$$\langle x\rangle = \int D\phi\, x\, e^{-S} \Big/ \underbrace{\int D\phi\, e^{-S}}_{Z}$$

无序 → 平均 相似点 $\langle x\rangle = \int D\phi\, x\, P(\phi)$ ，$P(\phi) = \frac{e^{-S}}{Z}$.

$\updownarrow$

$\boxed{\text{区}}$ → 平均 热力学平均 ⟺ 平衡态方式

平衡 ⟺ $D = k_B T \mu$.

无序 $\langle \eta\eta\rangle \sim \delta(x-x')\delta(t-t')$

Parisi 工作：（不用固定规范的做微扰）

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{g}{2!}\phi^3 - \frac{m}{2}\phi^2 \quad , \quad \partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} = \frac{\partial \mathcal{L}}{\partial \phi}$$

$$\ddot\phi = \partial^2\phi - m^2\phi - \frac{g}{2}\phi^2 - \boxed{\frac{1}{\mu}\dot\phi} + \xi . \qquad \text{overdamped regime} \quad \ddot\phi \sim 0 .$$

$$\boxed{\dot\phi = \partial^2\phi - m^2\phi - g\phi^2 + \eta}$$

$$g=0 ,\quad \frac{\partial\phi(k,t)}{\partial t} = -\mu_k \phi(k,t) + \eta(k,t) \quad , \mu_k = k^2 + m^2$$

方差，$t\to\infty$

$\bar\phi = 0$.

$$\phi(k,t) = \underbrace{e^{-\mu_k t}\phi(k,0)}_{\substack{\text{确定 平均 }\bar\phi(k,t)\\ \text{对方差无贡献}}} + \underbrace{\int_0^t e^{-\mu_k(t-t')}\eta(k,t')\,dt'}_{\text{涨落 随机}}$$

t 足够大时，$\phi(k,t) = \int_0^t e^{-\mu_k(t-t')}\eta(k,t')\,dt'$

$$\langle \phi(-k,t)\phi(k,t)\rangle = \int_0^t e^{-\mu_k(t-t_1)} e^{-\mu_k(t-t_2)} \langle \eta(k,t_1)\eta(-k,t_2)\rangle$$

$$= 2D \int_0^t e^{-2\mu_k(t-t')}dt'$$

$t\to\infty$ 时 $= \frac{2D}{2\mu_k} = \frac{D}{k^2+m^2}$ 取 $D=1$ ⟺ $\langle \phi(-k,t)\phi(k,t)\rangle = \frac{1}{k^2+m^2}$

非平衡系统（dynamical）

平均下来 propagator 一样，都是 $G_0 = \frac{1}{k^2+m^2}$.

经典场论中 $\int D\phi\, e^{-\beta H} \sim \int D\phi\, e^{-\beta(k^2+m^2)\phi_k^*\phi_k}$ ，$\langle \phi_k^*\phi_k\rangle = \frac{1}{\beta(k^2+m^2)} = \frac{k_B T}{k^2+m^2}$

$\dot\phi = \mu(\partial^2\phi - m^2) + \eta$ ⟺ $\mu_k = \mu(k^2+m^2)$

Einstein relation

$$\langle \phi(-k,t)\phi(k,t)\rangle = \frac{D}{\mu(k^2+m^2)} = \frac{k_BT\mu}{\mu(k^2+m^2)} \qquad (D=k_BT\mu)$$

$\langle X\rangle = \langle \phi_k^*, \phi_k\rangle = \frac{k_BT}{k^2+m^2}$ $\longrightarrow$ 相变 (critical phys).

$\downarrow$ $\qquad\qquad$ $\downarrow$

$\langle \phi(-k,t)\phi(k,t)\rangle = \frac{k_BT}{k^2+m^2}$ $\qquad$ 相变 (dynamical critical phys)

KPZ: 1986 $\quad g\phi^2 \longrightarrow g(\nabla\phi)^2$

$\mu=1,\ D=1$ | $\dot\phi = \partial^2\phi - m^2\phi - g\phi^2 + \eta$, $\quad \dot\phi(k,t) = -(k^2+m^2)\phi(k,t) + g\sum_q \phi(q,t)\phi(k-q,t) + \eta(k,t)$

$t\to\infty$，对应是frequency区间什么物理 $\boxed{\omega\to 0}$.

$f(t) = \sum_\omega f_\omega e^{i\omega t}$, $\quad \frac{1}{t}\int_0^t f(t)\,dt = f_0$ (低频有效) 低能物理.

接下来重点: 1) 认识Feynman图，了解规则.

