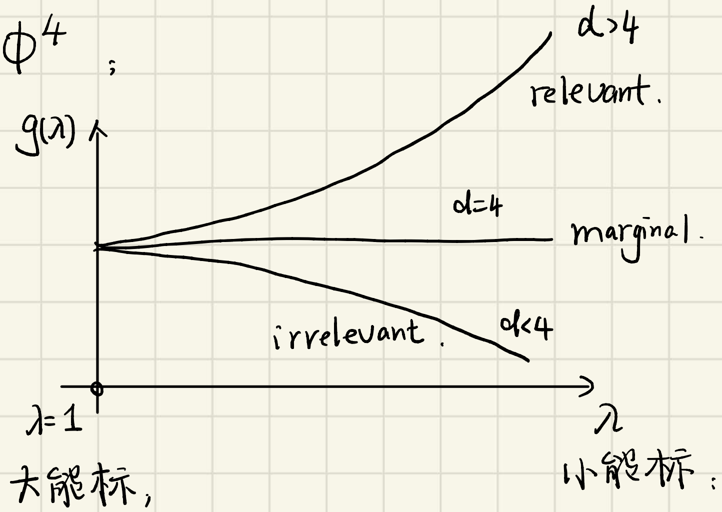
标度分析:

1) ϕ^4 理论.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!}\phi^4 ;$$

$$S = \int \mathcal{L}\, dx ;$$

$$\begin{aligned} x &\to \lambda x. \\ t &\to \lambda^2 t. \end{aligned} \Rightarrow g(\lambda) = g\lambda^{d-4}$$



2) Navier-Stokes eqn

$$\left| \begin{aligned} & u_t + u\cdot\nabla u + \nabla p - \Delta u = 0 \\ & \nabla\cdot u = 0 \\ & u|_{t=0} = u_0 \end{aligned} \right.$$

3) $$\frac{\partial h}{\partial t} = \nu\left(\frac{\partial^2 h}{\partial x^2}\right) + \eta \quad , \quad \text{kpz eqn.}$$

$$\frac{\partial h}{\partial t} = \nu\left(\frac{\partial^2 h}{\partial x^2}\right)^2 + \eta \quad , \quad \cdots$$

微分方程有标度分析;

2022.3.24. 第五周第2节课;

Review. RG of Ising model.

Step 1: Trace 掉偶格点.

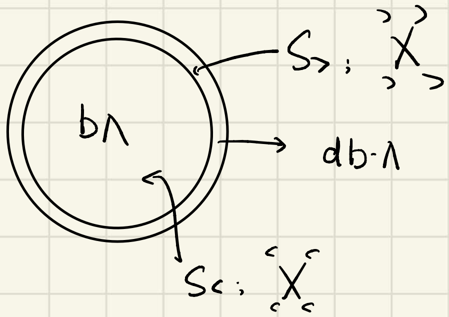
Step 2: rescaling.

ϕ^4 理论. $it \to \tau$.

$b = 1 - db$. $db \to 0$. $b \to 1$

$$Z = \int D\phi \, e^{-S}$$

$$S = S_< + S_> + \delta S$$



δS: X + X + X

$$\Rightarrow Z = \int D\phi_< D\phi_> e^{-S}$$

$$= \int D\phi_< e^{-S_<} \underbrace{\int D\phi_> e^{-S_> - \delta S}}_{S_> \text{单粒子}}$$

$$= \text{Const.} \times \underbrace{\langle e^{-\delta S} \rangle_>}_{\text{小量}} \times \int D\phi_> e^{-S_>}$$

$$= e^{-\langle \delta S \rangle_> - \frac{1}{2} \langle \delta S^2 \rangle_>^c}$$

$$= \int D\phi_< \, e^{-(S_< + \Delta S_{eff})}$$

$$\Delta S_{eff} = \langle \delta S \rangle_> + \frac{1}{2} \langle \delta S^2 \rangle_>^c$$

$$\sum_{|k| \le b\Lambda} (\cdots) + \sum_{|k| \le b\Lambda} (\cdots).$$

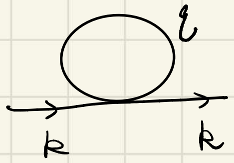
令 $\phi_k = \phi_{bq}$

$|bq| \le b\Lambda \iff |q| \le \Lambda$.

