2022.3.7 第3周第1节课.

Casimir effect.

1D: $E = \hbar c k$, $k_n = \frac{n\pi}{a}$

$$E = \frac{\hbar c}{2}\sum_n k_n = \frac{\hbar c}{2}\sum_{n=1}^{\infty}\frac{n\pi}{a}$$

ref. Zeta function. / 正规化.

$$\zeta(s) = \sum_{n=1}^{\infty}\frac{1}{n^s}.$$

$$\zeta(-1) = -\frac{1}{12}$$

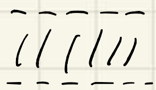
$$\Rightarrow \quad E = \frac{\hbar c}{2}\cdot\left(-\frac{\pi}{12a}\right) = -\frac{\hbar c\pi}{24a}$$


驻波.

$$E^f = \frac{1}{2}\hbar c\sum_k |k| = \frac{1}{2}\hbar c\,\frac{L}{2\pi}\int_0^{\infty} dk\,|k|$$

$$= \frac{\hbar c}{2}\,\frac{L}{\pi}\int_0^{+\infty} dk\cdot k$$

$$\tilde{E} = \frac{E^f}{L}\times a = \frac{\hbar c a}{2\pi\varepsilon^2}\int_0^{+\infty}\varepsilon k\,d\,\varepsilon k\;e^{-\varepsilon k}$$



$$= \frac{\hbar c a}{2\pi\varepsilon^2}.$$ ←没有板时真空的能量.

又 $E = -\frac{\hbar c}{2}\sum_{n=1}^{+\infty}\frac{n\pi}{a}$

$$= -\frac{\hbar c}{2}\sum_{n=1}^{+\infty}k_n \left| \begin{array}{l} e^{-k_n\varepsilon} \\ f(k_n\varepsilon) \\ e^{-k_n/\Lambda} \end{array} \right. , \quad (\varepsilon = 1/\Lambda).$$

$$= \frac{\hbar c a}{2\pi\varepsilon^2} - \underbrace{\frac{\hbar c\pi}{24a}}$$

有板存在时增加的能量.

$$\Rightarrow \text{Casimir Force} = -\frac{\partial E}{\partial a} = -\frac{\hbar c\pi}{24a^2}$$

耦合常数与选择的能标有关: $E = -\frac{\hbar^2\Lambda^2}{2m}e^{-\frac{4\pi}{\lambda}}$

可以观测到的能量越高，可以相互作用的激发态越多.

ϕ^4 理论
- 声子场. $\mathcal{L}=(\frac{\partial\phi}{\partial t})^2-(\frac{\partial\phi}{\partial x})^2-\frac{\lambda}{4!}\phi^4$
- 超导 / 超流.
- Spin / 磁性.

$$Z=\int D\phi\, e^{-iS[\phi]}$$

$$S=\int \mathcal{L}\,dx \quad , \quad x=(\vec{x},t).$$

$$\mathcal{L}=\frac{1}{2}(\frac{\partial\phi}{\partial t})^2-\frac{m^2}{2}(\frac{\partial\phi}{\partial x})^2-\underbrace{\frac{\lambda}{4!}\phi^4}_{-V}.$$

$$=T-V.$$

Ref. Peskin. Chap 9.
Shanker. Chap 4.
Two views of renormalization

m: mass.
λ: interaction.

$$Z=\int D\phi\, e^{-iS[\phi]}$$ 如何计算？

考虑1阶的情形:

$$S=\int dx\,\mathcal{L}_0;$$

$$\mathcal{L}_0=\frac{1}{2}(\frac{\partial\phi}{\partial t})^2-\frac{m}{2}(\frac{\partial\phi}{\partial x})^2-\frac{\mu}{2}\phi^2,$$

ϕ: real field;

$$\phi(x)=\frac{1}{V}\sum_k \phi_k e^{ik\cdot x} \Longleftrightarrow \quad \phi(x)=\phi^*(x). \quad \phi(k)=\phi^*(k).$$

$$\Rightarrow \quad S=\int (\quad)\phi_k\phi_{-k}\,dk \leftarrow \text{计算如下:}$$

$$\phi(x)=\frac{1}{V}\sum_k \phi(k)e^{ik\cdot x}, \quad k\cdot x=\vec{k}\cdot\vec{x}-\omega t;$$

Ref. Parseval 定理

$$\int\frac{1}{2}(\frac{\partial\phi}{\partial t})^2dx=\frac{1}{2V^2}\sum_{kk'}\phi_k\phi_{k'}(-i\omega_k)(-i\omega_{k'})\int e^{i(k+k')\cdot x}dx$$

$$=\frac{1}{2V^2}(2\pi)^d\sum_{k,k'}\phi_k\phi_{k'}(-i\omega_k)(-i\omega_{k'})\,\delta(k+k')$$

(d为积分维度.)

