Spin Glass: | Replica trick
Euwards-Anderson 原始论文.

1) 另一个图像
2) Anderson 直觉能力. 保留本质的东西 $J_{ij}\sigma_i\sigma_j$
Anderson 如何解释 Replica sym?

E-A paper 第二节: The mean correlation relation

$H = -\sum_{ij} J_{ij} S_i S_j$, $S_i = \pm 1$

$Z = Tr(e^{-\beta H}) = e^{-\beta F}$, 取 $\beta = 1$

$P(S) = e^{F - \sum_{ij} J_{ij}\sigma_i\sigma_j}$ 几率分布 $\Leftrightarrow \sum_S P(S) = 1$

2份 Copies | $S_i^{(1)}$ $t=0$ $S_i^{(2)}$ $t=\infty$

$$P(S^{(1)}, S^{(2)}) = e^{2F - \sum_{ij} J_{ij}(S_i^{(1)} S_j^{(1)} + S_i^{(2)} S_j^{(2)})} = e^{2F - \sum_i (S_i^{(1)} \xi_i^{(1)} + S_i^{(2)} \xi_i^{(2)})}$$

$\xi_i^{(1,2)}$: spin S_i 感受到的平均场.

{ Quened disorder
Annealed disorder $\Rightarrow$ J_{ij} 与时间无关.

① ξ_i 是完全随机的: $\Rightarrow \langle \xi_i \rangle = 0$ $\left(\langle \xi_i^{(1)} \xi_i^{(1)} \rangle = \langle J_{ij} S_j^{(1)} J_{ik} S_k^{(1)} \rangle = \sum_j J_{ij}^2 = J_0^2 \right)$

$\langle \xi_i^{(1)} \xi_j^{(1)} \rangle = 0$, $\langle \xi_i^{(1)} \xi_i^{(2)} \rangle = 0$.

$\Rightarrow P(S^{(1)}, S^{(2)}) = e^{2F - \sum_i (S_i^{(1)} S_i^{(1)} + S_i^{(2)} S_i^{(2)})}$

随机数 $\Rightarrow$ 完全解耦 (decoupled). $\boxed{\langle S_i^{(1)} S_i^{(2)} \rangle = 0}$

② 非完全随机 IF $\langle S_i^{(1)} S_i^{(2)} \rangle = q$.

$\langle \xi_i^{(1)} \xi_i^{(2)} \rangle = \langle J_{ij} S_j^{(1)} J_{ik} S_j^{(2)} \rangle = \langle J_{ij}^2 S_j^{(1)} S_j^{(2)} \rangle = q J_0^2 \neq 0$

$$P(S^{(1)},S^{(2)},\xi^{(1)},\xi^{(2)}) = N e^{-\frac{1}{k_BT}(S_i^{(1)}\xi_i^{(1)}+S_i^{(2)}\xi_i^{(2)})} P(\xi_i^{(1)},\xi_i^{(2)})$$

$\langle S_i^{(1)} S_i^{(2)}\rangle = q$ （在 ξ 下平均，考虑 $S_i=\pm 1$ ）

$$= \int d\xi_i^{(1)} d\xi_i^{(2)} P(\xi_i^{(1)},\xi_i^{(2)}) e^{-\beta(S_i^{(1)}\xi_i^{(1)}+S_i^{(2)}\xi_i^{(2)})} \cdot N$$

$$\propto \mathrm{Tr}\left(e^{-S_i^{(1)}S_i^{(2)}\beta q S_i^{(1)}S_i^{(2)}}\right)$$

why not $(S_i^{(1)})^2=1$, $(S_i^{(2)})^2=1$.

$$\int dx\,dy\, e^{-A(x^2+y^2)+(q_1x+q_2y)} \sim e^{\frac{q_1^2}{A}+\frac{q_2^2}{A}}$$

$$\int dx\,dy\, e^{-A(x^2+\alpha xy+y^2)+q_1x+q_2y} \sim e^{\frac{q_1^2}{A}+\frac{q_2^2}{A}+\frac{\alpha^2}{A}q_1\cdot q_2}$$

