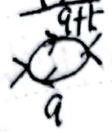
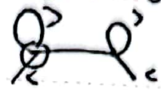
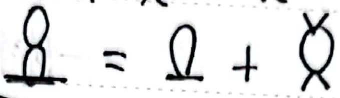
DATE

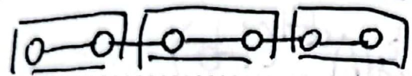
 $\sim \sum_q \frac{1}{q^2+m^2}$

 $\sim \sum_q \frac{1}{q^2+m^2} \cdot \frac{1}{(q+k)^2+m^2}$

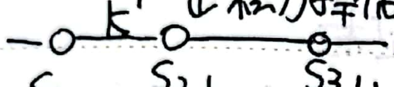
$\langle SS \rangle_c^?$ 4项　　　$*: \int x^{2n+1} e^{-ax^2} dx = 0$.

$\langle SS^2 \rangle_c^?$ 16项　　类似  节点处动量不守恒



RG: 

↓积分掉偶数项/高能



s_1　s_2　s_3　　$S_{2i-1} \to S_i$

↓ rescaling　　$k \to k_1 \to k_2$ or $k_{n+1} = R(k_n)$

逐步接近平均场的过程.

$\langle e^{xt} \rangle = e^{\sum_{n=0}^{+\infty} \frac{t^n}{n!} \langle x^n \rangle_c}$

$$☆\ \Omega = \sum_k \frac{1}{k^2+m^2} = \frac{1}{(2\pi)^d} \int dk \frac{1}{k^2+m^2} = \underbrace{\frac{\Omega_d}{(2\pi)^d}}_{S_d} \int_{b\Lambda}^{\Lambda} \frac{k^{d-1} dk}{k^2+m^2}$$

处理: ① $\Lambda \gg m$, $\int_{b\Lambda}^{\Lambda} \frac{k^{d-1}}{k^2} dk = \int_{b\Lambda}^{\Lambda} k^{d-3} dk = \frac{1}{d-2} k^{d-2} \Big|_{b\Lambda}^{\Lambda}$

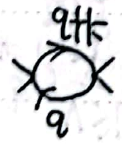
$b^{d-2} \approx e^{(d-2)\ln b}$　　$= \frac{\Lambda^{d-2}(1-b^{d-2})}{d-2} \approx \frac{\Lambda^{d-2}[1-1-(d-2)\ln b]}{d-2} = -\Lambda^{d-2}\ln b$

$1-b \approx -\ln b$ if $b \to 1^-$, $b = 1-dx$　$\ln(1-dx) = -dx$　$= \Lambda^{d-2}(1-b)$

② $\int_{b\Lambda}^{\Lambda} \frac{k^{d-1}}{k^2} dk = \Lambda^{d-3}(\Lambda - b\Lambda) = \Lambda^{d-2}(1-b)$

③ $\int_{b\Lambda}^{\Lambda} \frac{k^{d-1}}{k^2+m^2} dk = \int_{b\Lambda}^{\Lambda} \frac{k^{d-1}}{k^2}\left(1-\frac{m^2}{k^2}\right) dk$

$= \Lambda^{d-1}(1-b) - m^2 \Lambda^{d-3}(1-b)$ ← 有可能对书上结果有小修正.

☆  $\propto \sum_q \frac{1}{q^2+m^2} \cdot \frac{1}{q^2+m^2} = S_d \Lambda^{d-4}(1-b)$

$q \approx 0$

Sign问题: 一阶 —— : 成正比

区别　二阶微扰: 降低能量 ✓

三阶 —— :

$\int D\phi_< e^{-S_<}\int D\phi_> e^{-(S_>+\delta S)} = \int D\phi_< e^{-S_< - \langle\delta S\rangle_>^c + \frac{1}{2!}\langle\delta S^2\rangle_>^c}$

Yang-Mills 渐近自由理论， Sidney Coleman/Wilczek

$\Omega \propto \frac{\lambda^N}{4!} S_d \Lambda^{d-2}(1-b)$  $\propto S_d\Lambda^{d-4}(1-b)$

$\phi_k^*\phi_k$

$S_< = \sum_k \frac{k^2+m^2}{2}\phi_k^*\phi_k + \frac{\lambda}{4!}\sum_{q<b\Lambda}\phi\phi\phi\phi$

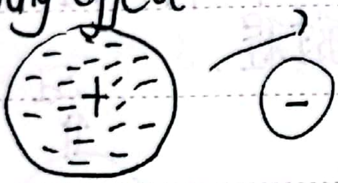
质量修正 $m'^2 = m^2+\delta m^2$ ， $\delta m^2 = +\frac{\lambda}{4!} S_d\Lambda^{d-2}(1-b)$

