$$\lambda(\Lambda)=\lambda_R^{(\mu)}+\delta\lambda(\Lambda) \xrightarrow{类比} m^2(\Lambda)=m_R^2(\mu)+\delta m^2(\Lambda)$$

$$m_R^2(\mu)=m^2(\Lambda)-\delta m^2(\Lambda)$$

$$\infty-\infty = \text{finite}$$

Wilson RG ① $\Lambda \to 0.99\Lambda \to 0.99^2\Lambda$

② cond' matt' $\to$ 相变

③ 无抵消项，无发散. $(\partial\theta)^2+A\cos(\theta(\vec{x}))$

重整化 Renormalization (R)

2d $\delta(\vec{x}) \leftrightarrow E=-\frac{\hbar^2\Lambda^2}{2m}e^{\frac{\hbar^2}{m\lambda(\Lambda)}}$

电荷 $e^*=\frac{(\vec{p}+e^*\vec{A})^2}{2m}$

两个能标 $\begin{cases}\mu & \lambda(\mu)\to 测量\\ \Lambda & \lambda(\Lambda) \ 不可测\end{cases}$

ϕ^4理论 λ $\begin{cases} m^2=m_0^2+\Omega\Lambda^2 = m_0^2+\lambda\sum_k\frac{1}{k^2+m_0^2} \\ \lambda=X+X\!\!\!\bigcirc\!\!\!X \ \ln\Lambda = \lambda_0+\lambda_0^2(\beta\sum_k\frac{1}{(k^2+m_0^2)^2})\end{cases}$

μ下观测结果. 重整化条件

$\mu\ll\Lambda$

重整化：目的：发散抵消发散

区分：m_R和m_0

系统的一套方法 $\leftarrow$ Renormalization Group (Peskin书 chap12-13)

$\phi=\sqrt{Z}\phi_R$, $\mathcal{L}=\mathcal{L}_R+\underbrace{\delta\mathcal{L}}_{抵消项}$

$\underset{R}{\underset{\downarrow}{m_R}}$ $\underset{RG}{\underset{\downarrow}{m(\Lambda)}}$ 从 Λ 降到 μ.

$\begin{cases}\text{Cond' matt' } \delta\mathcal{L} \text{ 无需} \\ \text{适于研究相变} \\ \text{来源于 Ising model.}\end{cases}$

发散方式不同 $C_v,\chi\propto\frac{1}{|T-T_c|^\nu}$

扫描全能王 创建

kadanoff · widom
发展了(RG)
wilson
发展到k空间

① RG ② 标度分析(基本功)

Ising model : $H=-J\sum_i S_i S_{i+1}$ 转移矩阵

Heisenberg $\sigma_z=\pm1$, model : $H=-J\sum_i \vec{\sigma}_i\cdot\vec{\sigma}_{i+1}$

Onsager prize 相变点附近标度 $C_V\sim|T-T_c|^{-\frac{1}{8}}$ ⇒ Yang

$\neq\frac{1}{2}$ 超越了朗道相变理论.

Topo相变，Anderson location,

$Z=Tr(e^{-\beta H})=\sum_{\{S_i\}}e^{k\sum_i S_iS_{i+1}}$ 其中 $S_i=\pm1$

$\sum_{S_2=\pm1}e^{kS_1S_2+kS_2S_3}$

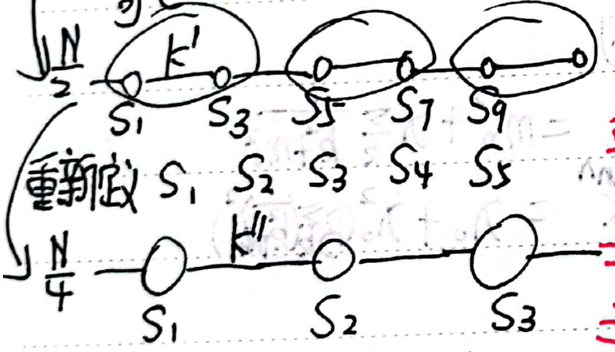
$=e^{k(S_1+S_3)}+e^{-k(S_1+S_3)}$

$=Ae^{k'S_1S_3}$

$=\sum_奇\sum_偶 e^{k\sum_i S_iS_{i+1}}$

$=\sum_奇 e^{\sum k' S_{2i+1}S_{2i-1}}$ (奇数格点间的相互作用)

$\begin{cases}e^{2k}+e^{-2k}=Ae^{k'}\\ 2=Ae^{-k'}\end{cases}$



平均⇒精度降低

⇒抹平细节 精度↓

⇒能标下降.

