$\frac{\partial S_\alpha}{\partial t} = -\beta \frac{1}{\mu}\frac{\partial H}{\partial S^\alpha} + \eta_\alpha \qquad f = -\nabla U$

Dynamical RG

$\frac{\partial h}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta$ perturbation → RG

$\frac{\partial h_0}{\partial t} = \nu \nabla^2 h_0 + \eta \qquad h = h_0 + \lambda h_1 + \lambda^2 h_2 + \cdots$

$\frac{\partial h_0}{\partial t} + \lambda \frac{\partial h_1}{\partial t} = \nu \nabla^2 h + \frac{\lambda}{2}(\nabla h_0)^2 + \cdots$

方差的标度行为

over damped

$m\ddot{x} = -\frac{1}{\mu}\dot{x} + f + \xi$

阻力 随机变量

$\dot{x} = \mu f + \eta$

$t \ll t_0$ η 不重要 $\quad x = x_0 + vt \quad$ distance ~ t^2

$t \gg t_0$ η 重要 $\quad \sigma = \langle x^2 \rangle - \langle x \rangle^2 = 2Dt$

$D = k_B T \mu$

$V = \frac{k}{2}x^2 \qquad f = -kx \qquad \dot{x} = -\mu k x + \eta$

$\langle \eta(t) \rangle = 0 \qquad \langle \eta(t)\eta(t') \rangle = 2D\delta(t-t')$

$t' = bt$ 标度不变性 $\quad x(t) = e^{-\mu k t} y \quad x(0)$

$y = e^{\mu k t}\eta(t)$

$x(t) = x(0) e^{-\mu k t} + \int_0^t e^{-\mu k (t-t')}\eta(t')\,dt'$

$\overline{x} = x(0) e^{-\mu k t}$ 系综平均

十 X

求 $\langle x^2\rangle - \langle x\rangle^2$　　　确定　不确定
$x(t) = x_1 + x_2$

$\overline{x^2} = \overline{x_1^2} + 2\underbrace{\overline{x_1 x_2}}_{0} + \underbrace{\overline{x_2^2}}_{\eta^2}$

$= x^2(0)\int_0^t \overline{e^{-\mu k(t-t_1)}\eta(t_1)e^{-\mu k(t-t_2)}\eta(t_2)}\,dt_1 dt_2$

$= x^2(0)\int_0^t e^{-\mu k(t-t_1)-\mu k(t-t_2)}\,2D\delta(t_1-t_2)\,dt_1 dt_2$

$= 2D x^2(0)\int_0^t e^{-2\mu k(t-t')}dt'$

$= 2D x^2(0)\int_0^t e^{-2\mu k t'}dt' = \frac{2D x^2(0)}{2\mu k}$

平衡时 $\rho = -e^{-\beta u} = e^{-\frac{\beta}{2}kx^2}$　$\langle x^2\rangle = \frac{1}{\beta k}x^2(0)$

$\frac{2D}{2\mu k} = \frac{1}{\beta k} \Longleftrightarrow D = k_B T\mu$　平衡时T定义

标度行为与不变性

$\frac{d}{dt}x(t) = -\mu x(t) + \eta(t)$

t　　$x(t),\ \mu, D, \eta(t)$

$t' = bt$　$x'(t')$　μ'　D'　η'

$\phi'(k') = \phi'(bk) = b^z\phi(k)$

$\frac{d}{dt'}x'(t') = -\mu' x'(t') + \eta'(t')$

令 $\eta'(bt) = b^6\eta(t)$

$\langle \eta'(t_1')\eta'(t_2')\rangle = 2D'\delta(b(t_1-t_2))$

$\delta(bx) = b^{-1}\delta(x)$　　$= 2D' b^{-1}\delta(t_1-t_2)$

$b^{2\sigma} \langle \eta(t_1)\eta(t_2)\rangle = 2Db^{2\sigma}\delta(t_1-t_2)$

$D' = Db^{2\sigma+1}$

if $\sigma = -1/2$, $D' = D$

$x'(t') = x'(bt) = b^z x(t)$

$$\frac{b^z}{b}\frac{d}{dt}x = -\mu' b^z x + b^\sigma \eta$$

$$\frac{d}{dt}x = -\mu' b x + b^{\sigma+1-z}\eta$$

if $\mu = \mu' = 0$, 则 $b^{\sigma+1-z} = 1$, $z = 1/2$. $\sigma = -1/2$

$\mu'(b)b = \mu \quad \mu'(b) = \mu/b$

$\langle x^2\rangle = 2Dt \qquad \langle x'(t')\rangle^2 = 2D't' = 2Dbt$

$b^{2z}\langle x^2\rangle = 2Dtb \leftrightarrow z = 1/2$

讨论 $\frac{\partial h(\vec{x},t)}{\partial t} = \nu\nabla^2 h(\vec{x},t) + \eta(\vec{x},t)$

$\langle \eta(\vec{x},t)\eta(\vec{x}',t')\rangle = 2D\delta(\vec{x}-\vec{x}')\delta(t-t')$

$\vec{x}' = bx \quad b' = b^z t \qquad h'(x',t') = b^\chi h(x,t)$

$\eta'(x',t') = b^\sigma \eta(x,t)$

$$b^{\chi-z}\frac{d}{dt}h(x',t) = \nu' b^{\chi-2}\partial_x^2 h + b^\sigma \eta$$

$$\frac{\partial h(x',t')}{\partial t'} = \nu'\frac{\partial^2}{\partial x^2}h(x',t') + \eta(x,t')$$