一阶修正 $\propto \lambda$ ； $\frac{\Lambda^{d-1}}{\Lambda^2}(\Lambda-b\Lambda)$

相互作用 $\lambda' = \lambda+\delta\lambda$ ， $\delta\lambda = -B\lambda^2 S_d\Lambda^{d-4}(1-b)$

二阶过程

Screening effect



$\frac{e^2}{4\pi\varepsilon_0\varepsilon_r r}$ $(\varepsilon_r>1)$

动量交换　产生粒子后湮灭回光子

$\int D\phi_< e^{-S_{eff}}$ ， $S_{eff} = \sum_{|k|\le b\Lambda}\frac{k^2+m'^2}{2}\phi_k^*\phi_k + \frac{\lambda'}{4!}\sum_{|k_1k_2k_3k_4|\le b\Lambda}\phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{k_4}\delta(k_1+k_2+k_3+k_4)$

RG 前 $|k|\le\Lambda$
后 $|k|\le b\Lambda$ } re-scaling { Ising $S_{2i-1}\to S_i$ ； $|k|\le b\Lambda \to |k|\le\Lambda$

标度分析

设 $k=bq$ ， $|k|\le b\Lambda \Rightarrow |q|\le\Lambda$ $\longrightarrow$ $\phi_k = \phi_{bq} = z\phi_q$

$\sum_k \frac{k^2}{2}\phi_k^*\phi_k = \left(\frac{1}{2\pi}\right)^d\int_{|k|\le b\Lambda} dk\, \frac{k^2}{2}\phi_k^*\phi_k = \underbrace{\boxed{b^{d+2}z^2}}_{=1}\left(\frac{1}{2\pi}\right)^d\int_{|q|\le\Lambda}\frac{q^2}{2}\phi_q^*\phi_q$

取 $z^2 b^{d+2}=1$ ∴ $z = b^{-1-\frac{d}{2}}$

$\propto m'^2\int dk\,\phi_k^*\phi_k = m'^2 b^d z^2\int dq\,\phi_q^*\phi_q = m'^2 b^{-2}\int_{|q|\le\Lambda} dq\,\phi_q^*\phi_q$

$m^2(b)$ 与之前的m相比变化了

$m^2(b) = b^{-2}[m^2 + \delta m^2]$

相互作用：$\lambda' \int dk_1 dk_2 dk_3\, \phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{-k_1-k_2-k_3}$

$$= \underbrace{\boxed{\lambda' z^4 b^{3d}}}_{\lambda(b)} \int_{|q_i|<\Lambda} dq_1 dq_2 dq_3\, \phi\phi\phi\phi$$

$$\lambda(b) = \lambda' z^4 b^{3d} = \lambda' b^{-4-2d} b^{3d} = \lambda' b^{d-4}$$

总结：把 $[b\Lambda, \Lambda]$ 积分 $\Rightarrow$ 重新标度变化，积分区间 $[0,\Lambda]$

$k_1 \to k_2 \to k_3 \to \cdots$ $\begin{cases} m^2(b) = b^{-2}(m^2 + \delta m^2) \\ \lambda(b) = b^{d-4}(\lambda + \delta\lambda) \end{cases}$

其中 $\begin{cases} \delta m^2 \propto \lambda \\ \delta\lambda \propto -\lambda^2 \end{cases}$ $\qquad \dfrac{1}{q^2+m^2} \sim \dfrac{1}{\Lambda^2}$

$$\lambda(b) = b^{d-4}[\lambda - B\lambda^2 S_d \Lambda^{d-4}(1-b)]$$

取 $b = 1 - dl$ 则 β-function

$$\lambda(1-dl) = \lambda - \frac{d\lambda}{dl} dl = \lambda - \beta dl$$

$$\lambda - \beta dl = (1-dl)^{d-4}(\lambda - B\lambda^2\Lambda^{d-4}dl) \qquad (1-dx)^n$$

$+k_3+k_4)$ $\qquad = [1 + (4-d)dl][\lambda - B\lambda^2\Lambda^{d-4}dl] \qquad = 1 - ndx$

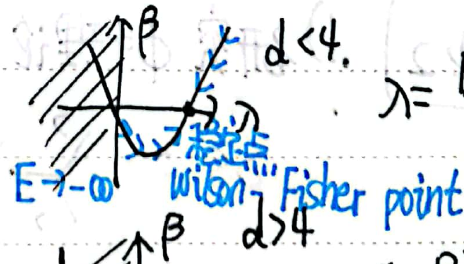
取 $dl = 0$，自然成立.

线性项：$-\beta dl = -B\lambda^2\Lambda^{d-4}dl + (4-d)\lambda dl$ 由此

$$\boxed{\beta = B\lambda^2\Lambda^{d-4} + (4-d)\lambda}$$

$\beta(\lambda)$ 函数

$d<4$. $\qquad \lambda = \dfrac{B\lambda\Lambda^{d-4}}{4-d} > 0$

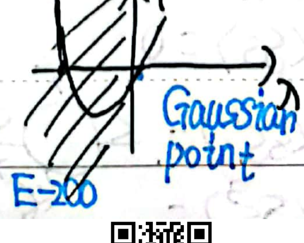


$\lambda < 0$ 是非物理区间

$\mathcal{L} = (\partial\phi)^2 + m^2\phi^2 + \lambda\phi^4$

$\because \lambda < 0$ 意味着系统无基态.

