Dyson eq: $g = g_0 + g_0\Sigma g \Rightarrow$ 右乘 $g^{-1} \Rightarrow 1 - g_0 g^{-1} = g_0\Sigma \Rightarrow$

$\begin{cases} g = \frac{1}{p^2-m^2} \\ g_0 = \frac{1}{p^2-m_0^2} \end{cases}$ 左乘 $g_0^{-1} \Rightarrow g_0^{-1} - g^{-1} = \Sigma$

即 $p^2 - m_0^2 - (p^2 - m^2) = \Sigma$

$(m_0 \Leftrightarrow \Lambda,\ m = m_R)$

$m^2 = m_0^2 + \Sigma$

观测值 Λ 自由能

真实质量在大$\Lambda \to m_0^2$ 观测值 $m^2 = m_0^2 + \Sigma$

$\langle O\rangle = \sum_{n=0}^{+\infty} \frac{i^n}{n!}\langle O S_I^n\rangle_0^c$ $S_I = -\frac{\lambda}{4!}\int \phi^4 dx$

$\langle\phi^*(p)\phi(p)\rangle$: ① $V=1 \Rightarrow \frac{1}{V}\sum$ 成对出现

② 舍弃许多图（哪些图）

③ 理解 Feynman diag /基本图 $\propto\Lambda^2$ $\propto\ln\Lambda$ sunrise diag

$S_I = -\frac{\lambda}{4!}\sum_{k_1k_2k_3}\phi(k_1)\phi(k_2)\phi(k_3)\phi(-k_1-k_2-k_3) = -\frac{\lambda}{4!}\sum_{k_1k_2} X_{k}$

$O = \phi^*(p)\phi(p)$, $\langle O\rangle = \langle O_0\rangle + \frac{i}{1!}(\frac{-\lambda}{4!})\sum_{k_1k_2k_3}\langle\phi^*(p)\phi(p) X\rangle_0^c +$

$= \frac{iV}{p^2-m^2-i\epsilon} + \frac{i}{1!}(-\frac{\lambda}{4!})4\times3 + \frac{i^2}{2!}(\frac{\lambda}{4!})^2\times4\times4\times3\times3\times4\times2$

$= g_0(p) - \frac{i\lambda}{2} g_0(p)\Sigma g_0(p) + \frac{(-i\lambda)^2}{(2!)^2}$

Remark: $= \Sigma$

$= g_0(p) + g_0(p)\Sigma g_0(p)$

传播子 vs 自由能

再考虑3阶 $\frac{i^3}{3!}(-\frac{\lambda}{4!})^3\times3!\langle\phi^*(p)\ X\ X\ X\ \phi(p)\rangle$

$= \frac{i^3}{3!}(-\frac{\lambda}{4!})^3\times3!\times4\times3\times4\times3\times3\times4$

$= (\frac{-i\lambda}{2!})^3$

$= -\frac{i\lambda}{2}\sum_k(\frac{i}{k^2-m^2})$

$= g_0\Sigma g_0\Sigma g_0\Sigma g_0$

相互作用: $-\frac{\lambda}{4!}$, $O = \langle\phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4)\rangle$, p_i任意

唯一要求 $\sum p_i = 0$

指任意允许的散射过程 $\langle O\rangle_0 = 0$

$\langle O\rangle = \langle O\rangle_0 + \frac{i^1}{1!}(-\frac{\lambda}{4!})\sum\langle\phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4)\cdot X\rangle_0^c$

$= \quad + \quad \times 4!\ X$ (p_1, p_2, p_3, p_4)

$-i\lambda X$

二阶: $\frac{i^2}{2!}(-\frac{\lambda}{4!})^2\sum\sum\langle\phi(p_1)\phi(p_2)\phi(p_3)\phi(p_4)\ X\ X\rangle_0^c$

$= \frac{i^2}{2!}(-\frac{\lambda}{4!})^2 C_4^2 C_4^2 C_4^2\times2\times2\times2$

$= -(\frac{3\lambda^2}{2})$

或 $\langle\phi\phi\phi\phi\ X\ X\rangle$

$\frac{i^2}{2!}(\frac{-\lambda}{4!})^2\sum\langle\phi^*(p)\phi(p)\ X\ X\rangle_0^c$ 或 $\langle\phi^*(p)\phi(p)\ X\ X\rangle$ 或 $\langle\phi^*(p)\phi(p)\ X\ X\rangle$

Λ^2 $\ln\Lambda$ $\phi(p)$ Σ_2 $\phi^*(p)$

$\propto\Lambda^2\ln\Lambda$ $g_0(p)\Sigma g_0(p)\Sigma g_0(p)$

$d=4$ $g_0(p)\Sigma_2 g_0(p)$ 二阶Σ贡献

$g_0(p)\Sigma_1 g_0(p)$ 1阶Σ贡献

$\langle O\rangle =$ 一阶 + 二阶

$= -i\lambda X - \frac{3\lambda^2}{2} \approx -i\lambda_{eff} X$

其中 $-i\lambda_{eff} = -i\lambda - \frac{3}{2}\lambda^2$ 解决方法

观测到的散射长度

$m^2 = m_0^2 + \Sigma \propto \Lambda^2$ (平方发散)

