Bosonization. (玻色化)

不同的 model 之间的关系：

BKT 相变 (2d. Classical) → Vortex excitation.

$$H = \frac{1}{2}(\partial_x \phi)^2 + (\partial_y \phi)^2 + A\cos\phi$$

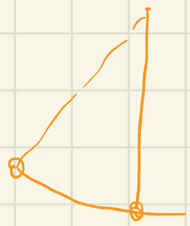
1d. Quantum. (Sine-Golden model)

$$\mathcal{L} = \frac{1}{2}\left[(\partial_t \phi)^2 + (\partial_x \phi)^2\right] + A\cos\phi \rightarrow \text{time vortex.}$$

Wick rotation.
$(x,y) \leftarrow (it, x)$

相位. (平移不变性)

$\psi \sim \psi e^{i\phi}$, $\phi \sim \phi + \text{const.}$ $\boxed{\cos\phi}$



在振子系统中,

$$\mathcal{L} = \frac{1}{2}(\partial_t \phi)^2 + \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4,$$

这里的 ϕ 是振幅. (x 点的) ;

Application of Sine-Golden model

☆ 物理. $1+1 = d = 2$ ⇔ 复数中的共形变换 / 场论.

☆ 1d. Fermion. ~ Dirac eqn. ⇔ Interacting Dirac eq.

☆ Sidney Coleman.

$$\text{SG} \overset{\text{dual}}{\simeq} \text{Thirring model}$$

☆ 物理系统中存在.

HE edge state.

Nanotube. Nanowire. Ultracold atom.

☆理论上证明：适用于任意粒子的物理.

① $d \geq 3$. F/B

② $d=1$, $d=2$ ⇒ Anyon.

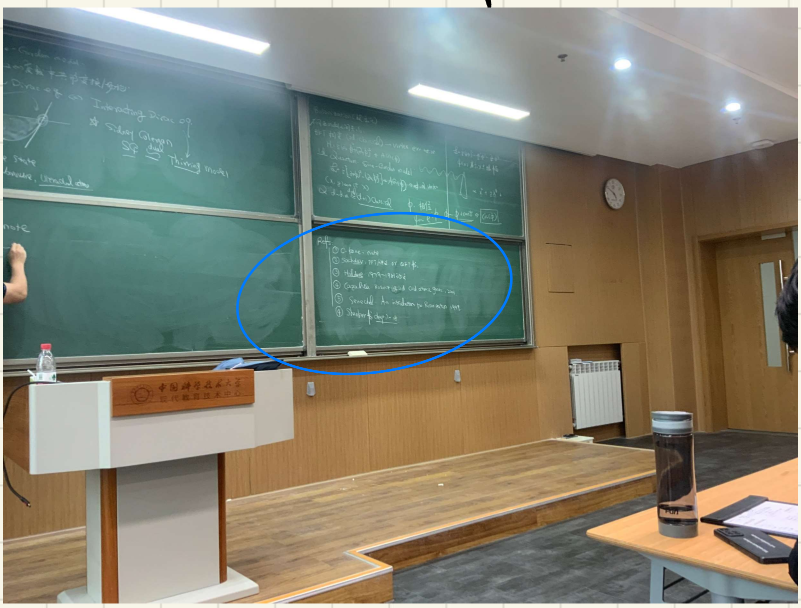
$d=1$. ⇒ 无法交换,

只会有 $\rho(x)$ 与 $\theta(x)$ 改变;

不同的统计都对应 Bose 激发. $\frac{\partial \rho}{\partial t} + \nabla_x j = 0$

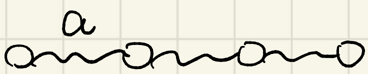
流体.

Refs.



几个例子

① C. Kane's Note



$$\mathcal{L} = \frac{m}{2}\sum_i \dot{x}_i^2 - \frac{k}{2}(x_i - x_{i+1})^2$$

⇓ 纵场论. $x_i = \sqrt{a}\,\phi(x_i,t)$

$\omega = v|q|$

$$N = \sum_q \frac{1}{e^{\beta v q}-1} = \underbrace{\int dq \frac{1}{e^{\beta v q}-1}}$$

红外发散. (你们会面临的问题).

Mermin–Wager Theorem.

1d chain.

$\omega = v|q|$

$q = \frac{n\pi}{L}$ (驻波)

$\omega_q = \frac{\pi V}{L} n$

$\Longleftrightarrow \quad H = \frac{V\pi}{L} \sum_{n\geq 1} n\, b_n^+ b_n \Leftrightarrow \omega_n = \frac{V\pi n}{L}$

Fermion:

$$H = \int \psi_k^+ \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\right)\psi_k$$

$$= \left(\frac{\hbar^2 k^2}{2m} - \mu\right)\psi_k^+ \psi_k, \qquad \frac{\hbar^2 k_F^2}{2m} = \mu;$$

