$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0^4}{4!}\phi^4 \quad , \quad (\lambda_0, m_0 \text{ at } \Lambda).$$

$$= \mathcal{L}_R(m_R, \lambda_R) + \Delta\mathcal{L}.$$

重整化.

(与 m_0, λ_0, m_R, λ_R 都有关, 用以抵消发散).

$\infty - \infty =$ infinite (任意). 由实验确定 λ_R, m_R. $\Rightarrow$ infinite (确定).

观测值 $\mu \Rightarrow m_R$, λ_R. 但 $\mu \ll \Lambda$.

重整化: $\phi = \sqrt{Z}\,\phi_R$

$$= \frac{1}{2}(\partial_\mu \phi_R)^2 - \frac{m_R^2}{2}\phi_R^2 - \frac{\lambda_R}{4!}\phi_R^4$$

$$\underbrace{+\frac{1}{2}\delta_Z(\partial_\mu\phi_R)^2 - \frac{\delta m^2}{2}\phi_R^2 - \frac{\delta\lambda}{4!}\phi_R^4}_{\text{小量}}$$

$$\left.\begin{array}{l}\phi^3.\\ \phi^5.\\ \phi_1^4+\phi_2^4\end{array}\right\} \Rightarrow \text{如何处理?}$$

2022.3.7. 第4周第2节课.

Dyson eqn.

$$g = g_0 + g_0 \Sigma g$$

$$g = \frac{1}{p^2 - m^2}$$

$$g_0 = \frac{1}{p^2 - m_0^2}$$

$m_0 \Leftrightarrow \Lambda$, 对应一个非常大的能标.

$m \Leftrightarrow m_R$.

右乘 g^{-1}

$$1 - g_0 g^{-1} = g_0 \Sigma$$

左乘 g_0^{-1}

$$g_0^{-1} - g^{-1} = \Sigma$$

$$(p^2 - m_0^2) - (p^2 - m_R^2) = \Sigma$$

$$m_R^2 - m_0^2 = \Sigma \qquad \text{(自由能)}$$

$$m = m_0^2 + \Sigma \leftarrow \text{散射对应的能量}$$

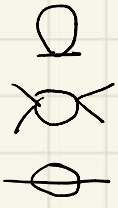
测量值; 不可测的能量;

$\langle \phi^*(p)\, \phi(p) \rangle$.

① 令 $V=1$, $\frac{1}{V}\sum_q \Rightarrow \sum_q$;

② 会画许多图; (哪些图).

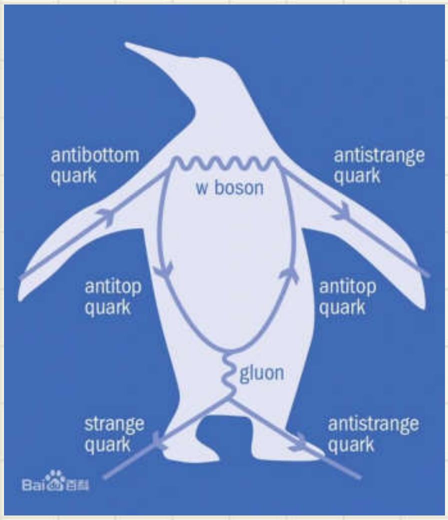
③ 理解 Feynman diag.



$\propto \Lambda^2$

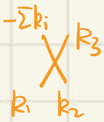
$\propto \ln \Lambda$

$\propto$? (Sunrise diag);



$O = \phi^*(p)\phi(p)$

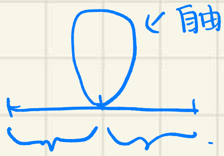
$$\langle O \rangle = \underbrace{\langle O \rangle_0}_{i/(p^2-m^2)} + \frac{i}{1!}\left(\frac{-\lambda}{4!}\right)\langle \phi^*(p)\phi(p)\, X \rangle_0^c \,;$$



$$= g_0(p) - \frac{i\lambda}{2} g_0(p) \underset{\substack{\Uparrow \\ \Sigma}}{\Sigma} g_0(p)$$

$$S_I = -\frac{\lambda}{4!} \sum_{k_i (i=1,2,3)} \phi(k_1)\phi(k_2)\phi(k_3)\phi(-k_1-k_2-k_3)$$

$$= -\frac{\lambda}{4!}\sum_k X$$



自由能.

