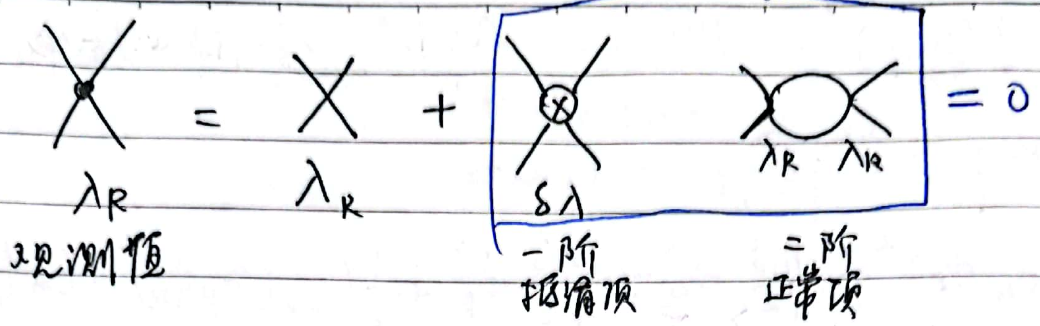
要求:

e.g.2



抵消项也抵消高阶项

$$\lambda(\Lambda) = \lambda_R(\mu) + \delta\lambda(\Lambda)$$

$$m^2(\Lambda) = m_R^2(\mu) + \delta m^2(\Lambda)$$

$$m_R^2(\mu) = m^2(\Lambda) - \delta m^2(\Lambda)$$

有限 $= \infty - \infty$
(观测值)

Wilson Renormalized Group Theory

① Λ 大 $\rightarrow \infty$
② 处理与相变有关的问题
③ 不要抵消项

3.21

重整化: | 目的: 发散 抵消发现
区分: m_R m_0
系统的一套方法

$$Z = Tr(e^{-\beta H})$$

$$= \sum_{\{S_i\}} e^{K\sum_i S_i S_{i+1}} \qquad S_i = \pm 1$$

$$\sum_{s_2=\pm1} e^{Ks_1s_2+Ks_2s_3} = e^{K(s_1+s_3)} + e^{-K(s_1+s_3)} \hat{=} A e^{K's_1s_3}$$

$$= \sum_{odd} \sum_{even} e^{K\sum_i s_i s_{i+1}}$$

even 的求和掉

$$= \sum_{odd} e^{\sum_i K' s_{2i-1} s_{2i+1}}$$

$(s_1, s_3) = (1,1),\ (-1,-1)$ $\Big\}$ $e^{2K} + e^{-2K} \hat{=} A e^{K'}$

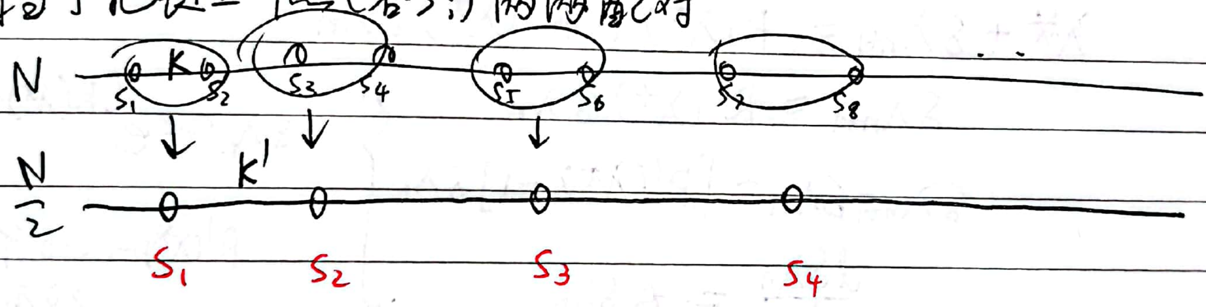
$(+1, -1),\ (-1, +1)$ $\Big\}$ $2 \hat{=} A e^{-K'}$