定义: $\psi(x) = \underbrace{e^{ik_F x}}_{\text{快变;}} \underbrace{\psi_R}_{\text{慢变;}} + e^{-ik_F x}\psi_L$

$$\Rightarrow \begin{cases} \rho = N/L. \\ k = \pi/a. \end{cases}$$

快变量: $\int e^{-2ik_F x}\psi_L^+ \psi_R \, dx \sim 0$; 费米面附近. $\frac{\hbar^2 k_F^2}{2m} - \mu \sim 0$;

$$\Rightarrow H = \int dx\, \psi_R^+ e^{-ik_F x}\, 2\left(-\frac{\hbar^2}{2m}\right)\frac{d}{dx}\left(e^{ik_F x}\right)\frac{d}{dx}\psi_R;$$

$$= \int dx\, \psi_R^+ \left(-\frac{\hbar^2 i k_F}{2m}\right)\frac{d}{dx}\psi_R;$$

$$= v_F \psi_R^+ \left(-i\frac{d}{dx}\right)\psi_R; \qquad v_F = \frac{\hbar^2 k_F}{2m} = \frac{E_F}{k_F}$$

Left:

$$-v_F \psi_L^+ \left(-i\frac{d}{dx}\right)\psi_L;$$

$$H = (\psi_R^+, \psi_L^+)\begin{pmatrix} -i\frac{d}{dx} & 0 \\ 0 & i\frac{d}{dx} \end{pmatrix}(\psi_R, \psi_L)$$

Exact duality of F/B in 1d;

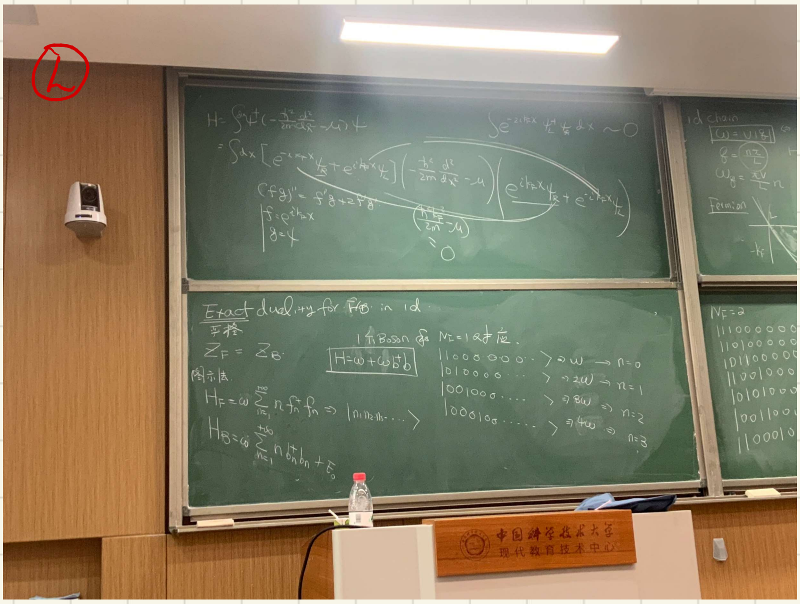
(1) $Z_F = Z_B$;

(2) 图示法;

$$H_F = V \sum_{n=1}^{+\infty} n f_n^\dagger f_n$$

$$H_B = V \sum_{n=1}^{+\infty} n b_n^\dagger b_n + E_0$$

电子→空穴 ⟺ 玻色子.



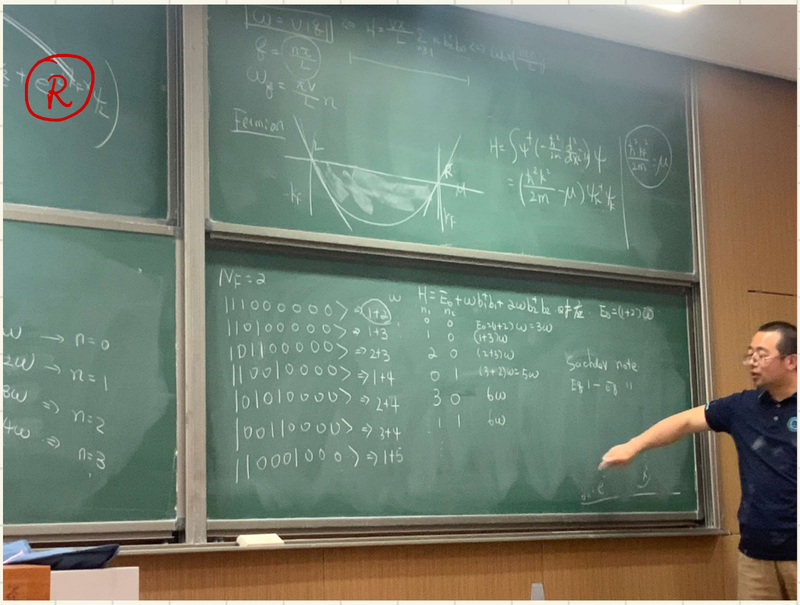
$|1 0 0 0 0 0 \cdots\rangle$ 0个 Boson

$|0 1 0 0 0 0 \cdots\rangle$ 1个 ...

$|0 0 1 0 0 0 \cdots\rangle$ 2个 ...

...

同理～.



作业:

Sachdev's note,

Eq1 ~ Eq11 (Proof);

$(Z_F = Z_B)$