$d>4$ $\qquad \lambda^* = \dfrac{B\lambda\Lambda^{d-4}}{4-d} < 0$

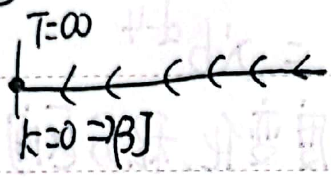


取 $\lambda \to 0$，所以 $\lambda^2 \to 0$　令 $\lambda^2 = 0$

$\boxed{\lambda(b) = b^{d-4}\lambda}$　讨论 $b = 0.99$，$d = 3 \Rightarrow \lambda(b) = \frac{1}{0.99}\lambda > \lambda$

$d = 5 \Rightarrow \lambda(b) = 0.99\lambda < \lambda$

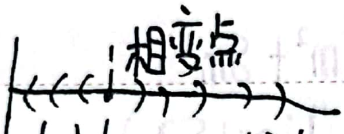
回顾 Ising model

1d 

$\int dx (\partial\phi)^2 + \int dx \lambda \phi^4$

$x \to \lambda x$　$[x] \sim 1 \Leftrightarrow d + 2[\phi] = 2$

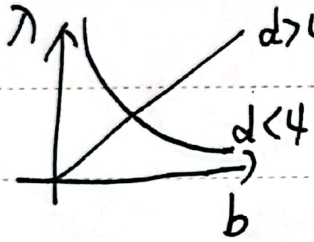
则 $[\phi] = 1 - \frac{d}{2}$

2d 

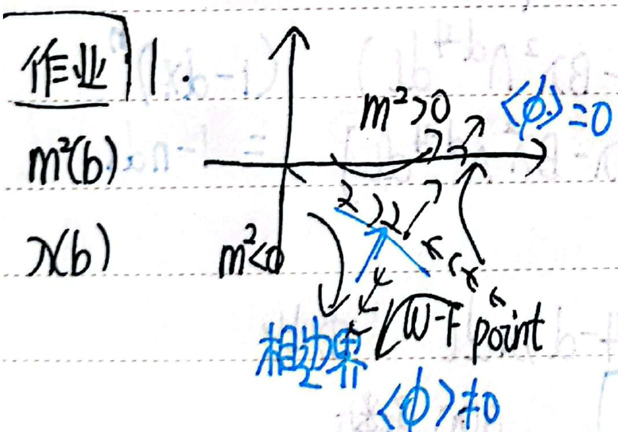
disordered phase　$\langle s \rangle \neq 0$

$\langle s \rangle = 0$　$\Leftrightarrow$ ordered phase

$[\lambda] + d + 4[\phi] = 0 \Leftrightarrow [\lambda] = d - 4$



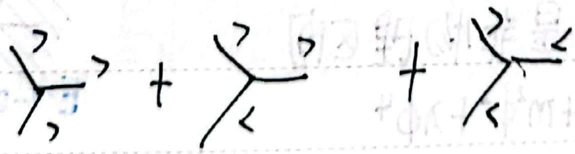
作业1.



$m^2 < 0$ 系统会自发对称破缺

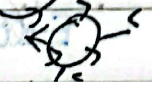
$(\partial\phi)^2 + m^2\phi^2 + \lambda\phi^4$

作业2　1) 研究 ϕ^3 理论 $d = 6 + \varepsilon$ 维的 RG flow

$\delta S =$ 

$\langle \delta S \rangle = 0$

$\langle \delta S^2 \rangle =$ 　给出 m^2 修正 $(-)$

$\langle \delta S^3 \rangle =$ 　$\sim \frac{1}{(k^2+m^2)^3} \propto S_d \Lambda^{d-6}(1-b)$　给出 $\delta\lambda$ 修正 $(+)$

DATE S M T W T F S

2) Spin model (RG), ref. kardor 书

$\phi^4 \Leftrightarrow$ 多分量 $\phi^2 = \sum_i \phi_i^2$

$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + m^2\phi^2 + \lambda\phi^4 + V\sum_i \phi_i^4$

Feynman 图只有相同分量才会有贡献.

Bosonization 玻色化 $\begin{cases} 1+1 \text{ 维} & \partial t, \partial x \to \text{Sine-Gordon model} \\ 2+0 \text{ 维} & \partial x, \partial y \to \text{BKT 相变} \end{cases}$

$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \lambda\cos(\beta\phi)$ $\qquad \psi = \psi e^{i2\pi}$

$\theta = \theta + 2\pi$