No.
Date

$$P(\sigma_i^{(0)}, \sigma_i^{(\infty)}) = e^{\beta F - \beta \sum_{ij} J_{ij} (\sigma_i^{(0)}\sigma_j^{(0)} + \sigma_i^{(\infty)}\sigma_j^{(\infty)})}$$
$$= e^{\beta F - \beta \sum_i \sigma_i^{(0)} \xi_i^{(0)} + \sigma_i^{(\infty)} \xi_i^{(\infty)}}$$

Spin Glass

Replica Trick

Edwards-Anderson 论文　另一种图像

关于 Replica Symmetry

的解释

E-A paper 第二节 The mean correlation Theory

$$H = -\sum_{ij} J_{ij} S_i S_j \qquad S_i = \pm 1$$

$$Z = Tr(e^{-\beta H}) = e^{-\beta F} \quad 取 \ \beta = 1$$

$$P(s) = e^{+\beta F - \sum_{ij} J_{ij} S_i S_j}$$

$$\sum_s P(s) = 1$$

2 份 Copies　$S_i^{(0)}$　$S_i^{(1)}$　(两者没有直接关联，不可能地跟踪)

联合分布 $P(S^{(0)}, S^{(1)}) = e^{2F - \sum_{ij} J_{ij} S_i^{(1)} S_j^{(1)} - \sum_{ij} J_{ij} S_i^{(0)} S_j^{(0)}}$

$$\Rightarrow e^{2F - \sum_i S_i^{(0)} \xi_i^{(0)} - \sum_i S_i^{(1)} \xi_i^{(1)}}$$

ξ_i 有效场

No.

Date

讨论 ① ξ_i 完全随机

$$\langle \xi_i \rangle = 0, \quad \langle \xi_i^{(1)} \xi_i^{(0)} \rangle = 0$$

$$\langle \xi_i^{(1)} \xi_j^{(1)} \rangle = 0, \quad \langle \xi_i^{(0)} \xi_j^{(0)} \rangle = 0$$

$$P(S_i^{(0)} S^{(1)}) = P(S^{(1)}) P(S^{(0)})$$ 无关联

② 非完全

IF $\langle S_i^{(0)} S_i^{(1)} \rangle = q$

$$\langle \xi_i^{(0)} \xi_i^{(1)} \rangle = \langle J_{ij} S_j^{(0)} J_{ik} S_k^{(1)} \rangle$$

$$\approx \langle J_{ij}^2 S_j^{(0)} S_j^{(1)} \rangle$$

$$= q J_{ij}^2 \neq 0$$

$$P(S^{(0)}, S^{(1)}, \xi^{(0)}, \xi^{(1)})$$

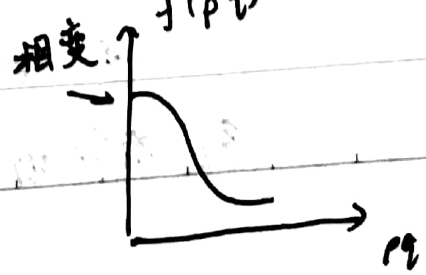
$$= N e^{-\beta(S_i^{(0)} \xi_i^{(0)} + S_i^{(1)} \xi_i^{(1)})} P(\xi_i^{(0)}, \xi_i^{(1)})$$

$$q = \langle S_i^{(0)} S_i^{(1)} \rangle = \mathrm{Tr} \int \cdots d\xi_i^{(0)} d\xi_i^{(1)}$$

$$\propto \mathrm{Tr}\left(e^{-S_i^{(0)} S_i^{(1)} \beta q} S_i^{(0)} S_i^{(1)}\right)$$

$$q = \langle S_i^{(0)} S_i^{(1)} \rangle = \frac{\int_{-1}^{1} \mu e^{-\mu \beta q} d\mu}{\int_{-1}^{1} e^{-\mu \beta q} d\mu} = \coth(\beta q) - \frac{1}{\beta q}$$

$$\frac{1}{\beta} = \left(\frac{\coth(\beta q) - \frac{1}{\beta q}}{\beta q} \right)$$



No.
Date

ρ 来自于 ξ 的关联 是和相作强度、温度有关的量

随机微分方程 (SDE, S- Stochastic

① $m\ddot{x} - \frac{1}{\mu}\dot{x} + f + \xi = 0 \quad f = -\nabla V$

当 $\frac{1}{\mu}$ 很大 (过阻尼 over damp)

质量项不重要 $\longrightarrow$ $\frac{1}{\mu}\dot{x} - f - \xi = 0$

无穷多粒 $\longrightarrow$ 场 $\phi(x,t)$

ρ

② Edwards - Wilkinson (E-W) model

(界面随机生成)

$$\frac{\partial\phi}{\partial t} = \nu\frac{\partial^2\phi}{\partial x^2} + \xi$$

$\xi = 0$ 扩散

$\nu = 0$ 布朗运动

③ Karder - Parisi - Zhang (KPZ) model

$$\frac{\partial\phi}{\partial t} = \nu\frac{\partial^2\phi}{\partial x^2} + \frac{\lambda}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 + \xi$$

④ K-S model

$$\frac{\partial\phi}{\partial t} = \nu\frac{\partial^2\phi}{\partial x^2} + \frac{\lambda}{2}\left(\frac{\partial\phi}{\partial x}\right)^2 - K\frac{\partial^4\phi}{\partial x^4} + \xi$$

No

Date

(kiramoto model)

这些模型与表面生成等过程有关，
并存在标度行为

随机量化

$\dot{x} = \mu(f+\xi)$ $f = -\nabla V$, $(V = \frac{1}{2}x^2$, $f = -x)$

$\dot{x} = -\mu x - \mu\xi$

$\langle\xi(t)\rangle = 0$ $\langle\xi(t)\xi(t')\rangle = 2D\,\delta(t-t')$

连续版 $\frac{\partial\phi}{\partial t} = \nu\frac{\partial^2\phi}{\partial x^2} + \xi$

$\rightarrow \frac{\partial\phi(k,t)}{\partial t} = -\nu k^2\phi^2 + \tilde{\xi}(k,t)$

① E-W model 是运量空间的布朗运动

② $\phi(x,t) \sim \frac{1}{A+Bk^2}$

$\langle\phi^2(x,t)\rangle \propto \sum_k \frac{1}{A+Bk^2}$ 发散 → 重整化

③ (Replica Trick)

$\langle e^{Ax}\rangle \sim e^{\frac{A^2}{u}}$

$\langle e^{\phi^2 x}\rangle \sim e^{\phi^4/u}$ 改变相互作用

通常为排斥

No.

Date

Ref: Kardar 统计场论 chp 9 (Dissipative dynamics)

若 $f=0$,

$$x = x(0) + \mu\int_0^t \eta(t')dt'$$

$$\sigma^2 = x^2 - x_0^2 = \mu^2 \int dt_1 dt_2 \langle \eta(t_1)\eta(t_2)\rangle$$

$$= 2\mu^2 D t$$

$$p(\eta(t)) \propto \exp\left[-\int \frac{1}{4D}\eta^2(t)dt\right]$$

$$\langle \eta(t)\rangle = 0, \quad \langle \eta(t_1)\eta(t_2)\rangle = 2D\,\delta(t-t')$$

而阻尼本身也是来自涨落(热运动)因此 μ 和描述热涨落强度的 D 不独立 $\cancel{\mu = k_B T D}$ $\mu = \beta D$