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微分方程 用到 RG:

1) 某些 exp. 有标度行为

2) KPZ, …, Nonlinear diff eq

3) perturbation ⇒ 发散 ⇒ Λ, λ_Λ ⇒ 有限

$m\ddot{x} = -\eta\dot{x} + Ax^3 + f\cos(\omega t)$ 发散 ⇒ 失效 ⇒ 新方法(数值)

$h(x,t) = h_0 + \lambda h_1$ $W = \langle (h(x,t) - \bar{h})^2 \rangle \sim t^{\beta}$

上一节课: overdamped

$m\ddot{x} = -\frac{1}{\mu}\dot{x} + f + \xi$ $\frac{1}{\mu}\dot{x} = f + \xi$

$\dot{x} = \mu f + \eta$ $\eta = \mu\xi$

1) $t \ll t_0$ η 不重要 $x = x_0 + vt$

2) $t \gg t_0$ ⇒ μ 不重要, 由 η 决定 ⇒ $\sigma = \langle x^2 \rangle - \langle x \rangle^2 = 2Dt$

可认定为温度 $p \sim e^{-\beta U}$ ⟺ $D = k_B T \mu$

(咋写着)

$V = \frac{k}{2}x^2$, $f = -kx$

$\dot{x} = -\mu k x + \eta$ ⇒ $x(t) = x(0)[e^{-\mu k t} + \int_0^t e^{-\mu k (t-t')} \eta(t') dt'$

$\langle \eta(t) \rangle = 0$ $\langle \eta(t)\eta(t') \rangle = 2D\delta(t-t')$

后面讨论 $t' = bt$ 时标度不变性 $\bar{x} = x(0)e^{-\mu k t}$

$x(t) = (e^{-\mu k t} y)\, x(0)$

$\dot{y} = e^{\mu k t}\eta(t)$ ⟺ $y = 1 + \int_0^t e^{\mu k t'} dt'$ (?)

$\langle x^2 \rangle - \langle x \rangle^2$ $x(t) = x_1 + x_2$. $\overline{x^2} = \overline{x_1^2} + 2\overline{x_1 x_2} + \overline{x_2^2}$

$= x^2(0) \int_0^t e^{-\mu k (t-t_1)} \eta(t_1)\, e^{-\mu k (t-t_2)} \eta(t_2)\, dt_1 dt_2$

$= x^2(0) \int_0^t e^{-\mu k (t-t_1) - \mu k (t-t_2)}\, 2D\, \delta(t_1 - t_2)\, dt_1 dt_2$

$= 2D x^2(0) \int_0^t e^{-2\mu k (t-t')} dt'$

$= 2D x^2(0) \int_0^t e^{-2\mu k t'} dt' = \frac{2D x^2(0)}{2\mu k}$

$p \sim e^{-\beta U} = e^{-\frac{\beta}{2} k x^2}$ $\langle x^2 \rangle = \frac{1}{\beta k} x^2(0)$

$\frac{2D}{2\mu k} = \frac{1}{\beta k}$ ⇒ $D = k_B T \mu$

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标度行为与不变性 $k=1$

$$\frac{d}{dt}x(t) = -\mu x(t) + \eta(t)$$

1) $t \Rightarrow$ $x(t)$, μ, D, $\eta(t)$

2) $t'=bt$, $x'(t')$ μ', D', $\eta'(t')$ 令 $\eta'(bt) = b^{\sigma}\eta(t)$

$$\frac{d}{dt'}x'(t') = -\mu' x'(t') + \eta'(t')$$

$$\langle \eta'(t_1')\eta'(t_2')\rangle = 2D'\delta(t_1'-t_2') = 2D'\delta(b(t_1-t_2))$$

$$= 2D'b^{-1}\delta(t_1-t_2) = 2Db^{2\sigma}\delta(t_1-t_2)$$

$\Rightarrow$ $D' = D b^{\cancel{2\sigma+1}\ 2\sigma+1}$ 取 $\sigma = -\frac{1}{2}$ $D'=D$

$$\cancel{\frac{d}{dt}x = -\mu' b x + b^{\sigma+1-z}\eta}$$

$$\frac{d}{dt'}x'(t') = -\mu'x'(t')+\eta'(t') \Rightarrow b^{z-1}\frac{d}{dt}x = -\mu' b^{z} x + b^{\sigma}\eta$$

$$\Rightarrow \frac{d}{dt}x = -\mu' b x + b^{\sigma+1-z}\eta \Rightarrow \frac{d}{dt}x(t) = -\mu x(t) + \eta(t)$$

讨论: $\frac{\partial h(\vec{x},t)}{\partial t} = \nu\nabla^2 h(\vec{x},t) + \eta(\vec{x},t)$

$$\langle \eta(\vec{x},t)\eta(\vec{x}',t')\rangle = 2D\delta(\vec{x}-\vec{x}')\delta(t-t')$$

$$\frac{\partial h'(x',t')}{\partial t'} = \nu'\frac{\partial^2}{\partial x'^2}h'(x',t') + \eta'(x',t')$$

$$b^{\chi-z}\frac{d}{dt}h(x,t) = \nu' b^{\chi-2}\partial_x^2 h + b^{\sigma}\eta$$

$$\frac{d}{dt}h = \nu' b^{z-2}\partial_x^2 h + b^{\sigma+z-\chi}\eta$$

不变 $\chi = z+\sigma$ $z=2$

$$\langle \eta'(x_1',t_1')\eta'(x_2',t_2')\rangle = 2D'\delta(x_1'-x_2')\delta(t_1'-t_2')$$

$$= 2D'\delta(b(x_1-x_2))\delta(b^{z}(t_1-t_2))$$

$$= 2D' b^{\cancel{-1-z}}\delta(x_1-x_2)\delta(t_1-x_2)$$

$-(d-1)-z$

$D=D'$ 不变 $\cancel{2\sigma} =$

$$\sigma = -\frac{1}{2}(d-1) - \frac{z}{2}$$

加 $\frac{\lambda}{2}(\nabla h)^2$ kpz eq $\frac{\lambda'}{2}b^{-d}(\nabla h)^2 \Rightarrow$ 越来越大