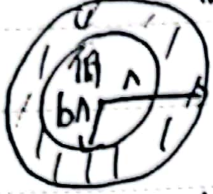
DATE

令 $V=1$ （∵ $\frac{1}{V}\sum_k$ 成对出现） $\frac{1}{V}\sum_k = \left(\frac{1}{2\pi}\right)^d \int dk$

$$S=\sum_{|k|<\Lambda}\frac{k^2+m^2}{2}\phi_k^*\phi_k+\frac{\lambda}{4!}\sum_{|k|,|k'|,|q|<\Lambda}\phi_k\phi_{k'}\phi_q\phi_{-k-k'-q}$$

shell 高能



$b=1-db$, $\boxed{db\to 0}$ shell很小，可以在δS里扰动

$$S=S^<+S^>+\delta S$$

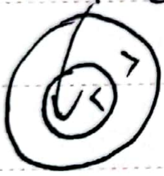
低能　高能

$$S^<=\sum_{|k|<b\Lambda}\frac{k^2+m^2}{2}\phi_k^*\phi_k+\frac{\lambda}{4!}\sum {}_<^<X_<^<$$

$$S^>=\sum_{b\Lambda<|k|<\Lambda}\frac{k^2+m^2}{2}\phi_k^*\phi_k$$

$$\delta S=\frac{\lambda}{4!}\sum\left({}_>^>X_>^>+{}_>^>X_<^>+{}_<^>X_<^>+{}_<^>X_<^<\right)$$

$$Z=\int D\phi_<e^{-S_<}\underbrace{\int D\phi_>e^{-(S_>+\delta S)}}$$



$S_>$全是单粒子

$$=\frac{\int D\phi_>e^{-(S_>+\delta S)}}{\int D\phi_>e^{-S_>}}\times\left(\int D\phi_>e^{-S_>}\right)$$

只与Λ, b, λ, m有关，与场无关

关注低能有效模型：

$$F=F(\phi)+\delta F(\text{const})\quad=\text{const}\times\langle e^{-\delta S}\rangle_>$$

$$\langle e^X\rangle=e^{\langle X\rangle_c-\frac{1}{2}\langle X^2\rangle_c}$$

$$e^{-\langle\delta S\rangle_>-\frac{1}{2}\langle\delta S^2\rangle_>^c}$$

$$\therefore Z=\int D\phi_<e^{-(S_<(\phi_<)+\Delta S_{eff}(\phi_<))}$$

$$\Delta S_{eff}=\langle\delta S\rangle_>+\frac{1}{2}\langle\delta S^2\rangle_>^c$$

$$\sum_{|k|\le b\Lambda}(\quad)+\sum_{|k|\le b\Lambda}(\quad)$$

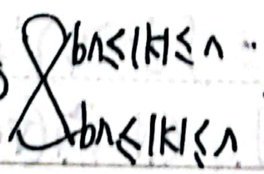
⇓ rescaling　令 $k=bq$　$\phi(bq)=\zeta\phi(q)$

$$\sum_{|q|\le\Lambda}(\quad)+\sum_{|q|\le\Lambda}(\quad)$$

$$\delta S=\overset{①}{{}_>^>X_>^>}+\overset{②}{{}_>^>X_<^>}+\overset{③}{{}_<^>X_<^>}+\overset{④}{{}_<^>X_<^<}$$

$$\langle\Delta S\rangle_>=\frac{\int D\phi_>^*D\phi_>e^{-S_>}\Delta S}{\int D\phi_>^*D\phi_>e^{-S_>}}$$

①



$\sim\sum\frac{1}{k^2+m^2}\cdot\frac{1}{k^2+m^2}$

（与场无关）

④ ② 无法形成配对，没有贡献.

③  $\phi_k^*\phi_k \sum_{b\Lambda<|q|<\Lambda}\left(\frac{1}{q^2+m^2}\right)$

定义 $\sum_{b\Lambda<|q|<\Lambda}\frac{1}{q^2+m^2} = \frac{1}{(2\pi)^d}\int dq \cdot \frac{1}{q^2+m^2}$

$$= \frac{\Omega^d}{(2\pi)^d}\int_{b\Lambda}^{\Lambda}\boxed{\frac{q^{d-1}}{q^2+m^2}}dq \quad \text{① 变角向积分}$$

$$= S_d \boxed{\frac{\Lambda^{d-1}}{\Lambda^2}}\int_{b\Lambda}^{\Lambda} dq$$

$$= S_d\,\Lambda^{d-3}\cdot(\Lambda-b\Lambda)$$

$$= S_d\,\Lambda^{d-2}(1-b)$$

则 $\sum_{b\Lambda<|q|<\Lambda}\left(\frac{1}{q^2+m^2}\right)^2 = S_d\,\Lambda^{d-4}(1-b)$

$\delta S = \frac{\lambda}{4!}\sum \phi_k\phi_{k'}\phi_q\phi_{-k-k'-q}$

$\langle \Delta S\rangle_> = \text{const} + \frac{\lambda}{4!}C_4^2\, \underbrace{\phi_k^*\phi_k}_{<}\ \underbrace{\text{Ω}}_{>}$

6个图

$$= \underline{\frac{\lambda}{4!}\times 6\times S_d\,\Lambda^{d-2}(1-b)}\sum_k^{<}\phi_{k<}^*\phi_{k<}$$

const, 即对质量的修正 m^2.

