z = \int D\phi\, e^{-S}$　　$\langle\phi(x)\phi(0)\rangle = \dfrac{\int D\phi\,\phi\phi\, e^{-S}}{\int D\phi\, e^{-S}}$

平均：场在 S 下平均

随机数

1) 构建 S,

2) 不用 S, 求平均　随机数 $\langle\eta\eta\rangle = $:

KPZ eq 1986

$$\frac{\partial h}{\partial t} = \nu\nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta \qquad \langle\eta(\vec{x},t)\eta(\vec{x}',t')\rangle = 2D\,\delta(\vec{x}-\vec{x}')\,\delta(t-t')$$

性质1：伽利略不变性

$t' = t$

$\vec{x}' = \vec{x} + \lambda\vec{\varepsilon}t$

$h' = h + \vec{\varepsilon}\cdot\vec{x}$

2) Burgers eq at $\eta = 0$

$\frac{\partial h}{\partial t} = \nu\partial^2 h + \frac{\lambda}{2}(\partial h)^2 + V(x)$

$W = e^{\frac{\lambda}{2\nu}h}$

$\Rightarrow \frac{\partial W}{\partial t} = \nu\partial^2 W + V(x)W$

性质3：　$\vec{v} = -\nabla h$

$\frac{\partial\vec{v}}{\partial t} + \lambda\vec{v}\cdot\nabla v = \nu\nabla^2\vec{v} - \nabla\eta$　($\nabla\eta$ 随机数)

$\langle\nabla\eta(\vec{x},t)\nabla\eta(\vec{y},t)\rangle \sim \delta(x-y)\,\delta(t-t')\,(\nabla)_i(\nabla)_j$

$\langle\eta(\vec{k},\omega)\eta(\vec{k}',\omega')\rangle \sim f(k,k')\,\delta(k+k')\,\delta(\omega+\omega')$

求解:

$$\frac{\partial h}{\partial t} = \nu \partial^2 h + \frac{\lambda}{2}(\partial h)^2 + \eta$$

无解析解. 只能用微扰 $\lambda = 0$

平均 $\langle \eta \eta \rangle$

$$\ddot{y} + y = \varepsilon(\dot{y} - \frac{1}{3}\dot{y}^3) \qquad y = y_0 + \varepsilon y_1 + \varepsilon^2 y^2 + \cdots \text{ 代入}$$

ε^0:

ε^1:

ε^2:

$$y_0 = A\cos(t+\theta)$$

Parisi - Wu model:

$$\dot{\phi} = \partial^2\phi - m^2\phi + g\phi^2 + \eta \qquad \langle \eta(x,t)\,\eta(x',t')\rangle = 2\,\delta(x-x')\,\delta(t-t')$$