Fourier trans: $\dot{\phi}(k,t) = -k^2\phi(k,t) - m^2\phi(k,t) + g\sum_q \phi(q,t)\,\phi(k-q,t) + \eta(k,t)$

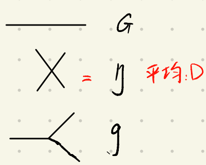
讨论: $\phi(x,t) = \int G(x-y, t-\tau)\left[\eta(y,\tau) + g\phi^2(y,\tau)\right] dy\, d\tau$

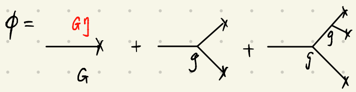
$g=0 \downarrow \quad = \int G(x-y, t-\tau)\,\eta(y,\tau)\, dy\, d\tau \Rightarrow \langle \phi(x,t)\,\phi(x',t')\rangle_{平均}$ S / Disorder

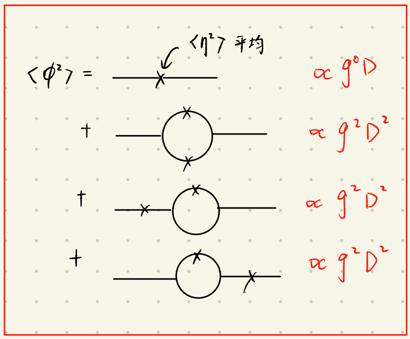
$\langle \phi(x,t)\,\phi(x',t')\rangle = \int G(x-y, t-\tau)\, G(x'-y', t'-\tau')\,\langle \eta(y,\tau)\,\eta(y',\tau')\rangle\, dy\, dy'\, d\tau\, d\tau'$

$= 2\int G(x-y, t-\tau)^2\, dy\, d\tau$

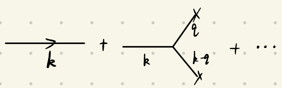
$g \neq 0$, 上面的解用 propagator 表示.





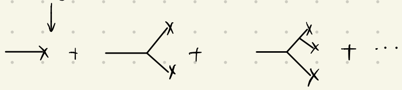


回到 KPZ. $h =$



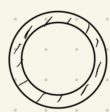
$h = G\eta - \frac{\lambda}{2} q(k-q)\, Ghh$

$= G\eta - \frac{\lambda}{2} q(k-q)\, G_0 \left[G_0\eta - \frac{\lambda}{2} q(k-q)\, G_0 hh\right]\left[G_0\eta - \frac{\lambda}{2} q(k-q)\, G_0 hh\right]$



如何平均: 1) $\int \langle h^2(x,t)\rangle dx = \sigma^2 \propto t^?$

$\int_0^{\Lambda e^{-dl}} dq + \int_{\Lambda e^{-dl}}^{\Lambda} dq$



Shang-Keng Ma 马上庚

(5.23) $\boxed{无序}$ → 平均
↕
$\boxed{区}$ → 平均

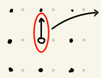
相似点
$\langle x\rangle = \int D\phi\, x\, e^{-S} / \int D\phi\, e^{-S}$

$= \int D\phi\, x\, P(\phi)$, $P(\phi) = e^{-S}/Z$

场平均: $P(\phi)$

无序平均: 热力学平均 ⟺ 平衡态方式.

$\langle \eta\eta\rangle = \delta(x-x')\,\delta(t-t')$

 → 平衡 ⇒ $D = k_B T \mu$ ← 等效温度

(运动由环境决定)

Parisi 工作（不用固定规范的微扰论） 1980.

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{g}{3}\phi^3 - \frac{m^2}{2}\phi^2$$

EOM: $\partial_\mu \frac{\partial \mathcal{L}}{\partial(\partial_\mu\phi)} = \frac{\partial \mathcal{L}}{\partial \phi} \Rightarrow \ddot{\phi} = \partial^2\phi - m^2\phi - g\phi^2 - \frac{1}{\mu}\dot{\phi} + \xi$ 随机力

考虑 overdamped regime: $\ddot{\phi} \sim 0$: $\dot{\phi} = \partial^2\phi - m^2\phi - g\phi^2 + \eta$

Brown motion: $m\ddot{x} = -\frac{1}{\mu}\dot{\phi} + f + \xi$

$g=0$. $\frac{\partial\phi(k,t)}{\partial t} = -\mu_k\phi(k,t) + \eta(k,t)$, $\mu_k = k^2+m^2$

$$\Rightarrow \phi(k,t) = \underbrace{e^{-\mu_k t}\phi(k,0)}_{\text{确定平均}} + \underbrace{\int_0^t e^{-\mu_k(t-t')}\eta(k,t')\,dt'}_{\text{涨落随机}}$$