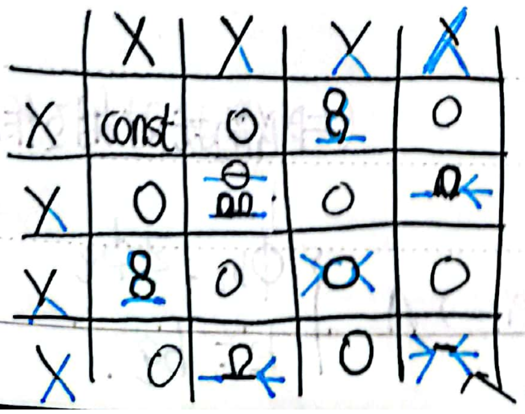
$S^< = \sum_{|k|<b\Lambda}\frac{k^2+m^2+\delta m^2}{2}\phi_k^*\phi_k + \frac{\lambda}{4!}\sum_{|k|<b\Lambda}\phi_k\phi_{k'}\phi_q\phi_{-k-k'-q}$

第1修正: $\delta m^2 = \frac{\lambda}{2}S_d\,\Lambda^{d-2}(1-b) \propto \lambda\ ,\ \lambda>0.$

符号: $e^{-(m^2\phi^*\phi)-\lambda\phi^4}$

第2修正: $\langle\Delta S^2\rangle_>^c$

$\langle\Delta S^2\rangle$



典型特点:

1) 新图 / 新相互作用

2) 高阶更复杂.

3) shell 中的散射是有限的.

6体相互作用 (动量不守恒)

DATE

$\langle \Delta S^2 \rangle_>^c$ | 连接图 有效散射 |

k_F $\quad C^+_{k_1} C^+_{k_2} C_{k_3} C_{k_4} \, V_{k_1 k_2 k_3 k_4}$

$|k_i| \sim k_F$

仅当 $k_1 = k_3 / k_4$, $k_2 = k_4 / k_3$ 时相等

此时 $C^+_{k_1} C^+_{k_2} C_{k_2} C_{k_1}$, $n_{k_1} n_{k_2} V(k_1 - k_2)$ ① Fermi Liquid

或 $C^+_{k_1} C^+_{-k_1} C_{k_2} C_{-k_2}$, ② cooper pair.

8 不重要: $\quad \left(\sum_{b\Lambda \le |q| \le \Lambda} \frac{1}{q^2+m^2} \right) \times \sum_{b\Lambda \le |q'| \le \Lambda} \frac{1}{(q'^2+m^2)^2}$

$$S_d \Lambda^{d-2}(1-b) \times S_d \Lambda^{d-4}(1-b) = S_d^2 \Lambda^{2d-6} \underbrace{(1-b)^2}_{=db^2}$$

$\Omega = \frac{\lambda}{2} S_d \Lambda^{d-2}(1-b) = \Delta m^2$ 高阶量 kardar 书.

考虑: $e^{-\frac{1}{2}\langle \Delta S^2 \rangle_>^c}$

$$= -\frac{1}{2} \frac{\lambda^2}{(4!)^2} \phi^<_{k_1} \phi^<_{k_2} \phi^<_{k_3} \phi^<_{k_4}$$

$$= -\frac{1}{2} \frac{\lambda^2}{(4!)^2} \sum_{k_1 k_2 k_3} \phi_{k_1} \phi_{k_2} \phi_{k_3} \phi_{-k_1-k_2-k_3} \sum_{b\Lambda \le |q| \le \Lambda} \frac{1}{q^2+m^2} \frac{1}{(q-k_1-k_2)^2+m^2}$$

假设 低能, $k_1 k_2 k_3 k_4 \ll \Lambda$ 则 $\approx \sum_q \left(\frac{1}{q^2+m^2}\right)^2 = S_d \Lambda^{d-4}(1-b)$

考虑常数 (; $6\times6\times2$)

$$= -\frac{\delta\lambda}{4!} \sum_{k_1 k_2 k_3} \phi_{k_1} \phi_{k_2} \phi_{k_3} \phi_{-k_1-k_2-k_3}$$

则 $\delta\lambda = \frac{1}{2} \cdot \frac{1}{4!} \times 6 \times 6 \times 2 \times$

$= \frac{3}{2}$

$= \frac{3}{2} S_d \Lambda^{d-4}(1-b)$ (二阶微扰对相互作用的修正).

rescaling $k \to k'$, $b\Lambda \to \Lambda$ $\begin{cases} \phi_{bq} = z \phi_q \\ k = bq. \end{cases}$