一个抽象的证明:

Ref: Methods of QFT in Condensed Matter. Abrikosov.

累积量展开的物理意义:在传播路径中,周围的物理与传播无关.

$$\langle O\rangle = \frac{\int D\phi\ O\ e^{-H_0-U}}{\int D\phi\ e^{-H_0-U}}$$

operator

iS_0 iS_I

$$= \frac{\langle O e^{-U}\rangle_0}{\langle e^{-U}\rangle_0}$$

用分母来抵消掉周围无相互作用项的影响;

$$= \sum_{n=1}^{+\infty}\frac{(-1)^n}{n!}\langle O e^{-U}\rangle^{c}_{H_0}$$

连接图;

2022.3.14. 第4周. 第1节课.

目标: ϕ^4 理论. 计算 m, λ.

$$\mathcal{L} = \underbrace{\frac{1}{2}(\partial_\mu\phi)^2 - \frac{1}{2}m^2\phi^2}_{\text{Gaussian}} - \underbrace{\frac{\lambda}{4!}\phi^4}_{\text{Perturbation}}.$$

1) Propogator. of $\mathcal{L}_0$

2) Operator. $\hat{O}$. $\langle\hat{O}\rangle = \frac{\int D\phi\, e^{iS}\, O}{\int D\phi\, e^{iS}}$

3) $\int D\phi\, e^{iS} d\phi(x_1)\cdots d\phi(x_\ell)$.

上节课回顾:

① Cumulant expansion:

$$Z = \langle e^{tX}\rangle = e^{\Omega}$$

Z: 特征函数 / 配分函数.

Ω: 自由能.

$$X = \frac{\lambda}{4!}\int \phi^4 dx.$$ X 是变量. 也可以是场.

$$\Omega = \sum_{r=1}^{+\infty} \frac{t^r}{r!} k_r.$$ k_r 为累积量.

$$\langle O \rangle = \frac{\int D\phi\, e^{i(S_0+S_I)}\, O}{\int D\phi\, e^{i(S_0+S_I)}} = \sum_{n=0}^{+\infty} \frac{i^n}{n!} \langle O S_I^{\,} \rangle_0^c$$

$$\langle O \rangle_H = \frac{\int D\phi\, e^{H_0-U}}{\int D\phi\, e^{H_0}} = \sum_{n=0}^{+\infty} \frac{(-1)^n}{n!} \langle O U^n \rangle_0^c$$

需要解决的问题:

① 分母问题 ⇒ 扣除无贡献的相互作用.

② 平均. $\langle O \rangle_S \longrightarrow \langle O \rangle_{S_0} = \langle O \rangle_0$

③ $\langle O U^n \rangle_0^c$

剩下的部分.

Connected diag.

Cumulant expansion.

Green's function. (S_0):

$$g_0(x-y) = \langle T\, \phi(x)\phi(y) \rangle$$

$$= \underbrace{\frac{1}{V}\sum_q}_{\text{成对出现}} \frac{i e^{iq(x-y)}}{q^2-m^2}$$ ← FT

⇐ $1/L$.

(S_0+S_I):

$\langle \phi(x)\phi(y) \rangle$

$$\text{表示} = \frac{\int D\phi\, e^{iS_0+iS_I}\, \phi(x)\phi(y)}{\int D\phi\, e^{iS_0+iS_I}}$$

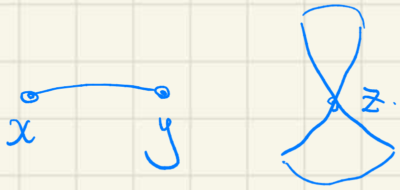
$$\hat{O} = \phi(x)\phi(y) = \sum_{i=0}^{+\infty} \frac{i^n}{n!} \langle O S_I^n \rangle_0^c$$

$$S_I = \frac{\lambda}{4!}\int \phi^4(z)\, dz$$

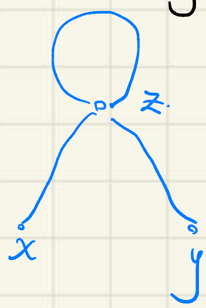
☆ $\langle O S_I \rangle_0$ 与 $\langle O S_I \rangle_0^c$ 的区别:

只包含连接图.

$$\frac{\int D\phi\, e^{iS_0}\, S_I^2\, \phi(x)\phi(y)}{\int D\phi\, e^{iS_0}} = -\frac{\lambda}{4!}\int dz \left[\frac{\int D\phi\, e^{iS_0}\, \phi(x)\phi(y)\phi^4(z)}{\int D\phi\, e^{iS_0}} \right]$$

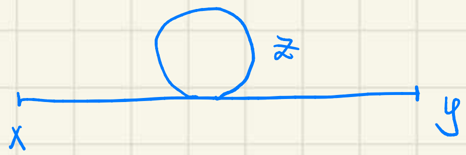


非连通.