2) 如何做平均 $\longrightarrow X$

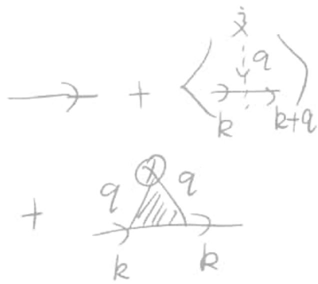
$H = H_0 + V(\vec{x})$

$V(\vec{x}) = \sum_i V_i\,\delta(\vec{x}-\vec{x}_i)$

$$\begin{cases}\langle V_i\rangle = 0 \\ \langle V_i V_j\rangle = 2D\delta_{ij}\end{cases}$$

phys, transport.

$$S = \underset{\text{2D Dirac}}{S_\psi} + \underset{\substack{\text{scalar field}\\ \frac12(\partial\phi)^2-\frac{m^2}{2}\phi^2}}{S_\phi} + \overset{g\phi\psi^\dagger\sigma_2\psi}{S_g} + \underset{\frac{\lambda}{4!}\phi^4}{S_\lambda} + \overset{S_I=\int V_i(x)\psi^\dagger\psi\,dx}{S_I}$$



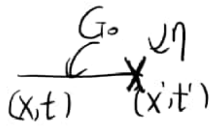
Replica trick $\Leftarrow$ spin Glass.

无序 $\Rightarrow$ 平均 $\quad$ 1) Replica trick, $n\to 0$

2) Brown motion. $\langle\eta\eta\rangle = 2D\,\delta(x-x')\,\delta(t-t')$

★ $\langle\eta\eta\eta\eta\rangle \Rightarrow$ All pairings.

3) Transport $\langle G\rangle = \frac{1}{N}\sum_i G(\xi_i)$.



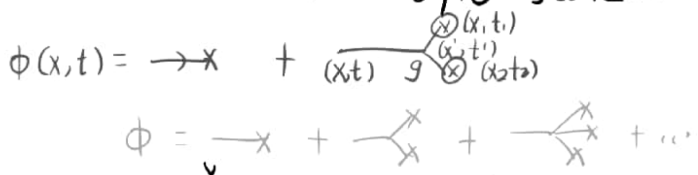
$\Leftrightarrow G\eta \Rightarrow G*\eta$ 卷积

$$\dot\phi = \partial^2\phi - m^2\phi^2 + g\phi^2 + \eta .$$

$$\phi(x,t) = \int G(x-x',t-t')\left[\eta(x',t') + g\phi^2(x',t')\right]dx'dt' .$$

$$= \int G(x-x',t-t')\,\eta(x',t')\,dx'dt' + g\int G(x-x',t-t')\,dx'dt'\int G(x'-x_1,t'-t_1)\left[\eta+g\phi^2\right]$$

$$G(x'-x_2,t'-t_2)\left[\eta+g\phi^2\right]dx_2dt_2dx_1dt_1 + \cdots .$$



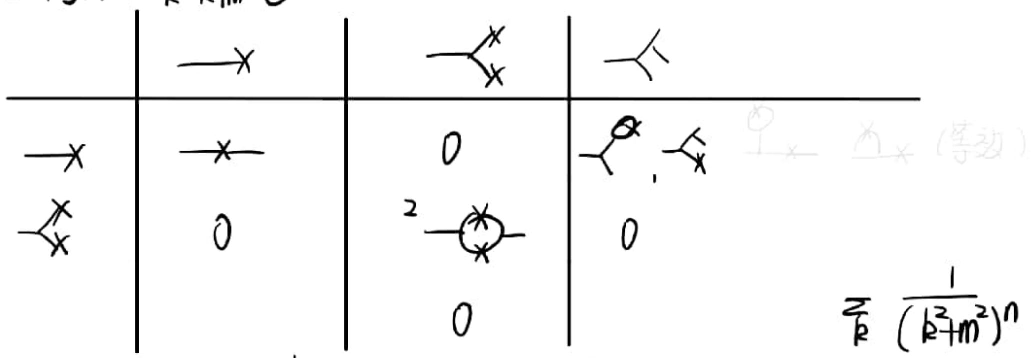
$\Leftrightarrow$ $\langle\phi^2(x,t)\rangle$, $\langle\phi(x,t_x)\phi(y,t_y)\rangle$

$\langle\phi^2(x,t)\rangle =$  $\quad t\to\infty \quad = \sum_k \frac{1}{u_k} = \sum_k \frac{1}{k^2+m^2}$

$\langle\phi(x,t_x),\phi(y,t_y)\rangle = \underset{(x,t_x)\qquad(y,t_y)}{\overset{(x',t')}{—}} = \sum_k \frac{1}{k^2+m^2}e^{ik(x-y)}$ 相差一个傅利叶变换

$\downarrow$

$\langle\phi(x)\phi(y)\rangle \sim \sum_k \frac{1}{k^2+m^2}e^{ik(x-y)}$



$$\sum_k \frac{1}{(k^2+m^2)^n}$$

$$G(x-x',t-t') = \frac{1}{i\omega-(k^2+m^2)}e^{i(kx-\omega t)}$$

$$\frac{D(k)}{u(k^2+m^2)} \propto \frac{k}{k^2+m^2} \sim \frac{1}{k}$$

$D(k)$重要.

$\boxed{\partial^4\phi}$ 为什么不讨论？ 这项对低能有效模型不起作用 $\because$ 其 $\propto k^4\phi$.

Navier - Stoke eq .