$\bar{\phi}(k,t)$, 对方差无贡献

方差: $t\to\infty$: $\bar{\phi} = 0$ $\quad \phi(k,t) = \int_0^t e^{-\mu_k(t-t')}\eta(k,t')\,dt'$

$$\langle\phi(-k,t)\phi(k,t)\rangle = \int_0^t e^{-\mu_k(t-t_1)} e^{-\mu_k(t-t_2)} \langle\eta(k,t_1)\eta(-k,t_2)\rangle$$

$$\langle\phi_k^*\phi_k\rangle = \langle\phi_{-k}\phi_k\rangle \qquad = 2D\int_0^t e^{-2\mu_k(t-t')}\,dt'$$

$$\overset{t\to\infty}{=} \frac{2D}{2\mu_k} = \frac{D}{k^2+m^2}$$

$$\overset{\text{取}D=1}{=} \frac{1}{k^2+m^2}$$

场的平均: $\langle\phi_k^*\phi_k\rangle = \frac{1}{k^2+m^2} \longleftrightarrow$  $= \sum_k \frac{1}{k^2+m^2}$ (Green fun)

非平衡系统 (Dynamical)

平均下来 propogator 一样. $G_0 = \sum_k \frac{1}{k^2+m^2}$

经典场论 $\int D\phi\, e^{-\beta H} \sim \int D\phi\, e^{-\beta(k^2+m^2)}\phi_k^*\phi_k$ $\qquad \dot{\phi} = \mu(\partial^2\phi - m^2) + \eta$

$\langle\phi_k^*\phi_k\rangle = \frac{1}{\beta(k^2+m^2)}$ $\qquad \Longleftrightarrow \mu_k = \mu(k^2+m^2)$

$$\langle\phi(-k,t)\phi(k,t)\rangle = \frac{D}{\mu(k^2+m^2)}$$

$\underline{\underline{\text{Einstein relation}}}$ $D = k_B T\mu$ $\qquad \frac{k_B T}{k^2+m^2}$

$Z \to F \to \chi, C_v$

相似点: $\langle X\rangle = \langle\phi_k^*\phi_k\rangle = \frac{k_BT}{k^2+m^2} \longrightarrow$ 相变 critical phys

$\langle\phi(-k,t)\phi(k,t)\rangle = \frac{k_BT}{k^2+m^2} \longrightarrow$ 相变 Dynamical critical phys.

Haprin equation $\quad g\phi^3$

KPZ: 1986. application. $g(\nabla\phi)^2$.

注: 交通流
网络

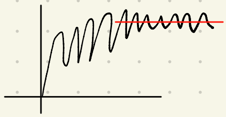
$\mu = 1.\ D = 1$

$$\dot\phi = \partial^2\phi - m^2\phi - g\phi^2 + \eta$$

$$\Rightarrow \dot\phi(k,t) = -(k^2+m^2)\,\phi(k,t) + g\sum_q \phi(q,t)\,\phi(k-q,t) + \eta(k,t)$$

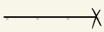
$t\to\infty$，对应 frequency 空间的什么物理？ $\omega = 0$. 低能

$$f(t) = \sum_\omega f(\omega)\, e^{i\omega t}.$$

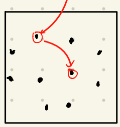


$$\frac{1}{t}\int_0^t f(t)\,dt = f(0)$$

重点：① 认识 Feynman 图，Feynman 规则
② 知道如何做平均



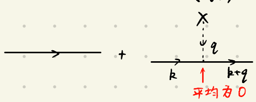
$\langle G \rangle$

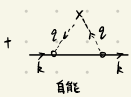


一个无序模型：$H = H_0 + V(\vec x)$

$$V(\vec x) = \sum_i V_i\,\delta(\vec x - \vec x_i)$$

$$\langle V_i \rangle = 0,\ \langle V_i V_j \rangle = 2D\delta_{ij}$$





ref: Uryszek PRB 2022

$$S = S_\psi + S_g + S_\lambda + S_\phi + S_I$$

2D Dirac eq

$g\phi\psi^\dagger\sigma_z\psi$

$\frac{\lambda}{4!}\phi^4$

scalar field. $\frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2$

$$S_I = \int V_i(\vec x)\,\psi^\dagger\psi\, d\vec x$$

Replica Trick：把 Fermi 场积掉.