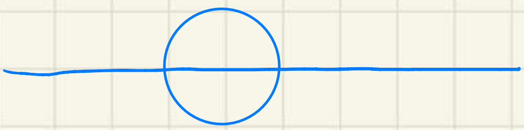
连通

⇓

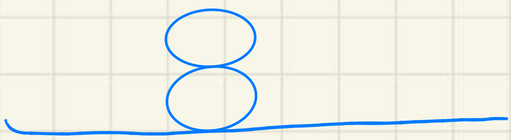


又比如有多分量 z_1, z_2 时.

$$\int dz_1 dz_2 \frac{\int D\phi\, e^{iS_0}\, \phi(x)\phi(y)\phi^4(z_1)\phi^4(z_2)}{\int D\phi\, e^{iS_0}}$$

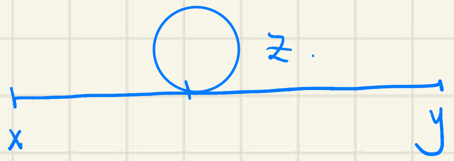


Sunrise diag.



一阶项. 只有一个图.

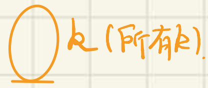
2d. $\delta(\vec{x})$ potential.



$$\frac{1}{V}\sum_k \frac{i}{k^2-m^2}$$

$$g\frac{1}{V}\sum_k \frac{1}{E^2-\frac{\hbar^2k^2}{2m}}$$

→ 发散. ⇒



↑ 短一点

$$\langle \phi(x)\phi(y)\phi^4(z)\rangle$$

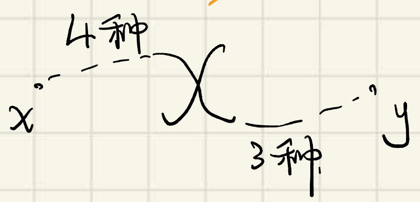
$$= \langle \phi(x)\phi(z)\rangle_0 \langle \phi(y)\phi(z)\rangle_0 \underbrace{\langle \phi(z)\phi(z)\rangle_0}_{\text{发散}}$$

★ $\langle \phi(z)\phi(z)\rangle_0$ 如何理解这一项?

$$\langle \phi(z)\phi(z)\rangle_0 = \frac{1}{V}\sum_k \frac{i}{k^2-m^2}$$

$$g(x-y) = g_0(x-y) - \underbrace{\frac{i\lambda}{4!}\times 4\times 3}_{-\frac{i\lambda}{2!}} \times g_0(x-z)\, g_0(z-z)\, g_0(z-y).$$

卷积.



动量空间:

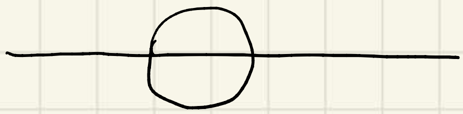
$$g = g_0 + g_0\Sigma g$$
$$= g_0 + g_0\Sigma g_0 + g_0\Sigma g$$
$$= g_0 + g_0\Sigma g_0 + g_0\Sigma g_0\Sigma g_0 + \cdots \quad \text{(Dyson equation).}$$

$$g_0\Sigma g_0 = \frac{1}{V}\sum_k \frac{i}{k^2-m^2-\Sigma}$$

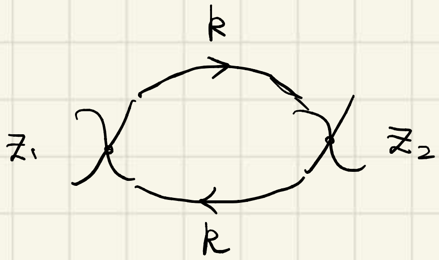
如果 g_0 是小量. 则可视为对 m 的修正.

但 g_0 会发散. 所以对 m 的修正为 ∞. 发散!

$$g(k) = g_0(k) - \frac{i\lambda}{2} g_0(k)\, \underbrace{g_0(0)}_{\text{发散.}}\, g_0(k).$$



$$\frac{1}{p^2 - m^2} \Rightarrow \frac{1}{(1+\delta)p^2 - m^2}$$



$$\frac{1}{V} \sum_k \left(\frac{1}{k^2 - m^2} \right)^2$$

$$k_0 \to i k_0 \Rightarrow \int \frac{dk_E}{(k_E^2 + m^2)^2}$$

$\boxed{\text{Wick Rotation.}}$

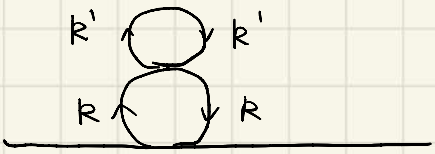
k_E (E: 欧氏).

$$\propto \int \frac{k_E^3\, dk_E}{(k_E^2 + m^2)^2}.$$

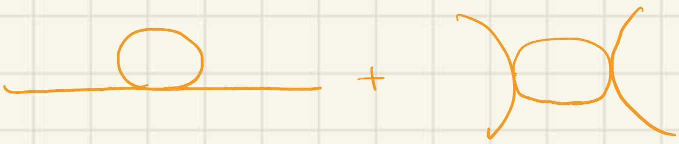
if $k_E \gg m$.