即有 $A e^{K'} = 2\cosh(K)$　　$A e^{-K'} = 2$

$\Longleftrightarrow e^{2K'} = \cosh(2K)$ or $2K' = \ln\cosh(2K)$

$\Longleftrightarrow K' = \frac{1}{2}\ln\cosh(2K)$

相当于把链上节点（各 S_i）两两配对



重新命名，对无穷大N来说，事实上没有任何变化，唯一的变化是耦合常数 $K \to K'$。如此可无穷做下去

总结：① 两个Block看作一个整体

② re-scaling：$2a \to a$

$$\begin{cases} K' = \frac{1}{2}\ln\cosh(2K) \\ K_0 = \beta J = \frac{J}{k_B T} \end{cases}$$

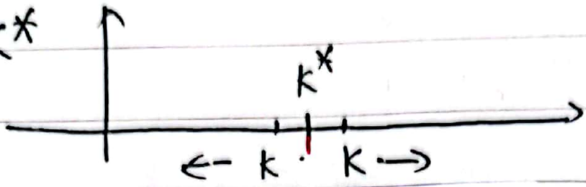
多变量 $\lambda_{n+1} = R(\lambda_n)$ 函数

极限：$K^* = \frac{1}{2}\ln\cosh(2K^*) \Longrightarrow K^* = 0$

因不论 K_0 是多少，最后都会收敛到 $K_n \to 0$

$2K \doteq \alpha \ln \cosh(2K)$

若 $K_0 < K^*$　　　　K会演化到0



对应 $T \to \infty$ ：无序(完全)

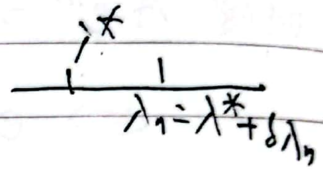
$K_0 > K^*$　　　　$K \to \infty$

对应 $T \to 0$

$K^* - 0^+ \Longrightarrow T = \infty$

$K^* + 0^+ \Longrightarrow T = 0$

这是一个典型的相变行为



1dim: $\lambda_{n+1} = R(\lambda_n)$　　令 $\lambda^* = R(\lambda^*)$

$\lambda_n = \lambda^* + \delta\lambda_n$

$\lambda^* + \delta\lambda_{n+1} = R(\lambda^*) + R'(\lambda^*)\,\delta\lambda_n$

$$\left.\begin{aligned} \delta\lambda_{n+1} &= R'(\lambda^*)\,\delta\lambda_n \\ \delta\lambda_{n+1} - \delta\lambda_n &= [R'(\lambda^*) - 1]\,\delta\lambda_n \\ \frac{d\delta\lambda_n}{dn} &= A\,\delta\lambda_n \end{aligned}\right\} \quad \delta\lambda_n \propto e^{[R'(\lambda^*)-1]n}$$

$R'(\lambda^*) - 1 < 0$ 稳

> 0 不稳

2dim: 略

以上合并过程，转到 k-space 去，实际上是能标 Λ 往下降的过程(Fourier trans: 高频项(高阶项)代表更细的细节，舍去高频项，即 $\Lambda \downarrow$)

标度分析

① ϕ^4 理论

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$$

e.g. $\int_{-\infty}^{+\infty} f(x)\,dx = \int f(\lambda x)\, d(\lambda x) = \int_{-\infty}^{+\infty} f(y)\,dy \qquad y=\lambda x$

$$S = \int \mathcal{L}\, dx$$

$$\int dx\, \frac{1}{2}(\partial_t \phi)^2 = \int dx'\, \frac{1}{2}\left(\partial_{t'} \phi(x')\right)^2 \cancel{= \lambda^d} \qquad t'=\lambda t,\ x=\lambda x$$

$$\phi(\lambda x) = \lambda^{\alpha} \phi(x)$$

$$= \lambda^{d-2+2\alpha} \int dx\, \frac{1}{2}\left(\partial_t \phi(x)\right)^2$$