2d $\delta(x)$ potential $\frac{g}{V}\sum_k\frac{1}{E-\frac{\hbar^2k^2}{2m}} = 1$, $E = -\frac{\hbar^2\Lambda^2}{2m}e^{\frac{\hbar^2}{mg(\Lambda)}}$

$\propto \frac{1}{V}\sum_k\frac{i}{k^2-m^2}$

重整化：① 系统化方法．

$\mu \ll \Lambda$ ② 谁抵消谁

③ 避免复杂的非线性相互作用．（方程）．

$\mathcal{L} = \frac{1}{2}(\partial_\mu\phi)^2 - \frac{m^2}{2}\phi^2 - \frac{\lambda}{4!}\phi^4$ ， $m_0, \lambda_0 \Rightarrow \Lambda$ 在能标 Λ 下定义

令 $\phi = \sqrt{Z}\phi_R$ ， $\langle\phi^*(p)\phi(p)\rangle = \frac{Z}{p^2-m^2} + \text{finite}$

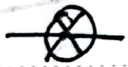
$$\mathcal{L} = \underbrace{\boxed{\frac{1}{2}(\partial_\mu\phi_R)^2 - \frac{m_R^2}{2}\phi_R^2}}_{\mathcal{L}_0} - \frac{\lambda_R}{4!}\phi_R^4 + \underbrace{\frac{1}{2}\left(\delta_Z(\partial_\mu\phi_R)^2 - \delta_m\phi_R^2\right) - \frac{\delta_\lambda}{4!}\phi_R^4}_{\text{抵消项}}$$

观测值（$\mu \ll \Lambda$） 实验可见

抵消项 $\delta_Z, \delta_m, \delta_\lambda$ 不可测

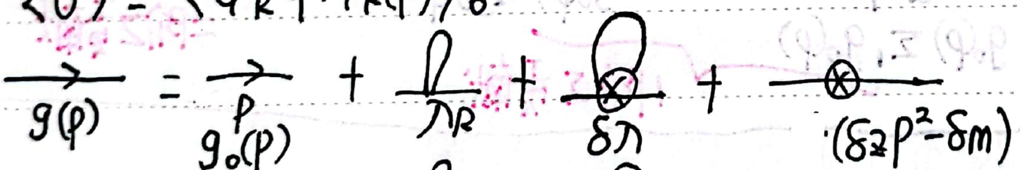
图 $\langle\phi_R^*(p)\phi_R^*(p)\rangle = \xrightarrow[p]{} = \frac{i}{p^2-m_R^2}$

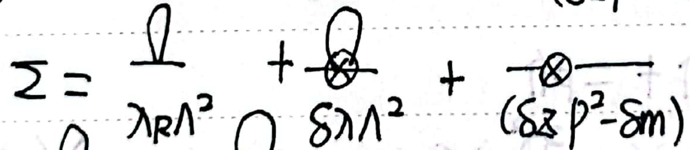
 $= -i\lambda_R$  $= -i\delta_\lambda$

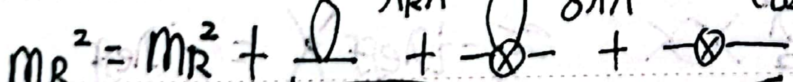
 $= (\delta_Z p^2 - \delta_m)$

$S_0 = \int \mathcal{L}_0 dx$ ， $S_I = -\frac{\lambda_R}{4!}\phi_R^4 - \frac{\delta_\lambda}{4!}\phi_R^4 + \frac{\delta_Z}{2}(\partial_\mu\phi)^2 - \frac{\delta_m}{2}\phi^2$

$\langle 0\rangle = \langle\phi_R^*(p)\phi_R(p)\rangle_0$



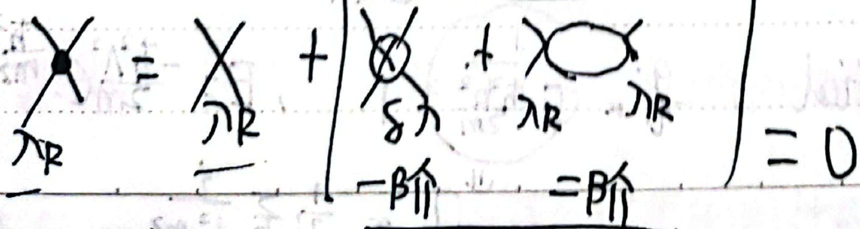




有限值 = 0．

*: $H = \boxed{\frac{p^2}{2m} + \frac{1}{2}m\omega_R^2x^2} + \lambda x^4 + \underbrace{\frac{1}{2}m\delta\omega^2x^2}_{V_I}$

$= \hbar\omega_R(n+\frac{1}{2}) + \langle n|V_I|n\rangle$



则 $\delta_\lambda = -$ 

$\propto \ln\Lambda$