扫描全能王 创建

DATE NOTES

$\frac{d}{dt}h = \nu' b^{x-2-x+z} \cdot \partial_x^2 h + b^{6+z-x} \eta$

$x = 2+6$
$z = 2$ → 标度不变性

$\langle \eta'(x_1', t_1') \eta'(x_2', t_2') \rangle \equiv 2D' \delta(x_1' - x_2') \delta(t_1' - t_2')$

$= 2D \delta(b(x_1 - x_2)) \delta(b^z(t_1 - t_2)) =$

$2D' b^{-d-z} \delta(x_1 - x_2) \delta(t_1 - t_2)$

$\eta' = b^6 \eta$

$b^{26} \langle \eta(x_1, t_1) \eta(x_2, t_2) \rangle = 2D b^{26} \delta(t_1 - t_2)$

$\delta(x_1 - x_2)$ $D = D'$ 不变 $\leftrightarrow 26 = -z-(d-1)$

$6 = -\frac{1}{2}(d-1) - \frac{z}{2}$ $x = z+6 = z - \frac{d-1}{2} - \frac{z}{2} = \frac{z-(d-1)}{2}$

$x = \frac{z-d}{2}$

$\frac{\lambda}{2}(\nabla h)^2$ term

(x', t') $\frac{\lambda'}{2}(\nabla' h')^2 = \frac{\lambda'}{2} b^{2x-2}(\nabla h)^2 =$

$\frac{\lambda'}{2} b^{z-d-2} = \frac{\lambda'}{2} b^{-d}(\nabla h)^2$ relevant

$\frac{\partial h(\vec{x}, t)}{\partial t} = \nu \nabla h + \eta$

$\frac{\partial h(k, t)}{\partial t} = -\nu k^2 h(\vec{k}, t) + \eta(\vec{k}, t)$

其中 $\langle \eta(k, t) \eta(k, t') \rangle = 2D \delta(t - t')$

Love study

$\mu = \nu k^2$

$h(\vec{k},t) = h(\vec{k},0)\cancel{t}\, e^{-\nu k^2 t} + \int_0^t e^{-\nu k^2(t-t')} \eta(k,t') dt'$

$\sigma = \frac{x\eta^2}{\mu}$

$\langle h^2(x,t)\rangle - \langle h(x,t)\rangle^2 = \sum_k \frac{h^2(k,0)}{\mu_k} = \sum_k \frac{1}{\nu k^2}$

$\langle \eta(k,t)\eta(k',t')\rangle = 2D\delta(t-t')\delta(k-k')$

考查$t\to\infty$行为

$\Omega_d \int \frac{dk\, k^{d-1}}{\nu k^2}$ $d=1$ 红

$d=2$ 红、紫 $d=3$ 紫

$h(\vec{k},t) = h(\vec{k},0)\cancel{t}\, e^{-\nu k^2 t} + \int_0^t e^{-\nu k^2(t-t')} \eta(k',t') dt'$

$\times$

$\langle h^2(x,t)\rangle = \sum_{kk'} \langle h(k,t)h(k',t)\rangle =$

$\sum_k \langle h(k,t)h(-k,t)\rangle$

$h = h_1 + h_2$ h_1, h_2 无关联

确定 不确定

$= \sum_k \int_0^t e^{-\nu k^2(t-t')} e^{-\nu k^2(t-t'')} dt' dt''$

$= 2D\sum_k \int_0^t e^{-2\nu k^2(t-t')} dt'$

$= 2D\sum_k e^{-2\nu k^2 t} \int_0^t e^{2\nu k^2 t'} dt'$

$$= 2D\sum_k e^{-2\nu k^2 t}\frac{1}{2\nu k^2}(e^{2\nu k^2 t}-1)$$

$$= 2D\sum_k \frac{1}{2\nu k^2}(1-e^{-2\nu k^2 t})$$

$$= 2D\left(\frac{1}{2\pi}\right)^d\int_0^\Lambda \frac{dk\, k^{d-1}}{\nu k^2}(1-e^{-2\nu k^2 t})$$

令 $\nu k^2 t = y^2 \rightarrow y = \sqrt{\nu t}\, k$

$$= 2D\left(\frac{1}{2\pi}\right)^d \frac{1}{\nu}(\sqrt{\nu t})^{d-2}$$

$$\propto \int_0^\infty \frac{y^{d-1}}{y^2}(1-e^{-2y})\,dy \quad t^{(\frac{d-2}{2})}$$

E-W model

$\frac{\partial h}{\partial t} = \nu\nabla^2 h + \eta$ 动量空间 Brown motion

$$\frac{\partial h(k,t)}{\partial t} = -\nu k^2 h(k,t) + \eta(k,t)$$

$$\frac{\partial f}{\partial t} = -\mu f + \eta \qquad \langle \eta(k,t)\eta(-k,t')\rangle = 2D(k)\,\delta(t-t')$$

$$f(t) = f(0)e^{-\mu t} + \int_0^t e^{-\mu(t-t')}\eta(t')dt'$$

$$\sigma^2 = \langle (h(x,t)-\bar{h}(x,t))^2\rangle \propto t^6$$

KPZ equation $\frac{\partial\phi}{\partial t} = \nu\partial^2\phi + \left(\frac{\lambda}{2}\right)(\partial\phi)^2 + \eta$

$$\dot{\phi} = \partial^2\phi - m^2\phi + g\phi^2 + \eta$$

(1998)

$$\partial_t\vec{v} + \lambda_1(\vec{v}\cdot\nabla)\vec{v} + \lambda_2(\nabla\cdot\vec{v})\vec{v} + \lambda_3\nabla\vec{v}^2 = \alpha\vec{v} + \beta|\vec{v}|^2\vec{v} -$$