$\begin{cases}e^{2k'}=\cosh(2k)\\ 2k'=\ln\cosh(2k)\\ \boxed{k'=\frac{1}{2}\ln\cosh(2k)}\end{cases}$

$k=\beta J=\frac{J}{k_BT}$

① 必须两个Block看作一个整体

② re-scaling $2a\to a$

$k_{n+1}=\frac{1}{2}\ln\cosh(2k_n)$ $k_1\to k_2\to k_3\cdots$ ⟺ $k^*=\frac{1}{2}\ln\cosh(2k^*)$

多变量 $\lambda_{n+1}=R(\lambda_n)$ 函数

极限 $\lim_{n\to\infty}a_n=a^*$, $\lambda^*=R(\lambda^*)$

$k^*=0$为唯一解 $J=0$ / $T=\infty$

当 $k_n\ll1$ 时 $k_{n+1}=\frac{1}{2}\ln\cosh(2k_n)\doteq k_n^2$

作业: 1) PRL, 31, 1411 (1973) by Nierneijer. etal

2) PRB, 15, 3460 (1977). by Dasgupta, etal

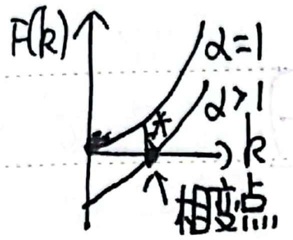
2d RG, spin.

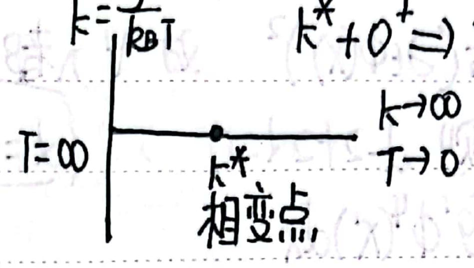
DATE

$2k' = \alpha \ln\cosh(2k)$, $\alpha > 1$

$F(k) = 2k - \alpha\ln\cosh(2k)$

→ PMRG





$k^* - 0^+ \Rightarrow T = \infty$

$k^* + 0^+ \Rightarrow T = 0.$

1d: $\lambda_{n+1} = R(\lambda_n)$ 令 $\lambda^* = R(\lambda^*)$

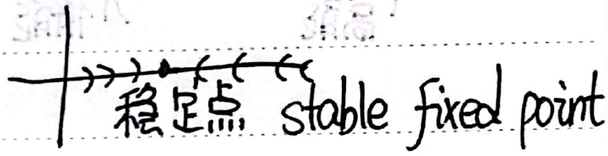
$\lambda_n = \lambda^* + \delta\lambda_n$ $\quad \lambda^* \Big|_{\lambda_n = \lambda^* + \delta\lambda_n}$ 泰勒展开

$\lambda^* + \delta\lambda_{n+1} = R(\lambda^*) + R'(\lambda^*)\delta\lambda_n$

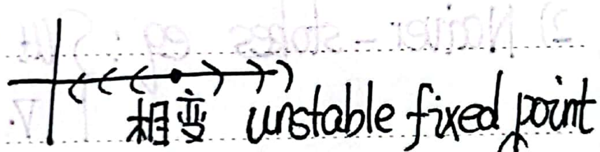
$\delta\lambda_{n+1} = R'(\lambda^*)\delta\lambda_n$

