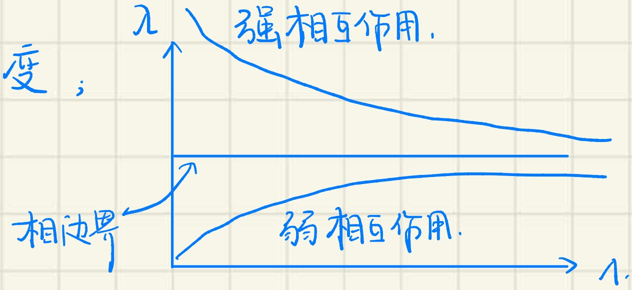
下节课预告.

Wilson. RG.

①. 从大能标 $\Lambda \to 0.99\Lambda \to 0.99^2\Lambda$;

②. Condensed matter. & 相变 ;



③. 不需要抵消项. (也没有发散的问题).

2022.3.21. 第五周第1节课.

① RG.

② 标度分析 (基本功).

Ising model. (Lenz). Weiss-Curise 平均场.

$H = -J \sum_{ij} S_i S_j$, 转移矩阵. Ising.

$H = -J \sum_i \vec{\sigma}_i \cdot \vec{\sigma}_{i+1}$, 自旋. Heisenberg. $\sigma_z = \pm 1$

Onsager.

Yang. $C_V \sim (T-T_c)^{-\frac{1}{8}} \neq -\frac{1}{2}$, 超越 Landau 相变理论;
Topo 相变;
Anderson localization;

Kardanoff.
wildom. $\to$ renormalization group.

动量空间. Wilson. RG $\Rightarrow$ 1981. Nobel. Phys. prize.

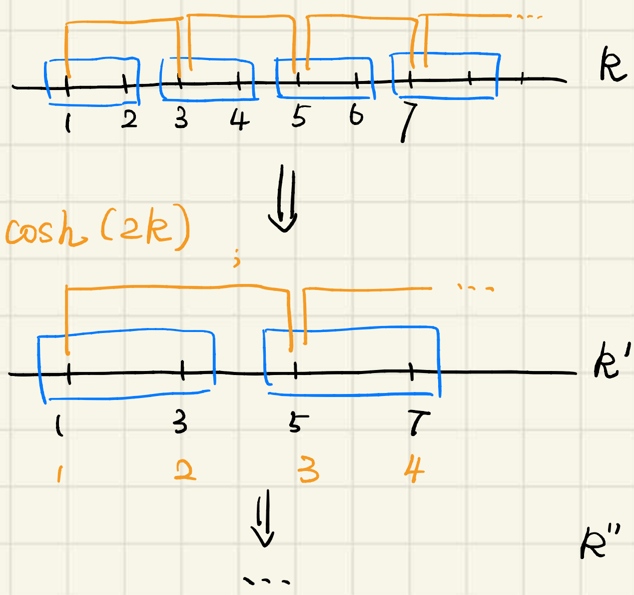
DMRG.

$$Z = \mathrm{Tr}\left(e^{-\beta H}\right)$$

$$= \sum_{\{S_i\}} e^{K\sum_i S_i S_{i+1}}$$ ，其中 $S_i = \pm 1$

$$= \sum_{奇} \sum_{偶} e^{K\sum_i S_i S_{i+1}}$$

$$= \sum_{奇} e^{\sum_i K' S_{2i-1} S_{2i+1}}$$ ，$K' = \frac{1}{2}\ln\cosh(2K)$；

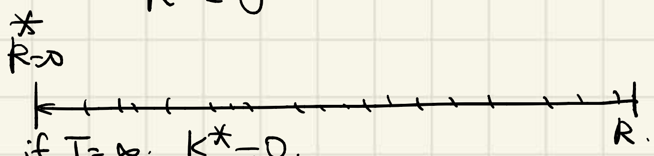


① 两个Block看作一个整体；

② re-scaling. $2a \to a$

$$K^* = \frac{1}{2}\ln\cos(2K^*)$$

$$K^* = 0$$



if $T=\infty$. $K^*=0$.

$\Rightarrow$ 一维没有自发磁化.

作业（2选1）.

1) PRL, 31, 1411 (1973). by Niemeijer. et al.

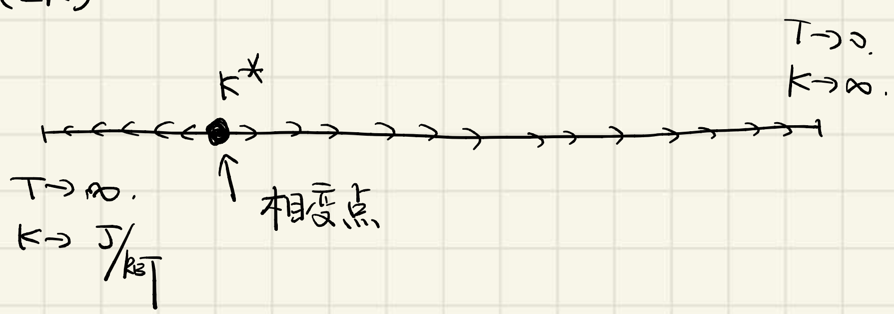
2) PRB, 15, 3460 (1977), by Dasgupta.