和差化积分.
$\sum_k (1) = \frac{L}{2\pi}\int dk$

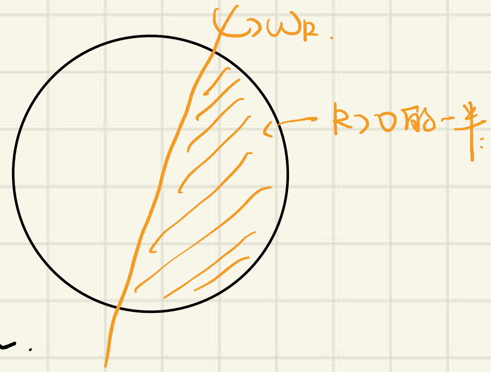
$$= \frac{1}{2V^2}(2\pi)^d \frac{V}{(2\pi)^d}\sum_k \int \phi_k \phi_{k'} (-i\omega_k)(-i\omega_{k'})\, \delta(k+k')\, dk'$$

$$= \sim$$

原则上: $k \to -k$

$\phi(x) \to \phi^*(x)$

$\phi_{-k} \to \phi^*_k$



$k\to -k$.没变化

所以($\times 2$) $= \frac{1}{V}\sum_{k\geq 0} \phi_k \phi_{-k} (-i\omega_k)(-i\omega_{-k})\, dk$.

$\omega_k = V|k|$.

$\omega_k = \omega_{-k}$

$(\frac{\partial}{\partial t})$ 贡献的仅仅是 $-i\omega_k$

同理 $\Rightarrow$ $(\frac{\partial}{\partial x})$ 贡献 k.

$$\Rightarrow S = \frac{1}{V}\sum_{k\geq 0} (-\omega_k^2 + m^2\vec{k}^2 - \mu)\, \phi_k^* \phi_k$$

(系数 $\frac{1}{2}$ 消失了)

复数场 (且,为对角).

总结起来.在 $\vec{k}$ 空间中.

$$Z = J \int D\phi_k\, e^{iS[\phi_k]}$$

$$S_0 = \sum_k \left(\frac{Q_k}{V}\right) \phi_k^* \phi_k . \quad \begin{cases} = 0 \\ = \infty \end{cases}$$

$$\prod_k D\phi_k = \prod_{k\geq 0} D\phi_k^* \phi_k$$

$$\int dx_1 \cdots dx_n\, e^{-\sum_i b_i x_i^2}$$

在计算关联时会有用:

$$\langle \phi(x)\phi(y) \rangle = \frac{\int D\phi\, e^{iS_0} \phi(x)\phi(y)}{\int D\phi\, e^{iS_0}}$$

$$= \sum_{q,q'} e^{iq\cdot x} e^{iq'\cdot y} \frac{\int e^{iS_0} \phi_q \phi_{q'}\, D\phi_q D\phi_{q'}}{\int e^{iS_0} D\phi_q D\phi_{q'}}$$

$$= \sum_{q,q'} e^{iq\cdot x + iq'y} \frac{1}{a} \delta(q,q').$$

$\left(a = \frac{Q_k}{V} \right).$

$$= \frac{1}{V} \sum_{q} e^{iq(x-y)} \frac{1}{Q_k}$$

2021.3.10 第三周第二节课.

parseval formula.

$$f(x) = \frac{a_0}{\sqrt{2}} + \sum_{n=1}^{\infty} a_n \cos(nx) + \sum_{n=1}^{\infty} b_n \sin(nx)$$

$$\langle f, f \rangle = \int f^2(x)\, dx$$

or $\int f^2(x)\, dx = ? \int f^2(k)\, dk.$

今天的内容: 1. Green's function.

Propogator. $D(x-y) = \frac{1}{V} \sum_{k} (\quad)$

图表示. 成对出现.

2. Interaction.

Cumulant Expansion (Moment Expansion).

物理: linked cluster expansion.

意义.

$\phi(x) = \frac{1}{V} \sum_{k} e^{ik\cdot x} \phi(k)$, $k = (k_0, \vec{k})$. 对应之前的 ω_k. $x = (t, -\vec{x})$.

$$S_0 = \int \mathcal{L}_0\, dx$$

$$\int \frac{1}{2} \left(\frac{\partial \phi}{\partial t}\right)^2 dx = \frac{1}{2V^2} \sum_{k,k'} \phi(k)\phi(k') (ik_0)(ik_0') \int e^{i(k+k')\cdot x} dx$$

$$= \frac{1}{2V^2} \sum_{k,k'} \phi(k)\phi(k')(ik_0)(ik_0')\, \delta(k+k') \cdot (2\pi)^d$$

$$= \frac{1}{2V^2} \sum_{k} \phi(k) \frac{V}{(2\pi)^d} \int \phi(k')(ik_0)(ik_0')\, \delta(k+k')\, dk'$$

要求 $k' = -k$. $\Rightarrow k_0' = -k_0$;

$$= + \frac{1}{2V} \frac{1}{(2\pi)^d} \sum_{k} \phi(k)\, \phi(-k)\, k_0^2$$