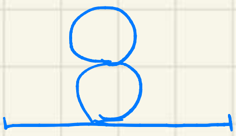
传播子. 传播子.

vs.

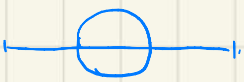
$g_0(p)$ $g_0(p)$

$$(\text{高阶}) \quad + \frac{i^2}{2!}\left(-\frac{\lambda}{4!}\right)\langle \phi^*(p)\phi(p)\, X\, X \rangle_0^c$$

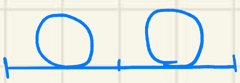
$d=4$



$\propto \Lambda^2 \ln \Lambda$ $\quad g_0(p)\, \tilde{\Sigma}_1\, g_0(p)$



$\propto$? $\quad g_0(p)\, \tilde{\Sigma}_2\, g_0(p)$

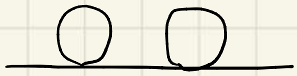


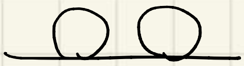
$\propto \underline{\Lambda^4} \cancel{X}$ $\quad g_0(p)\, \Sigma\, g_0(p)\, \Sigma\, g_0(p)$

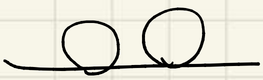
~~我写的,课后验算下~~.

系数的计算:

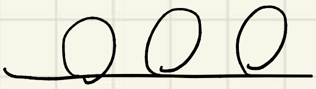
(2阶) $\frac{i^2}{2!}\left(-\frac{\lambda}{4!}\right)^2 \langle \underbrace{\phi^*(p)\ \times}_{4\times 3}\ \ \underbrace{\times\ \phi(p)}_{3\times 4} \rangle_0^c$ $\underline{\times 2}$ 交换.

$= \frac{i^2}{2!}\left(-\frac{\lambda}{4!}\right)^2 (4\times 3)^2 \times 2$ 

$= \frac{(-i\lambda)^2}{(2!)^2}$ 

$= \left(\frac{-i\lambda}{2!}\right)^2$ 

(3阶)

$= \left(\frac{-i\lambda}{2!}\right)^3$ 

 $\frac{-i\lambda}{2!}\sum_k\left(\frac{i}{k^2-m^2}\right)$, $g_0 \Sigma g_0$

 $g_0 \Sigma g_0 \Sigma g_0 \Sigma g_0$

Summary: 观测值;

$m^2 = m_0^2 + \Sigma$.

$m_0^2 \sim \Lambda$

真实质量;

高阶修正: $m_0^2 \Rightarrow m_0^2 + \underbrace{\Sigma}_{\text{发散}}$.

相互作用 $-\lambda/4!$

任意散射过程

$$O = \phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4)$$

唯一要求 $\sum_i p_i = 0$.

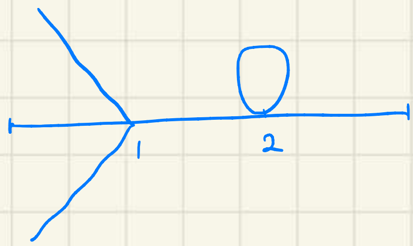
$$\langle O\rangle_0 = 0.$$

$$\langle O\rangle = \langle O\rangle_0 + \frac{i}{1!}\left\langle \phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4)\, X \right\rangle_0 \left(\frac{-\lambda}{4!}\right) + \cdots$$

$$= 0 + \underbrace{\frac{i}{1!}\left(\frac{-\lambda}{4!}\right)\cdot 4!\ {}^{p_1}_{p_3}X^{p_2}_{p_4}}_{-i\lambda X}$$

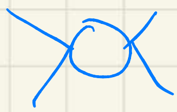
(高阶)
$$+ \frac{i^2}{2!}\left(-\frac{\lambda}{4!}\right)^2 \sum_k \sum_q \left\langle \phi\phi\phi\phi\ \underset{k}{X}\ \underset{q}{X} \right\rangle_0$$

$\bigcirc \times$



(舍弃计算).