无序 $\Rightarrow$ 平均：1) Replica trick：$n\to 0$

多体 & 相变.

2) Brown motion：$\langle\eta\eta\rangle = 2D\,\delta(x-x')\,\delta(t-t')$

$\langle\underbrace{\eta\eta}\,\underbrace{\eta\eta}\rangle \Rightarrow$ All pairings.

3) Transport：$\langle G \rangle = \frac{1}{N}\sum_i G(\xi_i)$

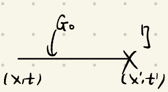
$$\langle F \rangle = \frac{1}{N}\sum_i F(\xi_i)$$

$$\dot\phi = \partial^2\phi - m^2\phi + g\phi^2 + \eta$$

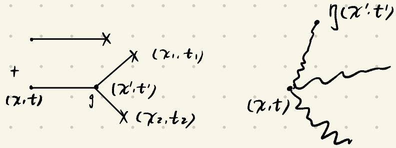
$$\phi(x,t) = \int G(x-x',t-t')\,[\eta(x',t') + g\phi^2(x',t')]\,dx'dt'$$

$$= \int G(x-x',t-t')\,\eta(x',t')\,dx'dt' + g\int G(x-x',t-t')\,dx'dt'$$

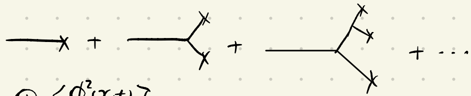
$$\int G(x'-x_1,t'-t_1)\,[\eta + g\phi^2]\,G(x'-x_2,t'-t_2)\,[\eta + g\phi^2]\,dx_1dt_1dx_2dt_2$$



$\Leftrightarrow G\eta \to G*\eta.$

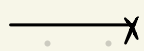
$\phi(x,t)$ 

$$g\int G(x-x',t-t')\,G(x'-x_1',t'-t_1)\,G(x'-x_2,t'-t_2)\,\eta(x_1,t_1)\,\eta(x_2,t_2)\,dx'dt'dx_1dt_1dx_2dt_2.$$

$\phi =$  $+ \cdots$

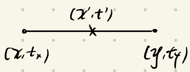
$\Longleftrightarrow$ ① $\langle\phi^2(x,t)\rangle$

② $\langle\phi(x,t_x)\,\phi(y,t_y)\rangle$

若 $\phi =$ 

$\langle\phi^2(x,t)\rangle =$ 

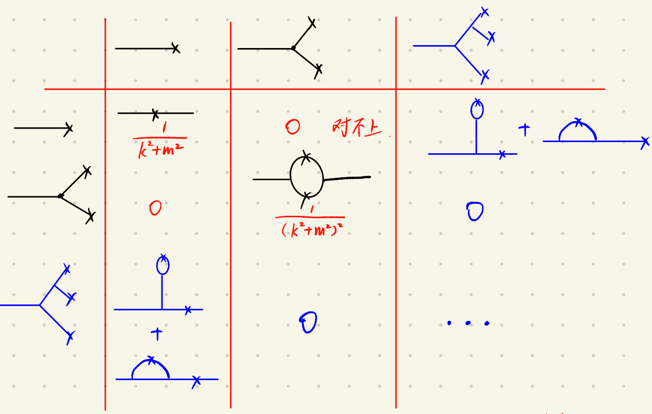
$,\ \xrightarrow{t\to\infty} \sum_k \frac{1}{\mu_k} = \sum_k \frac{1}{k^2+m^2}$ FT

$\langle\phi(x,t_x),\phi(y,t_y)\rangle =$ 

$= \sum \frac{1}{k^2+m^2}\, e^{ik\cdot(x-y)}$

若 考虑两阶.

若 三阶.



$\partial^4\phi \propto k^4\phi$ suppressed at low k.

$$G(x,t) = \frac{1}{i\omega-(k^2+m^2)}\, e^{i(kx-\omega t)}$$

$\sum_k \frac{1}{(k^2+m^2)^n}$, n越小,越不容易收敛.

$D = D(k)$? $\frac{D(k)}{k^2+m^2} \propto \frac{k^\rho}{k^2+m^2} \sim \frac{1}{k^{2-\rho}}$. δ function.

不是白噪声

Navier–Stokes eq.

下节课:计算临界行为

$D_\Lambda,\ \lambda_\Lambda,\ \upsilon_\Lambda$ G_k

kpz 原始论文