$\delta\lambda_{n+1} - \delta\lambda_n = [R'(\lambda^*) - 1]\delta\lambda_n = \frac{d\delta\lambda_n}{dn} = A\cdot\delta\lambda_n$

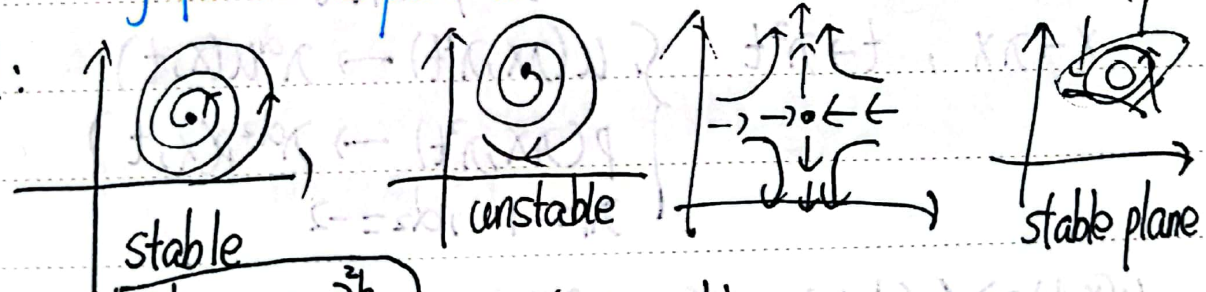
$\Rightarrow \delta\lambda_n \propto e^{[R'(\lambda^*)-1]n}.$



非线性物理.
Lyapunov exponents



2d:



$\boxed{\frac{\partial h}{\partial t} = D\frac{\partial^2 h}{\partial x^2}}$ 如何 rescaling

标度分析:

1) ϕ^4 理论: $\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$

$S = \int \mathcal{L}\, dx$

eg: $\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{+\infty} f(y)\,dy$

等价 $= \int f(\lambda x)\,\lambda\,dx$

$\int dx \frac{1}{2}(\partial_t\phi)^2 \quad t' \to \lambda t,\ x' \to \lambda x$

$= \int dx' \frac{1}{2}(\partial_{t'}\phi(x'))^2 \qquad \phi(\lambda x) = \lambda^{\alpha}\phi(x)$

$= \lambda^{d-2+2\alpha} \int dx \frac{1}{2}(\partial_t\phi(x))^2$ 对 $\forall\lambda$ 都成立

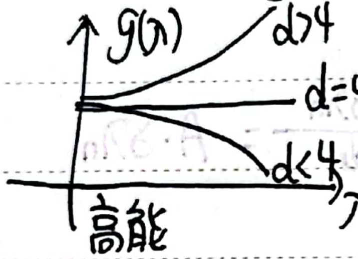
则 $d-2+2\alpha=0$，$\boxed{\alpha = \frac{2-d}{2}}$

$\int g\cdot\phi^4(x)\,dx = \int g'\phi^4(x')\,dx'$

$g' = \lambda^{\beta} g \qquad = g\lambda^{4\alpha+d+\beta}\int\phi^4(x)\,dx$

$\Leftrightarrow 4\alpha+d+\beta=0$，将 α 代入得 $\boxed{\beta = d-4}$

$g(\lambda) = g\lambda^{d-4}$



marginal

低能. $x' = \lambda x$

relevant 有关
marginal
irrelevant 无关

2) Naiver-stokes eq: $\begin{cases} u_t + u\cdot\nabla u + \nabla p - \Delta u = 0 \\ \nabla\cdot u = 0, \quad u|_{t=0} = u_0 \end{cases}$

$x \to \lambda x, \quad t \to \lambda^2 t \qquad \begin{cases} u(\lambda x, \lambda^2 t) \to \lambda^{d_1} u(x,t) \\ p(\lambda x, \lambda^2 t) \to \lambda^{d_2} p(x,t) \\ \lambda_1 = -1, \ d_2 = -2 \end{cases}$

$u(x,t) \Rightarrow u(x',t') = u'(\lambda x, \lambda^2 t)$

$u'_{t'} + u'\cdot\nabla' u' + \nabla' p' - \Delta' u' = 0, \quad t' = \lambda^2 t, \ x' = \lambda x, \ u' = \lambda u$

微分方程都可以作标度分析.