$\downarrow \qquad \downarrow$

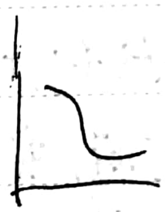
$S_1^2=1,\ S_2^2=1.$

$$q=\langle S_i^{(1)}S_i^{(2)}\rangle = \frac{\int_{-1}^{1} u e^{-u\beta q}du}{\int_{-1}^{1} e^{-u\beta q}du} = \coth(\beta q)-\frac{1}{\beta q}$$

$u=S_i^{(1)}S_i^{(2)}$, $|u|\le 1$

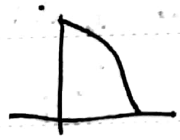
$$\frac{1}{\beta}\cdot\beta q = \coth(\beta q)-\frac{1}{\beta q}$$

$$\frac{1}{\beta}=\left(\frac{\coth(\beta q)-\frac{1}{\beta q}}{\beta q}\right)$$



$q^2=5\left[1-\left(\frac{T}{T_c}\right)^2\right]\left(\frac{T}{T_c}\right)^4$. BCS Theory.

$\Delta^2\propto\left[1-\left(\frac{T}{T_c}\right)^2\right]$



A single spin problem + Random field.

(stochastic diff eq. SDE) $\Rightarrow$ Random, Stochastic number, process.

随机微分方程：① 哪些？

① $m\ddot{x} = -\frac{1}{\mu}\dot{x} + f + \xi$, $f=-\nabla U$

if $\frac{1}{\mu}$ 很大（overdamped 过阻尼）

$\frac{1}{\mu}\dot{x} - f+\xi \Rightarrow \dot{x}=\mu(f+\xi)$

② 多个 $\dot{x}_i=\mu_i(f_i+\xi_i)$

DATE

③ 无穷多个：连续场：$x \Rightarrow \phi(x,t)$ 为随机数.

$$\frac{\partial\phi(x,t)}{\partial t} = \mu(x)\left(f(x)+\xi(x,t)\right) \sim \underbrace{\left(\frac{\partial^2\phi}{\partial x^2}+\lambda\phi+\lambda\phi^3\right)}_{\phi^4 \text{ theory}}+\xi$$

④ Edwards - Wilkinson (E-W model), 1982

$$\frac{\partial\phi}{\partial t} = \nu\frac{\partial^2\phi}{\partial x^2}+\xi$$

讨 $\xi=0 \Rightarrow$ 扩散方程

$\nu=0 \Rightarrow$ Brown motion.

⑤ Kardar - Parisi - Zhang model

$$\frac{\partial\phi}{\partial t} = \nu\frac{\partial^2\phi}{\partial x^2}+\frac{\lambda}{2}\left(\frac{\partial\phi}{\partial x}\right)^2+\xi.$$

⑥ K-S $\frac{\partial\phi}{\partial t} = \nu\frac{\partial^2\phi}{\partial x^2}+\frac{\lambda}{2}\left(\frac{\partial\phi}{\partial x}\right)^2-k\frac{\partial^4\phi}{\partial x^4}+\xi$.

Kuramoto (藏本) Kuramoto model

$$\frac{\partial\theta_i}{\partial t} = \omega_i+\frac{k}{N}\sum_j \sin(\theta_i-\theta_j)+\xi_i$$

① 生长过程, 实验

② 标度行为.

$$6 - w(t) \sim \frac{1}{L}\sum_x \langle(\phi-\bar{\phi})^2\rangle \propto t^{\beta}$$

随机量子化. 吴咏时 Utah, Fudan.

过阻尼 $\dot{x}=\mu(f+\xi)$; $f=-\nabla U$, $U=\frac{1}{2}x^2$

$f=-x$

$\dot{x}=-\mu x-\mu\xi$

$\langle\xi(t)\rangle=0$, $\langle\xi(t)\xi(t')\rangle=D\delta(t-t')$

$$\frac{\partial\phi(x,t)}{\partial t} = \nu\frac{\partial^2\phi}{\partial x^2}+\xi$$

$\langle\xi(x,t)\rangle=0$ 以及 $\langle\xi(x,t)\xi(x',t')\rangle=D\delta(x-x')\delta(t-t')$

$\phi(x,t)=\frac{1}{\sqrt{L}}\sum_k\phi(k,t)e^{ikx}$, $\xi(x,t)=\frac{1}{\sqrt{L}}\sum_k\xi(k,t)e^{ikx}$

取 $e^{ikx} \longrightarrow \frac{\partial\phi(k,t)}{\partial t} = -\nu k^2\phi(k,t) + \xi(k,t)$

关键点: ① E-W model = k-space Brown motion.

