Date. / /

另外一种: $\mathrm{Tr}\left[e^{\frac{\beta^2 J^2}{2}(m^{\alpha\beta})^2}\right] \propto \int dq^{\alpha\beta}\, e^{-X(q^{\alpha\beta})^2 + Y q^{\alpha\beta} m^{\alpha\beta}}$

$$\overline{Z^n} \propto \int Dq^{\alpha\beta}\, e^{-X\sum_{\alpha\beta}(q^{\alpha\beta})^2}\, \mathrm{Tr}\left(e^{\sum_{\alpha\beta} Y q^{\alpha\beta} m^{\alpha\beta}}\right)$$

$$= \left(\mathrm{tr}\left[e^{\sum_{\alpha\beta} Y q^{\alpha\beta} \sigma^\alpha \sigma^\beta}\right]\right)^N \propto \left[2\cosh(Yq^{\alpha\beta})\right]^N$$

$\langle \sum_i \sigma_i^\alpha \sigma_i^\beta \rangle = q \qquad \alpha = 1, 2, \cdots, n \quad \alpha \neq \beta \quad n \to 0$

Edward - Anderson 序参量 原始论文

$\sum_{ij} J_{ij}\sigma_i\sigma_j = \sum_i \sigma_i \xi_i \qquad P(\sigma) = e^{\beta F - \beta H} = e^{\beta F - \beta\sum_{ij} J_{ij}\sigma_i\sigma_j}$

$S_i^{(1)}$ 1 $\Rightarrow$ $t = \text{Now}$ 求 $\langle S_i^{(1)} S_i^{(2)} \rangle = q \neq 0$

$S_i^{(2)}$ 2 $\Rightarrow$ $t = +\infty$

$$P(s_i^{(1)}, s_i^{(2)}) = e^{2\beta F - \beta\sum_{ij} J_{ij}(s_i^{(1)} s_j^{(1)} + s_i^{(2)} s_j^{(2)})}$$

spin Glass

- Replica trick
- Edwards - Anderson 原始论文

1) 另一个图像

2) Anderson 直觉能力. 保留本质

Anderson 如何解释 Replica sym?

E-A paper 第二节: The mean correlation theory

$H = -\sum_{ij} J_{ij} S_i S_j \qquad S_i = \pm 1$

$Z = \mathrm{Tr}(e^{-\beta H}) = e^{-\beta F}$ 取 $\beta = 1$

$P(S) = e^{F - \sum_{ij} J_{ij} s_i s_j}$ 几率分布 $\Rightarrow \sum_S P(S) = 1$

2份 ~~Copy~~ copies | $S_i^{(1)}$ $t=0$ | $S_i^{(2)}$ $t=\infty$ | Quened disorder $\Rightarrow J_{ij}$ 和时间无关 ...

$$P(s^{(1)}, s^{(2)}) = e^{2F - \sum_{ij} J_{ij}(s_i^{(1)} s_j^{(1)} + s_i^{(2)} s_j^{(2)})} = e^{2F - \sum_i (s_i^{(1)}\xi_i^{(1)} + s_i^{(2)}\xi_i^{(2)})}$$

$\xi_i = \sum_j J_{ij} s_j$

① ξ_i 是完全随机的 $\langle \xi_i \rangle = 0 \qquad \langle \xi_i^{(1)} \xi_i^{(1)} \rangle = \sum_j \overline{J_{ij}^2} = J_0^2$

$\langle \xi_i^{(1)} \cdot \xi_j^{(1)} \rangle = 0 \qquad \langle \xi_i^{(1)} \cdot \xi_i^{(2)} \rangle = 0 \longleftarrow$ 随机数

$\Longleftrightarrow \quad P(s_i^{(1)}, s^{(2)}) = e^{2F - \sum_i (s_i^{(1)}\xi_i^{(1)} + s_i^{(2)}\xi_i^{(2)})}$

② 非完全 Random IF $\langle s_i^{(1)} s_i^{(2)} \rangle = q$

$$\langle \xi_i^{(1)} \xi_i^{(2)} \rangle = \langle J_{ij} s_j^{(1)} J_{ik} s_k^{(2)} \rangle = \langle J_{ij}^2 s_j^{(1)} s_j^{(2)} \rangle = q J_0^2 \neq 0$$

$$P(s^{(1)}, s^{(2)}, \xi^{(1)}, \xi^{(2)}) = N e^{-\frac{1}{k_B T}(s_i^{(1)}\xi_i^{(1)} + s_i^{(2)}\xi_i^{(2)})} P(\xi_i^{(1)}, \xi_i^{(2)})$$

$q = \langle s_i^{(1)} s_i^{(2)} \rangle$ 平均 ξ_i $\quad s_i = \pm 1 \quad = \mathrm{Tr} \int d\xi_i^{(1)} d\xi_i^{(2)} P(\xi_i^{(1)}, \xi_i^{(2)})$

$$e^{-\beta(s_i^{(1)} s_i^{(1)} + s_i^{(2)} s_i^{(2)})} N \quad \propto \quad \mathrm{Tr}\left(e^{-s_i^{(1)} s_i^{(2)} \rho g}\ s_i^{(1)} s_i^{(2)}\right)$$

$$q = \langle s_i^{(1)} s_i^{(2)} \rangle = \frac{\int_{-1}^{1} \mu e^{-\mu \rho g} d\mu}{\int_{-1} e^{-\mu \rho g} d\mu} = \coth(\rho g) - \frac{1}{\rho g}$$

$\mu = s_i^{(1)} s_i^{(2)} \quad |\mu| \leq 1$

$$\frac{1}{\rho} = \left(\frac{\coth(\rho g) - \frac{1}{\rho g}}{\rho g} \right)$$