对 $\forall \lambda$ 成立 $\therefore d-2+2\alpha=0 \Rightarrow \alpha = \frac{2-d}{2}$

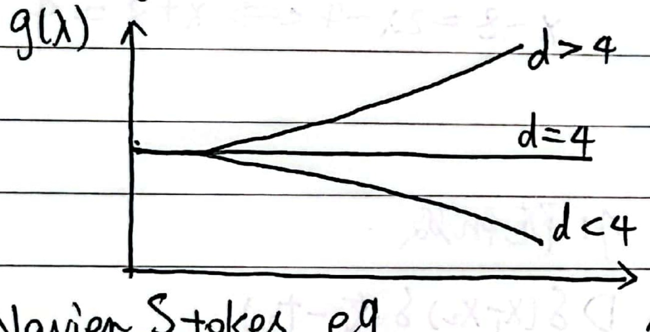
$$\int g\,\phi^4(x)\,dx = \int g'\,\phi^4(x')\,dx' = g\,\lambda^{4\alpha+d+\beta} \int \phi^4(x)\,dx$$

$$g' = g\lambda^{\beta}$$

$\therefore 4\alpha + d + \beta = 0$

$\therefore 4 - 2d + d + \beta = 0 \Rightarrow \beta = d-4$

$$g(\lambda) = g\lambda^{d-4}$$



Navier Stokes eq

$$\begin{cases} u_t + u\cdot\nabla u + \nabla P - \Delta u = 0 \\ \nabla\cdot u = 0 \\ u|_{t=0} = u_0 \end{cases} \qquad x\to\lambda x,\quad t\to\lambda^2 t$$

$$\begin{cases} u(\lambda x, \lambda^2 t) \to \lambda^{\alpha_1} u(x,t) \\ p(\lambda x, \lambda^2 t) \to \lambda^{\alpha_2} p(x,t) \\ \alpha_1 = -1 \quad \alpha_2 = -2 \end{cases}$$

此变换下该方程不变

$$u'_{t'} + u'\nabla u + \nabla' p - \Delta' u = 0$$

$$u(x,t) \Rightarrow u'(x',t') = u'(\lambda x, \lambda^2 t) \qquad t'=\lambda^2 t,\ x'=\lambda x,\ u'=\lambda u$$

这个例子(意在)说明很多微分方程都可以做标度分析

Kpz eq $\quad \frac{\partial h}{\partial t} = \nu \frac{\partial^2 h}{\partial x^2} + \eta$

$$\begin{cases} \frac{\partial h'}{\partial t'}(x',t') = \nu \frac{\partial^2 h'(x',t')}{\partial x'^2} + \eta'(x',t') \\ x' = bx \\ t' = b^z t \end{cases}$$

$$h'(x',t') = h'(bx, b^z t) = b^x h(x,t)$$

$$\left.\begin{aligned} \frac{\partial h'}{\partial t'} &= \frac{b^x}{b^z}\frac{\partial h}{\partial t} \\ \frac{\partial^2 h'}{\partial x'^2} &= \frac{b^x}{b^2}\frac{\partial^2 h}{\partial x^2} \end{aligned}\right\} \quad b^{x-z}\frac{\partial h}{\partial t} = \nu b^{x-2}\frac{\partial^2 h}{\partial x^2}$$

欲让此模型不变 只需令 $z=2$

$$b^{x-z}\frac{\partial h}{\partial t} = b^{2x-4}\left(\frac{\partial^2 h}{\partial x^2}\right)^2 \qquad x-z = 2x-4 \iff x+z=4$$

其它例子如 δ 函数

$\langle \eta(x,t) \rangle = 0 \qquad \eta$: 随机数

$\langle \eta(x_1,t_1)\,\eta(x_2,t_2) \rangle = D\,\delta(x_1-x_2)\,\delta(t_1-t_2)$

$\delta(bx) = \frac{1}{b}\delta(x)$

下节预告：转到动量空间：Wilson RG Theory