$$\frac{i^2}{2!}\left(\frac{-\lambda}{4!}\right)^2 C_4^2 C_4^2 C_4^2 \times 2\times 2\times 2 \quad \times$$



$$\| \\ -\frac{3\lambda^2}{2}$$

$\Rightarrow \quad \langle O\rangle = $ 一阶 + 二阶

$$-i\lambda_{eff} X = -i\lambda X - \frac{3\lambda^2}{2} \rangle\!\bigcirc\!\langle$$

$$-i\lambda_{eff} = -i\lambda X - \frac{3\lambda^2}{2} \rangle\!\bigcirc\!\langle \quad \propto \ln\Lambda.$$

观测的散射长度

$$m^2 = m_0^2 + \Sigma_{\propto\lambda^2}$$

重整化 $\Rightarrow$

$m_0 \Rightarrow m_0(\Lambda)$

$\lambda \Rightarrow \lambda(\Lambda)$ 与 $\rangle\!\bigcirc\!\langle$ 截断 (Λ 积分限) 相互抵消

重整化方法：① 系统化方法.

② 借抵消法.

小能标.
↓
$\mu \ll \Lambda$　　③ 避免复杂的非线性方程.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0}{4!}\phi^4 .$$

$m_0, \lambda_0 \Rightarrow \Lambda$ 下的质量和关联强度.

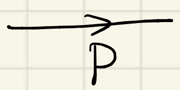
令 $\phi = \sqrt{Z}\,\phi_R$.

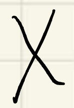
$$\langle \phi^* \phi \rangle = \frac{Z}{p^2 - m_0^2} + \cdots$$

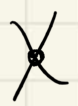
$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu \phi_R)^2 - \frac{m_R^2}{2}\phi_R^2 - \frac{\lambda_R}{4!}\phi_R^4}_{\text{观测值; } \mu \ll \lambda .}$$

$$+ \underbrace{\frac{1}{2}\left[\delta_Z (\partial_\mu \phi_R)^2 - \delta_m \phi_R^2\right] - \frac{\delta\lambda}{4!}\phi_R^4}_{\text{抵消项；}}$$

有抵消项的证据：如果没有这些项，那么就会发散（计算）.
但又没有观察到发散；

图：$\langle \phi_R^*(p)\, \phi_R^*(p) \rangle =$  $= \dfrac{i}{p^2 - m_R^2}$ ；

 $= -i\lambda_R$ ；

 $= -i\delta\lambda$ ；

抵消项 $= \frac{1}{2}(\delta_Z\, p^2 - \delta_m)\, \phi_R^*(p)\, \phi_R(p)$

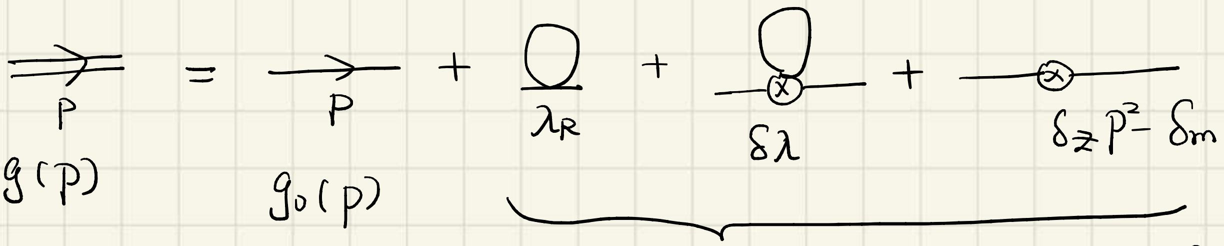
记为  $= (\delta_Z\, p^2 - \delta_m)/2$

$$S_0 = \int \mathcal{L}_0 \, dx$$

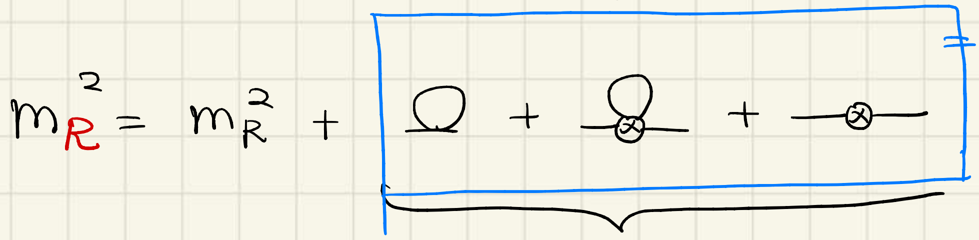
$$S_I = \int \frac{\lambda_R}{4!}\phi_R^4 - \frac{\delta\lambda}{4!}\phi_R^4 + \frac{\delta_Z}{2}(\partial_\mu \phi_R)^2 - \frac{\delta_m}{2}\phi_R^2$$

