$$= 2D\sum_k e^{-2\nu k^2 t}\frac{1}{2\nu k^2}(e^{2\nu k^2 t}-1)$$

$$= 2D\sum_k \frac{1}{2\nu k^2}(1-e^{-2\nu k^2 t})$$

$$= 2D(\frac{1}{2\pi})^d\int_0^\Lambda \frac{dk\, k^{d-1}}{\nu k^2}(1-e^{-2\nu k^2 t})$$

令 $\nu k^2 t = y^2 \rightarrow y = \sqrt{\nu t}\, k$

$$= 2D(\frac{1}{2\pi})^d \frac{1}{\nu}(\sqrt{\nu t})^{d-2}$$

$$\propto \int_0^\infty \frac{y^{d-1}}{y^2}(1-e^{-2y})\,dy \quad t^{(\frac{d-2}{2})}$$

E-W model

$\frac{\partial h}{\partial t} = \nu\nabla^2 h + \eta$ 动量空间 Brown motion

$$\frac{\partial h(k,t)}{\partial t} = -\nu k^2 h(k,t) + \eta(k,t)$$

$\frac{\partial f}{\partial t} = -\mu f + \eta \qquad \langle \eta(k,t)\eta(-k,t')\rangle = 2D(k)\delta(t-t')$

$$f(t) = f(0)e^{-\mu t} + \int_0^t e^{-\mu(t-t')}\eta(t')dt'$$

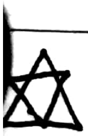
$$\sigma^2 = \langle (h(x,t)-\bar{h}(x,t))^2\rangle \propto t^{6}$$

KPZ equation $\frac{\partial\phi}{\partial t} = \nu\partial^2\phi + (\frac{\lambda}{2})(\partial\phi)^2 + \eta$

$$\dot{\phi} = \partial^2\phi - m^2\phi + g\phi^2 + \eta$$

(1998)

$$\partial_t\vec{v} + \lambda_1(\vec{v}\cdot\nabla)\vec{v} + \lambda_2(\nabla\cdot\vec{v})\vec{v} + \lambda_3\nabla\vec{v}^2 = \alpha\vec{v} + \beta|\vec{v}|^2\vec{v} -$$

100000...

OM OT OW OT OF OS OS

DATE NOTES

$\phi + \cdots + \vec{\eta}$

~~场的平均~~ ~~杂质平均~~

KPZ $\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta$

$\langle \eta(\vec{x}, t)\eta(\vec{x}', t')\rangle = 2D\,\delta(\vec{x}-\vec{x}')\,\delta(t-t')$

性质 1. 伽利略不变性

$t' = t$ $\quad\quad \vec{h}' = \vec{h} + \vec{\varepsilon}\cdot\vec{x}$

$\vec{x}' = \vec{x} + \lambda\vec{\varepsilon}\, t$

2. $\eta = 0$ Burgers equation 可解

$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2$

$W = e^{\frac{\lambda}{2\nu} h}$ $\quad \frac{\partial W}{\partial t} = \frac{\lambda}{2\nu}\frac{\partial h}{\partial t} W$

$\frac{\partial W}{\partial x} = \frac{\lambda}{2\nu}\frac{\partial h}{\partial x} W \rightarrow \quad \frac{\partial W}{\partial t} = \nu \partial^2 W$

$h = \frac{2\nu}{\lambda}\ln W$

引入势场 $V(x)$ $\quad \frac{\partial W}{\partial t} = \nu\partial^2 W + V(x)W$

$V(\vec{x}) \rightarrow \eta(\vec{x}, t)$ $\quad \eta \neq 0$ 随机势中的扩散问题

3. 定义 $\vec{v} = -\nabla h$ $\quad \frac{\partial \vec{v}}{\partial t} + \lambda\vec{v}\cdot\nabla v = \nu\nabla^2\vec{v} - \nabla\eta$

Navier-Stokes eq. $\quad \nabla\eta$ 随机数

$\langle \nabla\eta(\vec{x}, t)\,\nabla\eta(\vec{y}, t')\rangle \sim \delta(x-y)\,\delta(t-t')\,\nabla_i\nabla_j$

$\langle \eta(\vec{k}, w)\,\eta(\vec{k}', w')\rangle \sim f(k, k')\,\delta(k+k')\,\delta(w+w')$

correlated/colored noise

求解 perturbation $\lambda=0$ 附近 求解

平均 $\langle \eta\eta \rangle \sim \int D\phi\, \phi\phi e^{-S} / \int D\phi\, e^{-S}$

$\ddot{y}+y=\varepsilon(\dot{y}-\frac{1}{3}\dot{y}^3)$ Rayleigh eq.