$\phi_{bq} = \sqrt{z} \, \phi_q$

$\delta S =$ 

与场无关.　　去（无贡献）.　　√　　去



$$\phi_k^* \phi_k \sum_{\substack{b\Lambda \le |q| \le \Lambda \\ m \ll \Lambda}} \frac{1}{q^2+m^2}$$

$$\sum_{b\Lambda \le |q| \le \Lambda} \frac{1}{q^2+m^2} = \frac{1}{(2\pi)^d} \int_{b\Lambda}^{\Lambda} dq \frac{1}{q^2+m^2}$$

$$= \frac{\Omega}{(2\pi)^d} \int_{b\Lambda}^{\Lambda} dq \frac{q^{d-1}}{q^2+m^2}$$

$$= S_d \Lambda^{d-2} (1-b);$$

$$\Delta S = \frac{\lambda}{4!} \sum\nolimits^{>(\text{至少有一个})} \phi_k \phi_{k'} \phi_q \phi_{-(k+k'+q)}$$

$$\langle \Delta S \rangle = \frac{\lambda}{4!} S_d \cdot \Lambda^{d-2} (1-b) \times C_4^2 \, \phi_{k<}^* \phi_{k<}$$

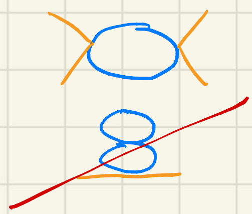
$$= \underbrace{\frac{\lambda}{4} S_d \cdot \Lambda^{d-2} (1-b)}_{\text{修正质量}} \phi_{k<}^* \phi_{k<}$$

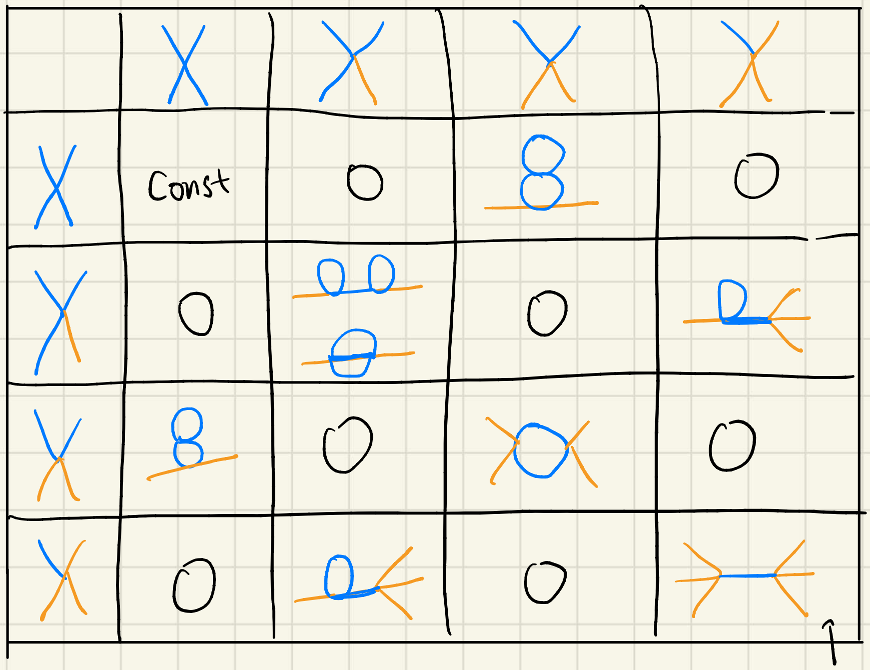
$$S^< = \sum_{|k| \le b\Lambda} \frac{k^2+m^2+\delta m^2}{2} \phi_{k\le b\Lambda}^* \phi_{k\le b\Lambda} + \frac{\lambda}{4!} \sum_{|k| \le b\Lambda} \phi_k \phi_{k'} \phi_q \phi_{-(k+k'+q)}$$

第1修正: $\langle \Delta S \rangle_>$　$\delta m^2 = \frac{\lambda}{2} S_d \Lambda^{d-2} (1-b)$, $\propto \lambda$. $\lambda > 0$.