有相互作用（质量修正，一般考虑到一阶即可）（m_R）.

$$\langle O \rangle = \langle \phi_R^*(p)\, \phi_R(p) \rangle_0$$



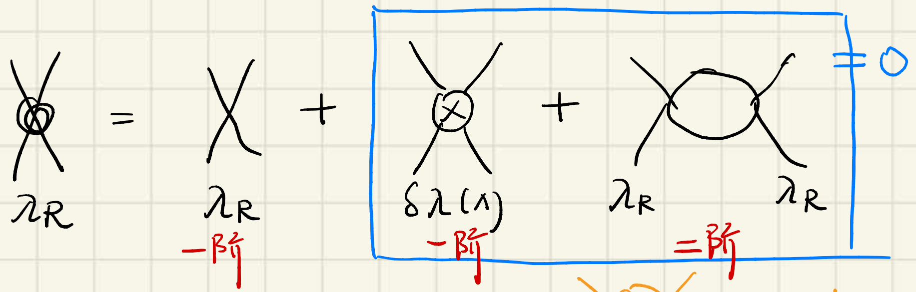
$$\Sigma = \lambda_R \Lambda^2 + \delta\lambda\, \Lambda^2 + \delta_Z p^2 - \delta_m$$

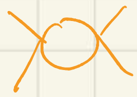


有限值=0

几组方程，求解各项系数

有相互作用（计算到2阶，得到 λ_R）.



$\delta\lambda(\Lambda) =$  $\propto \ln\lambda$

$$\lambda_R = \lambda_R^{(\mu)} + \delta\lambda(\Lambda)$$

$$m^2(\Lambda) = m_R(\mu) + \delta m^2(\Lambda) \Rightarrow m_R(\mu) = m^2(\Lambda) - \delta m^2(\Lambda)$$

$$\text{infinite} = \underset{\infty}{\wedge} - \underset{\infty}{\wedge}$$

Tips：用 latex 打 Feynmann 图的模板（包）.

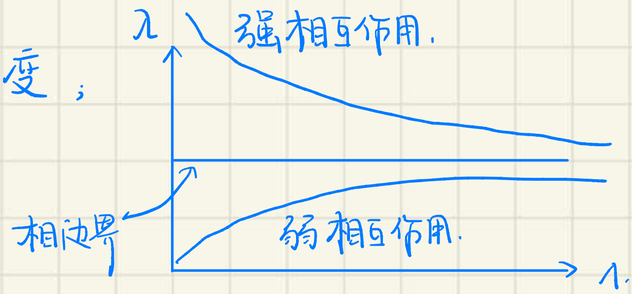
tikz-feynmann （package）.

下节课预告.

Wilson. RG.

①. 从大能标 $\Lambda \to 0.99\Lambda \to 0.99^2\Lambda$;

②. Condensed matter. & 相变 ;



③. 不需要抵消项. (也没有发散的问题).

2022. 3. 21. 第五周第1节课.

① RG.

② 标度分析 (基本功).

Ising model. (Lenz). Weiss-Curise 平均场.

$H = -J \sum_{ij} S_i S_j$, 转移矩阵. Ising.

$H = -J \sum_i \vec{\sigma_i} \cdot \vec{\sigma}_{i+1}$, 自旋. Heisenberg. $\sigma_z = \pm 1$

Onsager.

Yang. $C_v \sim (T-T_c)^{-\frac{1}{8}} \neq -\frac{1}{2}$, 超越 Landau 相变理论;

Topo 相变;

Anderson localization;

Kardanoff. / Wildom. → renormalization group.

动量空间. Wilson. RG ⇒ 1981. Nobel. Phys. prize.

DMRG.