$y=y_0+\varepsilon y_1+\varepsilon^2 y_2^2+\cdots$

代入对比 ε 系数 ε^0 $\ddot{y}_0+y_0=0$

ε^1 $\ddot{y}_1+y_1=\dot{y}_0-\frac{1}{3}\dot{y}_0^3$

ε^2 $\ddot{y}_2+y_2=\dot{y}_1-\square\dot{y}_0^2\dot{y}_1$

$y_0=A\cos(t+\theta)$

$\ddot{y}_1+y_1=A\cos(t+\theta)+\cdots$

$\ddot{y}_1+y_1=A\cos[(1+\eta)t+\theta)]$

$y_1=K\cos[(1+\eta)t+\theta]$ $[1-K(1+\eta)^2]=A$

$y_1=\frac{A}{1-(1+\eta)^2}\cos[(1+\eta)t+\theta]$ ~

$\cos(t+\theta)+t\sin(t+\theta)$

Singular perturbation theory

$h(x,t)=\sum_k e^{ikx} h(k,t)$

$\eta(x,t)=\sum_k e^{ikx}\eta(k,t)$

$\sum_k(\frac{\partial}{\partial t}h(k,t))e^{ik\cdot x}=-\sum_k \nu k^2 e^{ik\cdot x}h(k,t)+$

$\eta(k,t)e^{ikx}+\sum_k\sum_q \frac{\lambda}{2}(iq)(i(k-q))\, e^{ikx} h(q,t)$

$h(k-q,t)$ 取 e^{ikx}

$$\frac{\partial h(k,t)}{\partial t}=-\nu k^2 h(k,t)\eta(k,t)-\sum_q \frac{\lambda}{2} q(k-q)$$

$h(q,t)h(k-q,t)$

$\eta(k,t)$

$\langle \phi_{-k}\phi_k\rangle \qquad \langle \phi_k \phi_q\rangle \sim \frac{\delta_{k+q}}{k^2+m^2}$

$\langle \eta(k,t)\eta(k',t')\rangle$

$\eta(x,t)=\sum_k e^{ikx}\eta(k,t) \quad \eta(k,t)=\frac{1}{}\int e^{-ik\cdot x}\eta(x,t)\,dx$

$= \int e^{-ik\cdot x-ik'y} \underbrace{\langle \eta(x,t)\eta(y,t')\rangle}_{2D\delta(x-y)\delta(t-t')} dx\, dy$

$= \int e^{-i(k+k')x}\, 2D\, dx\ \delta(t-t')$

$= (2\pi)^d\, 2D\, \delta(k+k')\delta(t-t')$

$G_0 = e^{-\nu k^2 t}\Theta(t)$

$h(k,t)=G_0(k,t)h(k,0)+\int_0^t G_0(k,t-\tau)$

$\eta(k,\tau)d\tau - \frac{\lambda}{2}\sum_q q\cdot(k-q)\int_0^t G_0(k,t-\tau)$

$h(q,\tau)h(k-q,\tau)d\tau$

parisi-Wu model

$\dot{\phi} = \partial^2\phi - m^2\phi + g\phi^2 + \eta$

$\langle \eta(x,t)\eta(x',t')\rangle = 2\delta(x-x')\delta(t-t')$

$\dot{\phi}(k,t) = -k^2\phi(k,t) - m^2\phi(k,t) +$

$g\sum_q \square\ \phi(q,t)\phi(k-q,t) + \eta(k,t)$

when $g=0$ $\quad \phi(x,t) = \int G(x-y,t-\tau)\eta(y,\tau)\,dy\,d\tau$

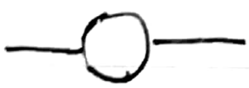
$\phi(x,t) = \int G(x-y,t-\tau)\,[\eta(y,\tau) + g\phi^2(y,\tau)]\,dy\,d\tau$