3) $\boxed{\frac{\partial h}{\partial t} = \nu\frac{\partial^2 h}{\partial x^2} + \eta}$ KPZ eq.

$\frac{\partial h'(x',t')}{\partial t'} = \nu\frac{\partial^2 h'(x',t')}{\partial x'^2} + \eta'(x',t')$ 坐标变换.

$x' = bx, \quad t' = b^z t, \quad h'(x',t') = h'(bx, b^z t) = b^{\chi} h(x,t)$

$\frac{\partial h'}{\partial t'} = \frac{b^{\chi}}{b^{z}}\frac{\partial h}{\partial t}$, $\frac{\partial^2 h'}{\partial x'^2} = \frac{b^{\chi}}{b^{2}}\frac{\partial^2 h}{\partial x^2}$

$$b^{\chi-z}\cdot\frac{\partial h}{\partial t} = \nu b^{\chi-2}\frac{\partial^2 h}{\partial x^2}$$

取 $\boxed{z=2}$ 即可

若将前提改为 $\frac{\partial h}{\partial t} = \nu\left(\frac{\partial^2 h}{\partial x^2}\right)^2+\eta$

则 $b^{\chi-z}\frac{\partial h}{\partial t} = \nu\cdot b^{2\chi-4}\frac{\partial^2 h}{\partial x^2}$

$\chi - z = 2\chi-4 \Rightarrow \boxed{\chi+z=4}$

$\langle\eta(x,t)\rangle=0$ $\quad \langle \eta(x_1t_1)\eta(x_2t_2)\rangle = D\delta(x_1-x_2)\delta(t_1-t_2)$

Review. RG of Ising Model:

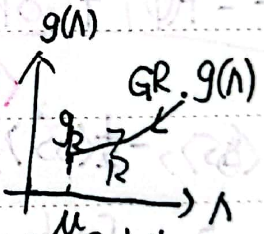
第一步: trace 掉偶数格点: $\quad -\sum_i k\sigma_i\sigma_{i+1}$

$\downarrow$ RG

第二步: rescaling $\sigma_{2i+1}\to\sigma_i$: $\quad -\sum_i k'\sigma_i\sigma_{i+1}$

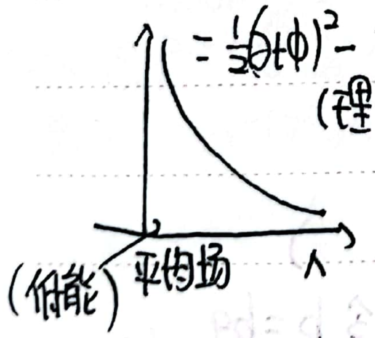
过程中将 ↑↑ ↑↓ ↓↑ ↓↓ $\xrightarrow{map}$ ↑ ↓ (会损失掉高能部分的信息)

ϕ^4理论: $Z=\int D\phi e^{iS}$, $S=\int \mathcal{L} dx$



$$\mathcal{L} = \frac12(\partial\phi)^2 - \frac{m}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$
$$= \frac12(\partial_t\phi)^2 - \frac12(\partial_x\phi)^2-\frac12(\partial_y\phi)^2-\frac12(\partial_z\phi)^2-\frac{m}{2}\phi^2-\frac{\lambda}{4!}\phi^4$$

(理想模型)



欧氏空间: $U(t)=e^{-iHt}$, $Z\sim e^{-\beta H}$, $it=\beta \Rightarrow t=-i\beta$

令 $t=-i\tau$ 则 $\int dt\, d\vec{x} = -i\int d\tau\, d\vec{x}$

则 $S=\int H dx = \int \frac12(\partial\phi)^2+\frac{m^2}{2}\phi^2+\frac{\lambda}{4!}\phi^4$

欧氏空间 $(\partial_t\phi)^2+(\nabla\phi)^2$