② $\phi(k,\omega) \propto \left(\frac{1}{A+Bk^2}\right)$

$\langle\phi^2\rangle \sim \sum_k \frac{1}{A+Bk^2}$

~ 0

③ Replica trick $\langle e^{A x\phi^2}\rangle \sim e^{\frac{A^2}{V}\phi^4}$ 改变相互作用. 排斥.

Ref: Kardar, 统计场论 chap 9. (Dissipative dynamics)

记住: 中心极限定理 $X = \frac{1}{N}(X_1 + \cdots + X_N)$

$\Leftrightarrow X \sim N(\mu, \frac{\sigma^2}{N})$, $Y = \sum_{i=1}^{N} X_i \Leftrightarrow Y \sim \text{Normal}(N\mu, N\sigma^2)$

$\langle X^2\rangle = 2Dt$, $\langle Y^2\rangle - \langle Y\rangle^2 = \sigma^2 N$.

$V \gg C_V \propto \langle E^2\rangle - \langle E\rangle^2$

$V \gg \chi \propto \langle M^2\rangle - \langle M\rangle^2$

讨论: $\dot{x} = \mu f + \eta$, $f = -\nabla U$, η为随机 (噪声)

& $f = 0$, $\dot{x} = \eta \Rightarrow x = x_0 + \int_0^t \eta(t')dt'$

$\bar{x} = x_0$, (方差) $\sigma^2 = \overline{x^2} - x_0^2 = \int_0^t dt_1 dt_2 \langle\eta(t_1)\eta(t_2)\rangle$

$= 2D\delta(t_1 - t_2) = 2D\int_0^t dt_1 = 2Dt$

$\Leftrightarrow \boxed{\sigma^2 = 2Dt}$ 中心极限定理. Functional form of Stochastic Process 随机过程的泛函

设 $\langle\eta(t)\rangle = 0$, 以及 $\langle\eta(t_1)\eta(t_2)\rangle = 2D\delta(t_1 - t_2)$

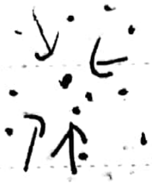
问 $\boxed{P(\eta(t)) = ? \propto \exp\left[-\int \frac{1}{4D}\eta^2(t)dt\right] \propto e^{-\sum_i \frac{\delta t_i}{4D}\eta^2(t_i)}}$

$\langle\eta(t_1)\eta(t_2)\rangle = 2D\delta(t_1 - t_2)$ $\begin{cases} \text{if } |t_1 - t_2| > \varepsilon, = 0. \\ |t_1 - t_2| \leq \varepsilon, \neq 0 \end{cases}$

$\Leftrightarrow \underbrace{\langle\eta(t_i)\eta(t_j)\rangle}_{\text{离散化}} \propto 2D\delta_{ij}/\delta t$

$\int xy\, e^{-A(x^2+y^2)} dx\, dy = 0.$

有两个物理量：μ和D，是否独立？ No.



碰撞 < 阻力 / 随机运动.

$m\ddot{x} = -\frac{1}{\mu}\dot{x} + f + \xi$.

$\frac{1}{\mu}$阻力

$\Longleftrightarrow \boxed{D = k_B T \mu}$

$U = \frac{1}{2}kx^2$ $\quad x(t) = \quad \longrightarrow \langle x^2 \rangle = \left(\frac{3D}{\mu k}\right)$

$\boxed{\dot{x} = -\mu k x + \eta}$ $\quad p \sim e^{-\beta U}$

$\langle x^2 \rangle$ $\quad D = k_B T \mu$. $\longrightarrow$ E-W model / KPZ eq $\left(\frac{\partial h}{\partial x}\right)^2$ 多体相互作用