$$E=\frac{1}{2}\hbar c\sum_{n=1}^{+\infty}\frac{n\pi}{a}$$
$$=\frac{1}{2}\hbar c\sum_{n=1}^{+\infty}k_n \quad \left|\begin{array}{l} e^{-k_n\varepsilon} \\ f(k_n\varepsilon) \\ e^{-\frac{k_n}{\lambda}} \\ \varepsilon=\frac{1}{\lambda}\end{array}\right.$$
$$=\frac{\hbar ca}{2\pi\varepsilon^2}-\frac{\hbar c\pi}{24a}+\cdots$$

$$E^f=\frac{1}{2}\hbar c\sum_k|k|=\frac{1}{2}\hbar c\cdot\frac{L}{2\pi}\int_{-\infty}^{\infty}dk|k|=\frac{\hbar cL}{2\pi}\int$$
$$\hat{E}=\frac{E^f}{L}\times a=\frac{\hbar ca}{2\pi\varepsilon^2}\int_0^{+\infty}\varepsilon k\, d\varepsilon k\cdot e^{-\varepsilon k}$$
$$=\boxed{\frac{\hbar ca}{2\pi\varepsilon^2}}$$

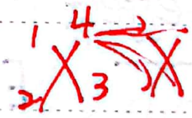
$$\Delta E=E-\hat{E}=-\frac{\hbar c\pi}{24a}$$

☆ $E=-\frac{\hbar^2\Lambda^2}{2m}e^{-\frac{4\pi}{\lambda}}$. （重整化的关键）

ϕ^4 理论：相关
- 声子：$(\frac{\partial\phi}{\partial t})^2-(\frac{\partial\phi}{\partial x})^2-\frac{\lambda}{4!}\phi^4$
- 超流/超导
- spin/磁性

$Z=\int D\phi\, e^{iS}$, $S=\int \mathcal{L}\, d\vec{x}\, dt=\int\mathcal{L}\, dx$. $x=(\vec{x},t)=(x,y,z,t)$

$\mathcal{L}=\frac{1}{2}(\frac{\partial\phi}{\partial t})^2-\frac{m^2}{2}(\frac{\partial\phi}{\partial x})^2-\frac{\lambda}{4!}\phi^4=T-V \leftarrow \frac{\lambda}{4!}\phi^4$

Feynman图  4!

Ref: Peskin chap 9

Shanker chap 14. two views of Renormalization.

m: mass

λ: Interation

Fourier 变换：常用公式：① $\int dx\, e^{-bx^2}=\sqrt{\frac{\pi}{b}}$

② $\int dx\, e^{-\frac{b}{2}x^2}=\sqrt{\frac{2\pi}{b}}$

③ $\frac{\int dx\, e^{-\frac{b}{2}x^2}\cdot x^2}{\int dx\, e^{-\frac{bx^2}{2}}}=\frac{1}{b}$

④ $\int dz\, d\bar{z}\, e^{-b\bar{z}z}=\frac{1}{b}$ ⑤ $\frac{\int dz\, d\bar{z}\, e^{-b\bar{z}z}\,\bar{z}z}{\int dz\, d\bar{z}\, e^{-b\bar{z}z}}=\frac{1}{b}$

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$Z=\int D\phi\, e^{S[\phi]}$ 如何计算?

1) 动量空间　　(2) 微扰 pertarbation $\frac{\lambda}{4!}\phi^4 \leftrightarrow \phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{k_4}$

$\int D\phi = \int \prod_{j=1}^{N} A\, d\phi(\vec{x}_j)$ 实空间 $\{\vec{x}_j\}$ → 动量空间 $\prod_{j=1}^{N} A' d\phi_{k_j}$

①J为常数 ②J与场无关.

Fourier transformation $\phi(x)=\frac{1}{V}\sum_k \phi_k e^{ikx} \Leftrightarrow \phi(k)=\int \phi(x) e^{-ikx} dx$

$kx=\vec{k}\cdot\vec{x}-\omega t$, $\sum_k f(k)=\frac{V}{(2\pi)^d}\int dk\, f(k)$

$\frac{1}{V}\sum_k f(k)=\frac{1}{(2\pi)^d}\int dk\, f(k)$

$\phi(x_j)=\frac{1}{V}\sum_n \phi_{k_n} e^{ik_n x_j}$

$\begin{pmatrix}\phi(x_1)\\ \phi(x_2)\\ \vdots\end{pmatrix}=\frac{1}{V}\left(e^{ik_n x_j}\right)\begin{pmatrix}\phi_{k_1}\\ \vdots\\ \phi_{k_N}\end{pmatrix}=U\begin{pmatrix}\phi_{k_1}\\ \vdots\\ \phi_{k_n}\end{pmatrix}$, U与φ无关 与m,λ均无关.