第2修正: $\langle \Delta S^2 \rangle_>^c$　连接图;

有效散射;





动量不守恒.

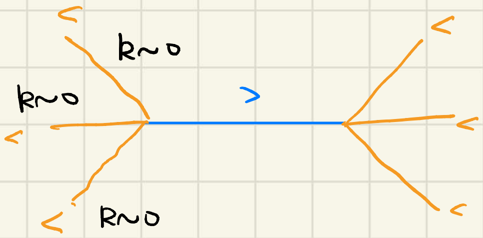
典型特点:

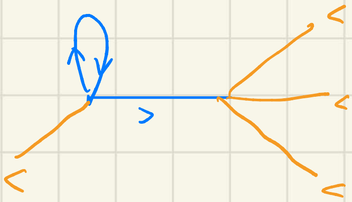
1) 新图/新相互作用.

2) 高阶 ⇒ 更复杂.

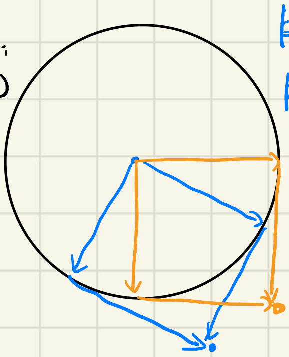
3) Shell中的散射有限.

1)的说明:





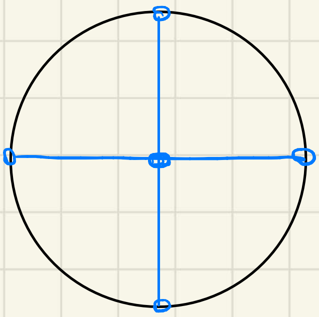
3)的说明:
(球形Fermi面)



$k_1 = k_3\ (k_4)$

$k_2 = k_4\ (k_3)$

Fermi Liquid.



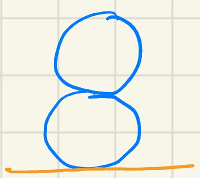
$k_1 = -k_3$

$k_2 = -k_4$

Cooper pair.

$$C^\dagger_{k_1} C^\dagger_{k_2} C_{k_3} C_{k_4} U_{k_1 k_2 k_3 k_4}.$$

又



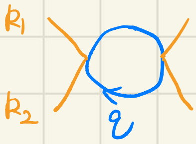
$$S_d \Lambda^{d-2}(1-b) \cdot S_d \Lambda^{d-4}(1-b)$$

$$= S_d^2 \Lambda^{2d-6}(1-b)^2$$

$$= db^2 \to 0$$

贡献也很小 $(\to 0)$.

则.只留下:



$$-\frac{1}{2}\langle \Delta S^2 \rangle_>^c$$

$$= -\frac{1}{2}\frac{\lambda^2}{4!}\sum_{k_1,k_2,k_3 \le b\Lambda} \phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{-(k_1+k_2+k_3)} \cdot \underbrace{\sum_{b\Lambda \le |q| \le \Lambda} \frac{1}{q^2+m^2}\frac{1}{(q-k_1-k_2)^2+m^2}}$$

$$\left(= \sum_q \left(\frac{1}{q^2+m^2}\right)^2 = S_d \Lambda^{2d-4}(1-b)\right)$$

$$\times 6 \times 6 \times 2 \quad = -\frac{\delta\lambda}{4!}\sum_{k_1k_2k_3}\phi_{k_1}\phi_{k_2}\phi_{k_3}\phi_{-(k_1+k_2+k_3)}$$

$\Rightarrow$ $\delta\lambda = \frac{3}{2}$ 

$$= \frac{3}{2} S_d \Lambda^{d-4}(1-b)$$