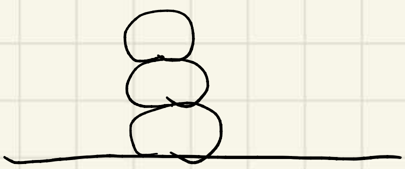
$$\Rightarrow \int^{\Lambda} \frac{dk_E}{k_E} = \ln \Lambda. \quad \text{发散!}$$

$$\Rightarrow \quad m(\Lambda),\ \lambda(\Lambda)$$



$$\propto \lambda^2 \underbrace{\left(\frac{1}{V} \sum_k \frac{1}{k^2 - m^2} \right)}_{(\propto \Lambda^2)} \underbrace{\left(\frac{1}{V} \sum_{k'} \frac{1}{k'^2 - m^2} \right)^2}_{(\propto \ln \Lambda)}$$





…… 发散同样存在于更高阶的圈中.

没有讲的内容：

① 算子.

② 如何计算（技巧性质的处理）. ref: Peskin书.

Λ 截断.

维度正规化. $d \to d-\varepsilon$.

$$\int_0^{+\infty} \frac{k_E^{d+\varepsilon}\, dk_E}{(k_E^2+m^2)^2}$$

Feynmann 参数化.

③ 流守恒.

下节课的铺垫：

$$H = \frac{p^2}{2m} + \frac{1}{2} m\omega^2 x^2 + \lambda x^4.$$

$$H_0 = \frac{p^2}{2m_r} + \frac{1}{2} m\omega_r^2 x^2, \quad V = \frac{1}{2} m\,\delta\omega^2 x^2 + \lambda x^4 + \frac{p^2}{2\mu}$$

$$, \quad \delta\omega^2 = \omega^2 - \omega_r^2$$

renormalization

$$|n\rangle_r = \cdots$$

$$E_n^r = \hbar\omega_r\left(n+\tfrac{1}{2}\right) + {}_r\langle n|V|n_r\rangle + \cdots$$

观测值.

目标：让这一块 $\to 0$.

（P.S. 这种情况下无发散，只是举例～）.

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{m_0^2}{2}\phi^2 - \frac{\lambda_0^4}{4!}\phi^4 , \quad (\lambda_0, m_0 \text{ at } \Lambda).$$

$$= \mathcal{L}_R(m_R, \lambda_R) + \Delta\mathcal{L}.$$

(与 m_0, λ_0, m_R, λ_R 都有关. 用以抵消发散).

重整化.

$\infty - \infty =$ infinite (任意). 由实验确定 λ_R, m_R. $\Rightarrow$ infinite (确定).

观测值 $\mu \Rightarrow m_R$, λ_R. 但 $\mu \ll \Lambda$.

重整化: $\phi = \sqrt{Z}\,\phi_R$

$$= \frac{1}{2}(\partial_\mu \phi_R)^2 - \frac{m_R^2}{2}\phi_R^2 - \frac{\lambda_R}{4!}\phi_R^4$$

$$\underbrace{+\frac{1}{2}\delta_Z(\partial_\mu\phi_R)^2 - \frac{\delta m^2}{2}\phi_R^2 - \frac{\delta\lambda}{4!}\phi_R^4}_{\text{小量}}$$

$$\left.\begin{array}{l} \phi^3 \\ \phi^5 \\ \phi_1^4 + \phi_2^4 \end{array}\right\} \Rightarrow \text{如何处理？}$$

2022.3.17. 第4周第2节课.

Dyson eqn.

$$g = g_0 + g_0 \Sigma g$$

$$g = \frac{1}{p^2 - m^2}$$

$$g_0 = \frac{1}{p^2 - m_0^2}$$

$m_0 \Leftrightarrow \Lambda$, 对应一个非常大的能标.

$m \Leftrightarrow m_R$.

右乘 g^{-1}

$$1 - g_0 g^{-1} = g_0 \Sigma$$

左乘 g_0^{-1}

$$g_0^{-1} - g^{-1} = \Sigma$$

$$(p^2 - m_0^2) - (p^2 - m_R^2) = \Sigma$$

$$m_R^2 - m_0^2 = \Sigma \quad \text{(自由能)}$$

$$m = m_0^2 + \Sigma \leftarrow \text{散射对应的能量}$$

测量值; 不可测的能量.