$\langle\phi(x,t)\phi(x',t')\rangle$ $<^{s}$ disorder 平均

$\langle\phi(x,t),\phi(x',t')\rangle = \int G(x-y,t-\tau)G(x'-y',t'-\tau')$

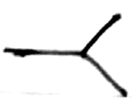
$\langle\eta(y,\tau)\eta(y',\tau')\rangle\,dy\,dy'\,d\tau\,d\tau'$

$= 2\int G(x-y,t-\tau)^2\,dy\,d\tau$

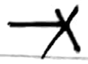


$g\neq 0$ propagator

propagator G $\quad$ ⊗ : η



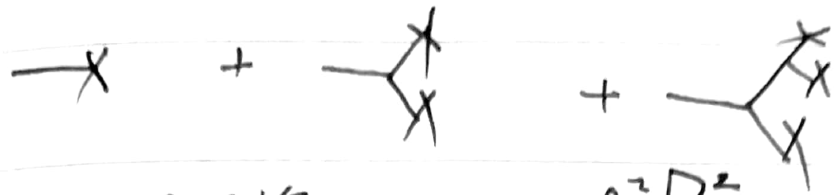
g



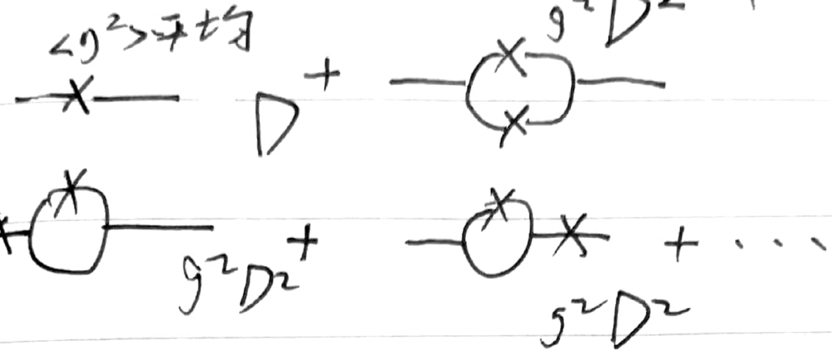
$G\eta$ $\quad \int G(x-y,t-\tau)\eta(y,\tau)\,dy\,d\tau$

× 杂质平均

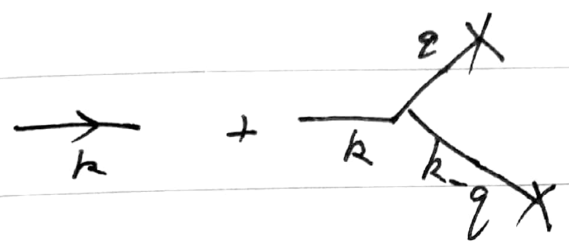
$\phi :=$



$\langle \phi^2 \rangle =$



$h =$



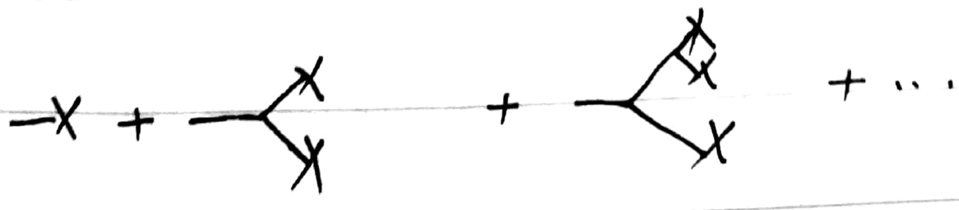
时间长 $G_0 k$ 不重要 time-scale RG time

$G = G_0$

$h = G_0 y - \frac{\lambda}{2} q(k-q) G_0 h \cdot h$

$= G_0 y - \frac{\lambda}{2} q(k-q) G_0 [G_0 y - \frac{\lambda}{2} q(k-q) G_0 h h]$

[. . .]



平均 $\int \langle h^2(x,t) \rangle dx = \sigma^2$