2d. RG. spin.

$$2K' = \alpha\ln\cosh(2K)$$

$$\alpha > 1$$

$$F(K) = 2K - \alpha\ln\cos(2K)$$



1d.

$$\lambda_{n+1} = R(\lambda_n)$$

令 $\lambda^* = R(\lambda^*)$.

$$\lambda_n = \lambda^* + \delta\lambda_n$$

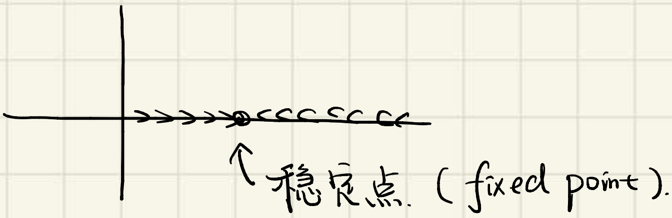
$$\lambda^* + \delta\lambda_{n+1} = R(\lambda^*) + R'(\lambda^*)\cdot\delta\lambda_n$$

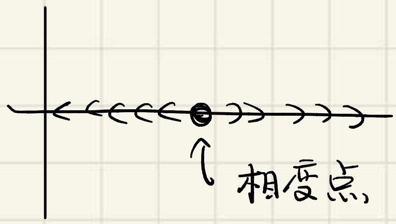
$$\delta\lambda_{n+1} = R'(\lambda^*)\cdot\delta\lambda_n$$

$$\delta\lambda_{n+1} - \delta\lambda_n = [R'(\lambda^*)-1]\cdot\delta\lambda_n$$

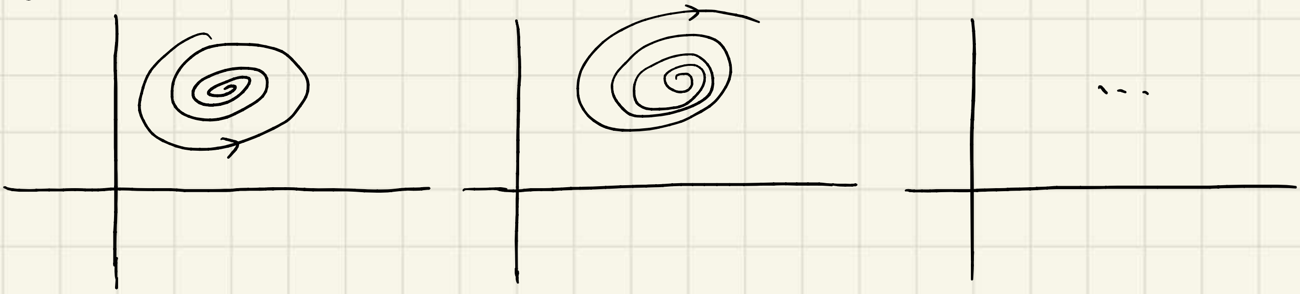
$$\frac{d\delta\lambda_n}{dn} = A\cdot\delta\lambda_n.$$

$$\Rightarrow \qquad \delta\lambda_n \propto e^{[R'(\lambda^*)-1]\cdot n}$$





2d



以上的过程对离散的体系，都很直白。

但，如果是连续的，比如

$$\frac{\partial}{\partial t}P = D\frac{\partial^2}{\partial x^2}P$$

如何做 Rescaling？

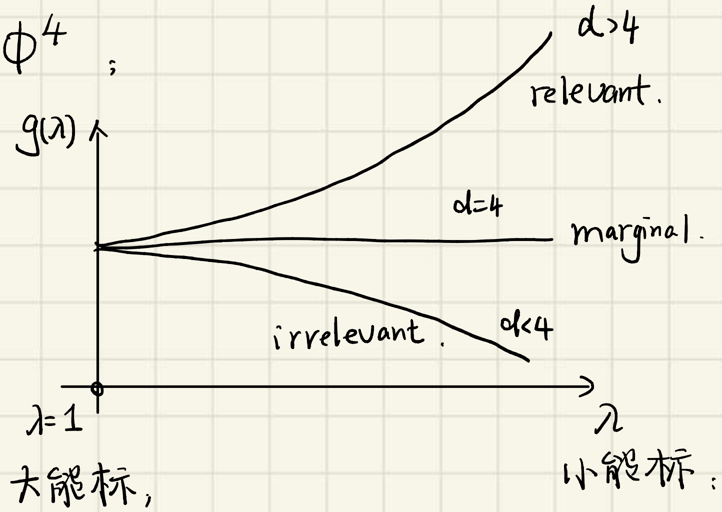
标度分析:

1) ϕ^4 理论.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 ;$$

$$S = \int \mathcal{L}\, dx ;$$

$$\begin{aligned} x &\to \lambda x \\ t &\to \lambda^2 t \end{aligned} \quad \Rightarrow \quad g(\lambda) = g\lambda^{d-4}$$



2) Navier-Stokes eqn

$$\left| \begin{aligned} & u_t + u\cdot\nabla u + \nabla p - \Delta u = 0 \\ & \nabla\cdot u = 0 \\ & u|_{t=0} = u_0 \end{aligned} \right.$$

3) $$\frac{\partial h}{\partial t} = \nu\left(\frac{\partial^2 h}{\partial x^2}\right) + \eta \quad , \quad \text{KPZ eqn.}$$

$$\frac{\partial h}{\partial t} = \nu\left(\frac{\partial^2 h}{\partial x^2}\right)^2 + \eta \quad , \quad \cdots$$

微分方程有标度分析;