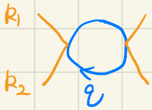
则. 只留下:

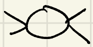


$$-\frac{1}{2}\langle \Delta S^2 \rangle_>^c$$

$$= -\frac{1}{2}\frac{\lambda^2}{4!}\sum_{k_1,k_2,k_3 \le d\Lambda} \phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{-(k_1+k_2+k_3)} \cdot \sum_{b\Lambda\le|q|\le\Lambda} \frac{1}{q^2+m^2}\frac{1}{(q-k_1-k_2)^2+m^2}$$

$$\left(= \sum_q \left(\frac{1}{q^2+m^2}\right)^2 = S_d \Lambda^{2d-4}(1-b)\right)$$

$$\times 6\times 6\times 2 \quad = -\frac{\delta\lambda}{4!}\sum_{k_1k_2k_3}\phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{-(k_1+k_2+k_3)}$$

$\Rightarrow$ $\delta\lambda = \frac{3}{2}$ 

$$= \frac{3}{2} S_d \Lambda^{d-4}(1-b)$$

2022.3.28. 第六周第1节课.

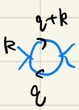
 $\propto \sum_q \frac{1}{q^2+m^2}$ $\quad = \int_{b\Lambda}^{\Lambda}\frac{k^{d-1}}{k^2}dk$

if $\Lambda \gg m$; ① Peskin书: $\Rightarrow \int_{b\Lambda}^{\Lambda} k^{d-3}dk = \Lambda^{d-2}(1-b)$;

②.

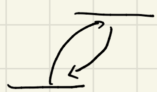
③.

$k\simeq 0$



$$\propto \sum_q \frac{1}{q^2+m^2}\frac{1}{q^2+m^2} = S_d\Lambda^{d-4}(1-b)$$

Sign 问题: 区别
- 一阶. 正比
- 二阶. 降低
- 三阶. ...



基态的二阶微扰修正为负;

$$\int D\phi_< e^{-S} = \int D\phi_< e^{-(S_<+\delta S)} = \int D\phi_< e^{-S_< - \delta S + \frac{1}{2}(\delta S)^2}$$

Sign ⊕.

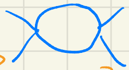
质量修正：

$$m'^2 = m^2 + \delta m^2$$

$$= m^2 + A\lambda S_d \Lambda^{d-2}(1-b)$$

相互作用修正：

$$\lambda' = \lambda + \delta\lambda$$



$$= \lambda + B\lambda^2 \Lambda^{d-4}(1-b)$$

Screening effect.

$$\begin{cases} \int D\phi_< e^{-S_{eff}} \\ S_{eff} = \sum_{|k|\le b\Lambda} \frac{k^2+m'^2}{2}\phi_k^*\phi_k + \frac{\lambda'}{4!}\sum_{|k_1,k_2,k_3,k_4|\le b\Lambda} \phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{k_4}\,\delta(k_1+k_2+k_3+k_4) \end{cases}$$

RG前 $|k| \le \Lambda$.

RG后 $|k| \le b\Lambda$.

令 $k = bq$

$$|k| \le b\Lambda \iff |bq| \le b\Lambda \iff |q| \le \Lambda$$

$$\phi_k = \phi_{bq} = z\,\phi_q$$

如何确定这个 z？ 让动能项不发生改变；

标度分析：

动能项 $\sum_k \frac{k^2}{2}\phi_k^*\phi_k$

$$\overset{k=bq}{=\!=} \left(\frac{1}{2\pi}\right)^d \int dk\, \frac{k^2}{2}\phi_k^*\phi_k = \left(\frac{1}{2\pi}\right)^d \underbrace{b^{d+2} z^2}_{\equiv 1} \int_{|q|\le\Lambda} dq\, \frac{q^2}{2}\phi_q^*\phi_q$$

$$b^{d+2} z^2 \equiv 1 \iff z = b^{-1-\frac{d}{2}};$$

质量项. $\propto m'^2 \int dk\, \phi_k^* \phi_k$

$$= m'^2 b^d z^2 \int dq\, \phi_q^* \phi_q$$

$$= m'^2 b^{-2} \int dq\, \phi_q^* \phi_q$$

$$= m^2(b) \int dq\, \phi_q^* \phi_q . \qquad \left(m^2(b) = b^{-2}[m^2 + \delta m^2] \right).$$

相互作用项.

$$\lambda' \int dk_1 dk_2 dk_3 dk_4\, \phi_{k_1} \phi_{k_2} \phi_{k_3} \phi_{k_4}\, \delta(k_1+k_2+k_3+k_4)$$

$$= \lambda' z^4 b^{3d} \int dq_1 dq_2 dq_3 dq_4\, \phi_{q_1} \phi_{q_2} \phi_{q_3} \phi_{q_4}\, \delta(q_1+q_2+q_3+q_4)$$

$$= \lambda(b) \int (\cdots)$$

where. $\lambda(b) = b^{d-4}(\lambda + \delta\lambda)$

$m^2(b) = b^{-2}(m^2 + \delta m^2)$

总结: 把 $[b\Lambda, \Lambda]$ 积分 $\Rightarrow$ 重新标度.