$\phi_k^*\phi_k(-i\omega_k)^2-m^2\vec{k}^2\phi_k^*\phi_k-\mu\phi_k^*\phi_k$

$S=\int dx\, \mathcal{L}_0$, $\mathcal{L}_0=\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2-\frac{m}{2}\left(\frac{\partial\phi}{\partial x}\right)^2-\frac{\mu}{2}\phi^2$

ϕ: real field $\phi(x)=\frac{1}{V}\sum_k \phi_k e^{ikx} \Leftrightarrow \phi(x)=\phi^*(x)\cdot \phi_k^*=\phi_k$

积分计算: parseval定理. $\int n(x)dx=\int n(k)dk$

$\int f^2(x)dx=\frac{1}{L^2}\sum_{kq}\int e^{i(k+q)} f_k f_q\, dx=\frac{1}{L^2}\sum_{kq} f_k f_{-k}\,\delta(k+q=0)$

$=\frac{1}{L^2}\sum_{kq} f_k f_{-k}\,\delta(k+q)=\frac{1}{L}\sum_k f_k f_{-k}\int \delta(k+q)dq$

$=\frac{1}{L}\int f^2(k)dk$

$S=\int dx\,\mathcal{L}_0 \to \int dk(\quad)\phi_{-k}\phi_k$ or $\frac{1}{V}\sum_k(\quad)\phi_k\phi_k$

$e^{ikx}=e^{i(\vec{k}\cdot\vec{x}-\omega t)}$

$\phi(x)=\frac{1}{V}\sum_k \phi(k)e^{ikx}$, $\mathcal{L}=\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2-\frac{m^2}{2}\left(\frac{\partial\phi}{\partial x}\right)^2-\frac{\mu}{2}\phi^2$.

$\int\frac{1}{2}\left(\frac{\partial\phi}{\partial t}\right)^2dx=\frac{1}{2V^2}\sum_{kk'}\phi_k\phi_{k'}\, e^{i(k+k')x}(-i\omega_k)(-i\omega_{k'})dx=\frac{1}{2V^2}\sum_{kk'}\phi_k\phi_{-k}(-i\omega_k)(-i\omega_{k'})\int e^{i(k+k')x}dx$

$=\frac{1}{2V^2}\sum_{kk'}\phi_k\phi_{-k}(-i\omega_k)(-i\omega_{k'})$

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由 $\int e^{ikx}dx = 2\pi\delta(k)$， $= \frac{1}{2V^2}(2\pi)^d \sum_{k,k'} \phi_k \phi_{k'} (-i\omega_k)(-i\omega_{k'})\delta(k+k')$

和差化积分 $\sum_k (1) = \frac{1}{2\pi}\int dk$ $= \frac{1}{2V^2}(2\pi)^d \sum_k (\frac{V}{2\pi})\int \phi_k \phi_{k'} (-i\omega_k)(-i\omega_{k'})$

$= \frac{1}{2V}\sum_k \phi_k \phi_{-k} (-i\omega_k)(-i\omega_{-k})$ 与一样 取一半

原则上 $k \to -k$ $\phi(x) = \phi^*(x)$， $\phi_{-k} = \phi_k^*$

$\sum_k \phi_k \phi_k (-i\omega_{-k})(-i\omega_k)$

$= \frac{1}{V}\sum_{k\geq 0} \phi_k^* \phi_k (-i\omega_k)^2$

则 $S_0 = \frac{1}{V}\sum_{k\geq 0}(\phi_k^*\phi_k(-i\omega_k)^2 - m^2(i\vec{k})^2\phi_k^*\phi_k - \mu\phi_k^*\phi_k)$

$= \frac{1}{V}\sum_{k\geq 0}(-\omega_k^2 + m^2\vec{k}^2 - \mu)\phi_k^*\phi_k$ 对角复数场.

总结起来，在 $\vec{k}$ 空间 $Z = \int D\phi_k e^{iS_0}$ ，$S_0 = \frac{1}{V}\sum_k (\frac{Q_k}{V})\phi_k^*\phi_k$.

$\prod_k D\phi_k = \prod_{k\geq 0} D\phi_k^* D\phi_k$

$\int dx_1 \cdots dx_N e^{-\sum_i b_i x_i^2}$

$\langle\phi(x)\phi(y)\rangle = \frac{\int D\phi e^{iS_0}\phi(x)\phi(y)}{\int D\phi e^{iS_0}} = \sum_{qq'} e^{iqx+iq'y} \cdot (\frac{Q_q}{V})\delta_{q+q'=0} = \frac{1}{V}\sum_q \left(\frac{e^{iq(x-y)}}{Q_k}\right)$

只有 $\phi^*(q)\phi(q)$ $q+q'=0$ 有贡献

$\langle x\rangle = \frac{\int P(x)\cdot x\,dx}{\int P(x)dx}$

$= \sum_{qq'} e^{iqx+iq'y} \frac{\int D\phi e^{iS_0}\phi(q)\phi(q')}{\int D\phi e^{iS_0}}$

对角化

eg: $\frac{\int dz_1 dz_2 dz_3 d\bar{z}_1 d\bar{z}_2 d\bar{z}_3 e^{-(a_1\bar{z}_1 z_1 + a_2\bar{z}_2 z_2 + a_3\bar{z}_3 z_3)}(\bar{z}_1 z_1)}{\int Dz D\bar{z} e^{-(\ )}}$

只有对角元素有贡献.

$= \frac{\int d\bar{z}_1 dz_1 e^{-a_1\bar{z}_1 z_1}\bar{z}_1 z_1}{\int d\bar{z}_1 dz_1 e^{-a_1\bar{z}_1 z_1}}$

$= \frac{1}{a_1}$