随机微分方程: Random stochastic

① $m\ddot{x} = -\frac{1}{\mu}(\dot{x}) + f + \xi$ if $\frac{1}{\mu}$ 很大 $\frac{1}{\mu}\dot{x} = f + \xi$

② 多个 $\dot{x}_i = \mu_i (f_i + \xi_i)$

③ ∞ 多个: 连续场 $x \Rightarrow \phi(x)$ $\phi(x,t)$ 为随机数

$$\frac{\partial \phi(x,t)}{\partial t} = \mu(x)\left(f(x) + \xi(x,t)\right) \sim \frac{\partial^2 \phi}{\partial x^2} + \wedge\phi + \wedge\phi^3 + \xi$$

④ Edwards - Wilkinson (E-W model): 1982

$$\frac{\partial \phi}{\partial t} = \nu \frac{\partial^2 \phi}{\partial x^2} + \xi$$

if $\xi = 0$ 扩散

$\nu = 0$ 布朗

⑤ KPZ $\quad \frac{\partial \phi}{\partial t} = \nu \frac{\partial^2 \phi}{\partial x^2} + \frac{\lambda}{2}\left(\frac{\partial \phi}{\partial x}\right)^2 + \xi$

⑥ K-S

⑦ $\frac{\partial \theta_i}{\partial t} = \omega_i + \frac{K}{N} \sum_j \sin(\theta_i - \theta_j) + \xi_i$

① 实验 → 生长过程 ② 标度行为

$$6 = W(t) \sim \frac{1}{L} \sum_x \langle (\phi - \bar{\phi})^2 \rangle$$

过阻尼 $\dot{x} = \mu(f + \xi)$ $\quad f = -\nabla U \quad U = \frac{1}{2}x^2 . \quad f = -x$

$\dot{x} = -\mu x - \mu \xi \qquad \langle \xi(t) \xi(t') \rangle = D\,\delta(t - t')$

$\langle \xi(t) \rangle = 0$

$$\frac{\partial \phi(x,t)}{\partial t} = \nu \frac{\partial^2 \phi}{\partial x^2} + \xi$$

$$\phi(x,t) = \frac{1}{V} \sum_k \phi(k,t) e^{ik\cdot x}$$

$$\xi(x,t) = \frac{1}{V} \sum_k \xi(k,t) e^{ik\cdot x}$$

$$\frac{\partial \phi(k,t)}{\partial t} = -\nu k^2 \phi(k,t) + \xi(k,t)$$

关键点：① E-W model = k-space

② $\phi(k,w) \propto \left(\frac{1}{A+Bk^{2}}\right)$ $\langle\phi^2\rangle \sim \sum_k \left(\frac{1}{A+Bk^2}\right)^n$

③ Replica trick $\langle e^{\phi^2 x}\rangle \sim e^{\frac{\phi^4}{U}}$

kardar 统计场论. 中心极限定理

讨论 | $\dot{x} = \mu f + \eta$ η随机（$\xi \to \eta$）
| $f = -\nabla U$

☆ $f=0$ $\dot{x} = \eta$ $x = x_0 + \int_0^t \eta(t')dt'$

$x = x_0$

方差 $\sigma^2 = x^2 - x_0^2 = \int_0^t dt_1 dt_2 \langle \eta(t_1)\eta(t_2)\rangle = 2D\int_0^t dt_1 = 2Dt$

$\langle \eta(t_1)\eta(t_2)\rangle = 2D\,\delta(t_1-t_2)$

说 $\langle\eta(t)\rangle = 0$ 以及 $\langle\eta(t_1)\eta(t_2)\rangle = 2D\,\delta(t_1-t_2)$

$p(\eta(t)) \propto \exp\left[-\int \frac{1}{4D}\eta^2(t)\,dt\right] \propto e^{-\sum_i \frac{\delta t}{4D}\eta^2(t_i)}$

$\langle\eta(t_1)\eta(t_2)\rangle = 2D\,\delta(t_1-t_2) \Longleftrightarrow \langle\eta(t_i)\eta(t_j)\rangle \propto 2D\,\delta_{ij}/\delta t$

| if $|t_1 - t_2| > \varepsilon$ $=0$
| $\leq \varepsilon$ $\neq 0$

有两个物理量 μ 和 D 是否独立？No. $D = k_B T \mu$

$x(t) \Rightarrow \langle x^2\rangle = \left(\frac{3D}{\mu k}\right)$ $p \sim e^{-\beta V}$

Wilson RG 1971-1972

Shang-keng Ma 1973-1974 随机非线性方程

$$\frac{\partial S_\alpha}{\partial t} = -\beta\frac{1}{\mu}\frac{\partial H}{\partial S_\alpha} + \eta_\alpha$$ ~~摩擦力~~ 随机力

$f = -\nabla U$

表面生长. 细菌生长

$$w^2(L,t) = \frac{1}{L}\int_0^L dx\,(h(x,t) - \bar{h})^2$$

$w \sim L^\chi f\left(\frac{t}{L^z}\right)$ $\chi = \frac{1}{2}$ $z = \frac{3}{2}$ $\chi = \frac{2-d}{2}$ $z = 2$

☆ $$\frac{\partial h}{\partial t} = \nu\nabla^2 h + \frac{\lambda}{2}(\nabla h)^2 + \eta$$

$h = h_0 + \lambda h_1 + \lambda^2 h_2 + \lambda^3 h_3 + \cdots$ $\frac{\partial h_0}{\partial t} + \lambda\frac{\partial h_1}{\partial t} = \nu\nabla^2 h + \frac{\lambda}{2}(\nabla h_0)^2$

h_0 $\frac{\partial h_0}{\partial t} = \nu\nabla^2 h_0 + \eta$