使 $[0, b\Lambda]$ 变化回积分区间 $[0, \Lambda]$

$$k_1 \to k_2 \to k_3 \to \cdots$$
$$(q)$$

$$\begin{cases} m^2(b) = b^{-2}(m^2 + \delta m^2) \\ \lambda(b) = b^{d-4}(\lambda + \delta\lambda) \end{cases}$$

其中. $\delta m^2 \propto \lambda$

$\delta\lambda \propto -\lambda^2$. $\qquad \dfrac{1}{q^2+m^2} \sim \dfrac{1}{\Lambda^2}$

$$\lambda(b) = b^{d-4}\left(\lambda - B\lambda^2 S_d \Lambda^{d-4}(1-b)\right).$$

取 $b = 1 - dl$.

β.function.

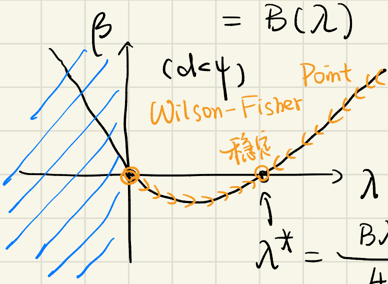
$$\lambda(1-dl) = \lambda - \frac{d\lambda}{dl}dl = \lambda - \beta dl$$

$$\lambda - \beta dl = (1-dl)^{d-4}\left(\lambda - B\lambda^2 S_d \Lambda^{d-4}(1-b)\right)$$
$$= \left[1+(4-d)dl\right]\left[\lambda - B\lambda^2\Lambda^{d-4}dl\right]$$

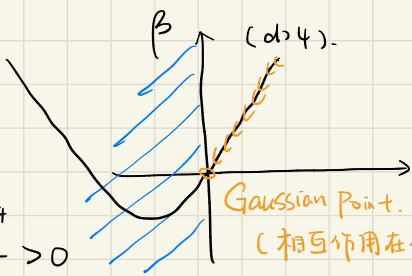
if $dl = 0$. 自然成立. $\lambda \equiv \lambda$.

$$-\beta dl = -B\lambda^2\Lambda^{d-4}dl + (4-d)\lambda dl.$$ 保留线性项相同.

$$\beta = B\lambda^2\Lambda^{d-4} - (4-d)\lambda$$
$$= B(\lambda)$$



$$\lambda^* = \frac{B\lambda\Lambda^{d-4}}{4-d} > 0$$



$\lambda<0$

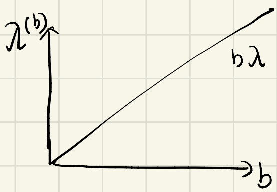
非物理: $(\partial\phi)^2 + m^2\phi^2 + \frac{\lambda}{4!}\phi^4$

没有基态 $(\lambda<0)$

取 $\lambda \to 0$. 则 $\lambda^2 \to 0$.

则 $\lambda(b) = b^{d-4}\lambda$

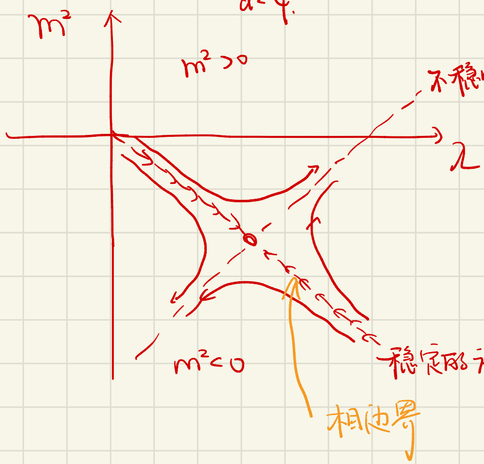
if $d=5$. $\lambda(b) = b\lambda$



if $d=3$. $\lambda(b) = \frac{1}{b}\lambda$

作业. ① $\begin{cases} m^2(b) \\ \lambda(b) \end{cases}$ 画图.

$d<4$.



$\langle\phi\rangle=0$.

自发对称破缺;

(自发磁化);

ordered. $\langle\phi\rangle\neq 0$.

② (2选1)

(1) ϕ^3 理论 $d=6+\epsilon$ 情的 RG flow;

(2) Spin model. (RG). ref. Kardar 书;

$\phi^4 \Longleftrightarrow$ 伪合号;